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NONLINEAR CONTROL APPROACH APPLIED TO PVTOL WITH RESTRICTED CARTESIAN DYNAMIC

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Abstract

This work is directed toward stabilizing a PVTOL by two rotors with restricted spatial mobility. The problem of controlling the system is solved using a new method proposed by Astolfi and Ortega, named Immersion and Invariance stabilization. The method consists of a controller which is strengthened by immersion in a system with better performance characteristics and its invariance ensures that it will converge to a point of stability. This is validated by numerical software simulation.

Key words

Stabilizing, controlling, inmersion and invariance, PVTOL.

1 Introduction

In recent years several studies have been made concerning the development of more efficient unmanned aerial vehicles, with better performance and higher yields. Today, there is a need for aerospace vehicles having a smaller size and wide range of applications ranging from vehicular traffic monitoring, inspection of hazardous areas, border surveillance, weather measurement applications. For instance, in military war zones there is a necessity to survey the area without risking the lives of pilots.

One of the main objectives for researchers now is to achieve automated flight dynamics of such aircraft so as to be stable and perform in the neighborhood of a desired trajectory.

The problem addressed in this paper is to stabilize the decoupling between the longitudinal and lateral dynamics. A study is then made on a dynamic PVTOL with Cartesian restricted conditions. The system is described as with two propellers arranged at a distance above the longitudinal plane (x) and restricted translational dynamics.

This system obtains a mathematical model describing the dynamics of a rod actuated by two rotors for which a control algorithm was designed that stabilizes the system. Such algorithms are validated through software simulations. The paper is organized as follows, section 1 is a brief introduction and discusses the background, section 2 gives the mathematical model to study and analyzes their properties, while section 3 presents the control algorithm, section 4 show the simulations and section 5 the conclusions.

During the last decade efforts have been taken by the scientific and technological community oriented towards stabilization and trajectory tracking of rotary wing aircraft.Altug et al. [Altug *et al.*, 2005] propose a control algorithm to stabilize the cuatrirotor using artificial vision and a camera as the main sensor. They studied two methods, using in the first a control algorithm. Their results were successfully tested in simulations.

In their paper Heredia et al. [Heredia *et al.*, 2008] deals with the problem of controlling an autonomous helicopter and do so by computer simulations of control strategies based on fuzzy logic and non-linear tracking control of two possible scenarios; vertical ascent and simultaneous longitudinal and lateral movement. The controller consists of a MIMO (Multiple Inputs Multiple Outputs) inner loop for stabilization and four ties leading to SISO (Simple Input Simple Output) in speed and position.

In their paper Pounds et al. [Pounds *et al.*, 2006] conceived and developed a control algorithm for a four-rotor prototype. They used an inertial measurement unit (IMU) for measuring the angular velocity and acceleration. Using the linearization technique they conceived a dynamic model control. The results were tested by simulation.

Vissiere and Petite [Vissiere and Petit, 2008] considered the problem of developing a modular system for real-time embedded control applications in UAVs. Their efforts lead them toward programming and they proposed the control strategies. To test reported results an extended Kalman filter was implemented at $75Hz$ used for estimating the states of a small helicopter.

Chowdhary and Lorenz [Chowdhary and Lorenz, 2005] conceived stabilization of a VTOL UAV by considering the feedback states in line as a simple linear control technique with the only problem being the flight envelope. The problem is often accentuated due to an improper linear model, measurement noise in the sensors and external shocks. They presented a control architecture based on a valid extension of the linear optimal control law for entire feedback states. An extended Kalman filter was used in the problem state and parameter estimation. Based on the estimation of the parameters feedback state gain is calculated by solving the Riccati equation for quadratic optimization control online.

In his doctoral thesis Arda [Arda, 2006] addresses the problem of designing an embedded system for a vehicle equipped with air cuatrirotor inertial sensors. The control system is developed in Matlab/Simulink and implemented in real time using a Simulink module Real Time Windows Target. They designed a linear quadratic regulator for stabilization of the attitude flight. The hardware integrates a data acquisition card, DC drives, a set of sensors, DC motors and Draganflyer V Ti platform. Now Salazar et al. [Salazar-Cruz *et al.*, 2007] described the design of an embedded control system for an unmanned aerial vehicle (UAV) capabilities cuatrirotor rate for stationary flights. The vehicle dynamic model is presented using Euler-Lagrange and proposed a control strategy based on integer saturation. Embedded control system architecture describes stationary autonomous flight. The main system components are a micro-controller, an inertial measurement unit (IMU), a global positioning system (GPS), and infra-red sensors. Euler angles are calculated using a data fusion algorithm. Experimental results show that the control system works for indoor flying autonomous vehicles.

Abdigail [Adigbil, 2007] presented the development of a reliable remote control to assist in a mini air robot with four rotors and capabilities to ensure a stable flight. In the first phase a dynamic model was obtained by Euler-Lagrange equations and tested by three different types of control laws feedback states, sliding modes and backstepping for stabilization and all UAV positions. The author mentioned that all of them were compared in simulations but did not describe the advantages of each.

Mian and Wang [Mian and Wang, 2008] proposed a nonlinear controller for stabilizing a helicopter. The strategy was based on the saturation of integrators. Due to the positive achievements that have this type of strategy coupling conditions were taken into account. The controller simulations showed good results with respect to other controlers. Thanks to embedded sensors and control it was capable of autonomous flight in real time. Their results showed that the control strategy was able to perform tasks autonomously such as take-off, landing and hovering.

Finally, Ollero and Merino [Ollero and Merino, 2004] discussed methods and technologies that have been applied in aerial robotics and several UAVs, summarizing the control techniques including control architectures and control methods.

2 Mathematical Model

2.1 Dynamic Model

For obtaining the dynamic model we consider that the two rotors produce a force normal to the system horizontal plane. Because the system has two rotors that

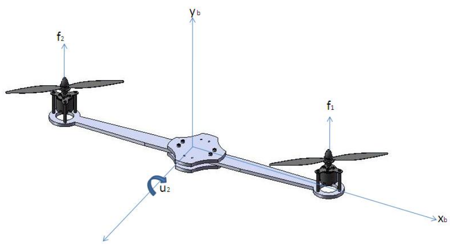

Figure 1. System model

provide the thrust force, the total thrust is given by, $T_t = \sum_{1=1}^{2} T_i$, where T_1 and T_2 are the forces caused by the action of the motors which produce a torque about the center of gravity. The total rotational torque is given by the following expression $u_\theta = (T_2 - T_1)l$, where l is the distance from the center of gravity of the system to the engine axis.

Making a forces analysis, applying Newton's second law and considering state variables to the energy accumulating elements, which in this case is a mass that rotates at a speed about its center of gravity, the following ordinary differential equation models the dynamic of the system.

$$J\ddot{\theta} + B\dot{\theta} = u \tag{1}$$

where u is an unknown control function that depends on time, using the fact that $u = \phi(u)$, we can re-write equation 1 as,

$$J\ddot{\theta} + B\dot{\theta} = \phi(u) \quad (2)$$

The representation in first-order equations was changed, and now we obtain the following system of ODE

$$\dot{\theta}_1 = \theta_2 \quad (3)$$
$$\dot{\theta}_2 = -\frac{B}{J}(\theta_2) + \frac{1}{J}\phi(u) \quad (4)$$

This model is represented in state variables, which are defined by the following states $x_1 = \theta_1, x_2 = \theta_2$. Using the later then it is possible to obtain the following matrix representation,

$$\dot{x} = f(x,u) = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{J} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \quad (5)$$

The J value was obtained from an analysis of the model made in SolidWorks®and B is the damping of the system modeled by the equation

$$B = -F \mid \dot{\theta} \mid \dot{\theta}$$

The stability of the system was analysed by calculating the characteristic polynomial of the open loop system which is

$$\lambda^2 + \lambda\frac{B}{J} \quad (6)$$

For equation 6 it is possible to obtain the system eigenvalues, given by $\lambda_1 = 0$ and $\lambda_2 = -\frac{B}{J}$.
So, any value of this system will have a zero eigenvalue and the other negative. This means that we can only inferred a internal stability since the A matrix it is not Hurwitz [Chi-Tsong, 1999] [Khalil, 2009].

3 Control Algorithm Design

The control algorithm used to stabilize the bar system is based on the method proposed by Astolfi and Ortega [Astolfi and Ortega, 2003]. This work uses asymptotic stabilization for design adaptive control laws of nonlinear systems.
Let consider a system of the form,

$$\dot{x} = f(x,u)$$

where the basic problem it is to find a stabilizing control law $u = u(x)$ (i.e. when it is possible) so that the closed loop system be either locally (globally) asymptotically stable. The proposed procedure for solving this problem consists of two steps. First, find a target dynamic system $\dot{\xi} = \alpha(\xi)$ to be locally (globally) asymptotically stable with dimension strictly less than x, a mapping $x = \pi(\xi)$, and a function $c(x)$, such that

$$f(\pi(x), c(\pi(\xi))) = \frac{\partial\pi}{\partial\xi}(\xi)\alpha(\xi)$$

that is any trajectory $x(t)$ of the system $\dot{x} = f(x, c(x))$ is the image by mapping $\pi(\cdot)$ of a path in the target system. Note that the mapping $\pi : \xi \to x$ is an immersion, i.e. the range of π is equal to the dimension of ξ. Secondly, implement a control law that contributes to the attract variety $x = \pi(\xi)$ and maintains bounded closed-loop trajectories. Thus, it follows that the closed-loop system asymptotically behaves as a desired objective system and then stability is ensured.

Considering equations 3 and 4 we can re-write using the fact that $\xi(t) = -\frac{B}{J}(\theta_2)$ and $u = \phi(u)$ this yields,

$$\dot{\theta}_1 = \theta_2 \quad (7)$$
$$\dot{\theta}_2 = \xi(t) + \frac{1}{J}u \quad (8)$$

Where $\xi(t)$ is an unknown function that depends on time. Now taking into account the following full-order target system: a new controller is designed for this system, due to $u = u(t)$ is any stabilizing control law for the system by feedback

$$z_1 = \theta_2 - \hat{\theta_2} + \beta_1(\theta_1) \quad (9)$$
$$z_2 = \xi(t) - \rho_1 + \beta_2(\theta_1) \quad (10)$$
$$z_3 = \dot{\xi}(t) - \rho_2 + \beta_3(\theta_1) \quad (11)$$

to obtain the state variables of interest for immersion in the higher-order system, one must find a function $\psi(x, z)$ that preserves the bounded trajectories and asymptotically stable zero dynamics

$$\theta_2 = z_1 + \hat{\theta_2} - \beta_1(\theta_1) \quad (12)$$
$$\xi(t) = z_2 + \rho_1 - \beta_2(\theta_1) \quad (13)$$
$$\dot{\xi}(t) = z_3 + \rho_2 - \beta_3(\theta_1) \quad (14)$$

Taking $\ddot{\xi} \approx 0$, and appling the following control input,

$$u = J[(\rho_1 - \beta_2) + k_p\theta_1 + k_d\theta_2]$$

we can replaced the equations 7 and 8 in order to obtain the following system

$$\dot{\theta}_1 = \theta_2 \quad (15)$$
$$\dot{\theta}_2 = -\xi + \rho_1 - \beta_2 + k_p\theta_1 + k_d\theta_2 \quad (16)$$

Now consider equation 10, then equation 16 could be write as follows

$$\dot{\theta}_2 = -z_2 + k_p\theta_1 + k_d\theta_2 \tag{17}$$

In order to begin the construction of the observer it is necessary obtain the derivatives of the system

$$\dot{z}_1 = \xi + \frac{1}{J}u - \dot{\hat{\theta}}_2 + \frac{\partial\beta_1}{\partial\theta_1}\theta_2 \tag{18}$$

$$\dot{z}_2 = \dot{\xi} - \dot{\rho}_1 + \frac{\partial\beta_2}{\partial\theta_1}\theta_2 \tag{19}$$

$$\dot{z}_3 = -\dot{\rho}_2 + \frac{\partial\beta_3}{\partial\theta_1}\theta_2 \tag{20}$$

it now we substitute ξ, $\dot{\xi}$ and θ_2 in the above equations we obtain the following system

$$\dot{z}_1 = z_2 + \rho_1 - \beta_2 + \frac{1}{J}u - \dot{\hat{\theta}}_2 + \frac{\partial\beta_1}{\partial\theta_1}(z_1 + \hat{\theta}_2 - \beta_1) \tag{21}$$

$$\dot{z}_2 = z_3 + \rho_2 - \beta_3 - \dot{\rho}_1 + \frac{\partial\beta_2}{\partial\theta_1}(z_1 + \hat{\theta}_2 - \beta_1) \tag{22}$$

$$\dot{z}_3 = -\dot{\rho}_2 + \frac{\partial\beta_3}{\partial\theta_1}(z_1 + \hat{\theta}_2 - \beta_1) \tag{23}$$

Solving for $\dot{\hat{\theta}}_2$, $\dot{\rho}_1$ and $\dot{\rho}_2$ we get,

$$\dot{\hat{\theta}}_2 = \rho_1 - \beta_2 + \frac{1}{J}u + \frac{\partial\beta_1}{\partial\theta_1}(\hat{\theta}_2 - \beta_1) \tag{24}$$

$$\dot{\rho}_1 = \rho_2 - \beta_3 + \frac{\partial\beta_2}{\partial\theta_1}(\hat{\theta}_2 - \beta_1) \tag{25}$$

$$\dot{\rho}_2 = \frac{\partial\beta_3}{\partial\theta_1}(\hat{\theta}_2 - \beta_1) \tag{26}$$

This system complies with the condition that $\psi(\theta, z)$ preserves bounding the trajectories and stabilizing at zero asymptotically. Now in order to solve the problem of finding a function $\xi(t)$ and a control u such that both describe the invariant manifold . We require solve a partial differential equation. The following system,

$$\dot{z}_1 = \frac{\partial\beta_1}{\partial\theta_1}z_1 + z_2 \tag{27}$$

$$\dot{z}_2 = \frac{\partial\beta_2}{\partial\theta_1}z_1 + z_3 \tag{28}$$

$$\dot{z}_3 = \frac{\partial\beta_3}{\partial\theta_1}z_1 \tag{29}$$

derivating 27 and substituted into 28 yields

$$\ddot{z}_1 = \frac{\partial\beta_1}{\partial\theta_1}\dot{z}_1 + \dot{z}_2 = \frac{\partial\beta_1}{\partial\theta_1}\dot{z}_1 + \frac{\partial\beta_2}{\partial\theta_1}z_1 + z_3 \tag{30}$$

using the same procedure whit 30 and 29, we obtain

$$\dddot{z}_1 = \frac{\partial\beta_1}{\partial\theta_1}\ddot{z}_1 + \frac{\partial\beta_2}{\partial\theta_1}\dot{z}_1 + \dot{z}_3 = \frac{\partial\beta_1}{\partial\theta_1}\ddot{z}_1 + \frac{\partial\beta_2}{\partial\theta_1}\dot{z}_1 + \frac{\partial\beta_3}{\partial\theta_1}z_1 \tag{31}$$

which is equals to zero **??**. Finally we obtain a polynomial wich terms are partial differential equations

$$z_1^{(3)} + \beta_1\ddot{z}_1 + \beta_2\dot{z}_1 + \beta_3 z_1 = 0 \tag{32}$$

The final result is the system and the controller, where the variables of the controller are obtained from the equations 24, 25 and 26, so that the asymptotically stable system is described by the following equations,

$$\dot{\theta}_1 = \theta_2 \tag{33}$$

$$\dot{\theta}_2 = -\frac{F}{J} + \frac{1}{J}u \tag{34}$$

$$u = J[(\rho_1 - \beta_2) + k_p\theta_1 + k_d(\hat{\theta}_2 - \beta_1)] \tag{35}$$

4 Results

4.1 Tuning to the Controller Gains and the Observer

This section validates the control algorithm through numerical simulation. The above results were made in *Matlab*®and *Simulink*®.

A first set of simulations was made in order to observe the behavior of both the observer gains and the PD controller gains. Figure 2 makes a first visualization of the behavior with different gains for the controller, while the observer remains constant at 1.0. It is clear that the optimal gain is over 100 and under 1000. Therefore, values are plotted in that interval. The figure 3 shows the behavior of different values. It is easy to conclude that higher values show the negative overshoot. The system is smaller, but the figure 2 shows that if the system is large it does not stabilize at zero. Therefore, the controller gain used is 400.

As shown in figure 4 the constant controller gains are 1.0 and were varied observer gains. It can be seen that the optimal gain is 1.35.

4.2 First Simulation of the Closed-Loop System

A first simulation of controller parameters and observer are shown in table 1

Controller poles	Observer poles
1.0	1.0
1.0	1.0
1.0	1.0

Table 1. System parameters for the first simulation

The figures 5 and 6 shows the position and the rotational speed of the closed-loop system. It is noticeable that both stabilize after a series of oscillations. Looking at figure 7, it can be noticed that the variable $\hat{\theta}_2$ is estimated and shows a similar behavior to the variable θ_2. The figure 8 shows a comparison between the two variables. It is shown that the observer estimates the variable θ_2 in 4.5 seconds. The figure 9 shows the estimation error of the observer and the figure 10 shows the control signal which stabilizes the system at zero degrees.

4.3 Second Simulation of the Closed-Loop System

This section gives a second simulation taking into account the poles obtained from the tuning gains. The parameters used for this simulation are from table 2

Controller poles	Observer poles
400.0	1.35
400.0	1.35
400.0	1.36

Table 2. System parameters for the second simulation

The figures 11 and 12 show the position and the rotational speed of the closed-loop system. It can be noted that the stabilization is immediate. Looking at figure 13, notes that the variable estimated $\hat{\theta}_2$, shows a behavior equal to the variable θ_2. Figure 14 shows a comparison between the two variables. It shows that the observer estimates the variable θ_2 immediately. Figure 15 shows the estimation error of the observer and figure 16 shows the control signal which stabilizes the system at zero degrees. It can be concluded that properly tuning the observer shows an excellent performance, as it stabilizes the system immediately. It should also be noted that the dynamic quickest driver is having a significant impact on the performance of the control-loop and the dynamics the observer must be just a little faster, because otherwise the system becomes unstable. This due to saturation in the control loop.

5 Conclusions

This paper develops a control algorithm for a system consisting of a rod actuated by two rotors. The proposed algorithm is based on the theory proposed by Astolfi and Ortega [Astolfi and Ortega, 2003].This controller showed an excellent performance in the simulations when the gains were appropriate. The simulations presented show that the controller gain is the one with greater presence in the system dynamics. However, a good choice of observer gains ensures a dynamic fast enough for the correct estimation of the state θ_2. Caution is needed because a too rapid dynamics saturates the loop and leads to instabilities. The initial conditions are not an important result in the stabilization of the system when stored below 45 degrees above the horizontal.
Graphs 9 and 15 shows the estimation error, which can be verified as tends to be zero. Again proper tuning makes gains error dynamics faster.

6 Acknowledgements

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Appendix A Graphics

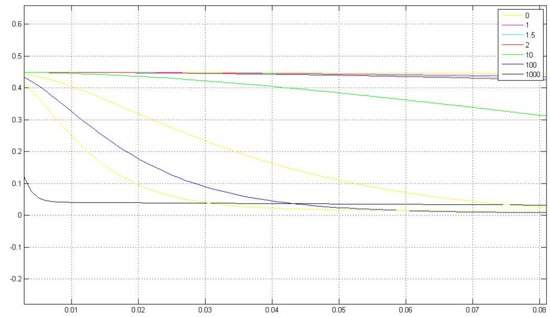

Figure 2. Observer gains

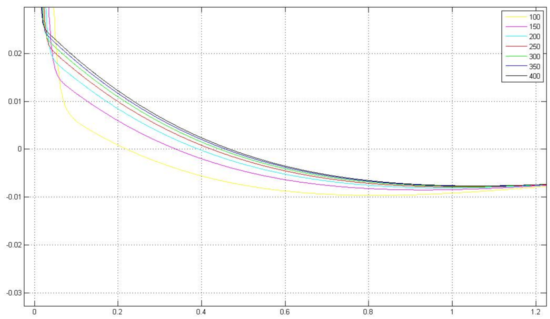

Figure 3. Controller gains in the 100 to 400 interval

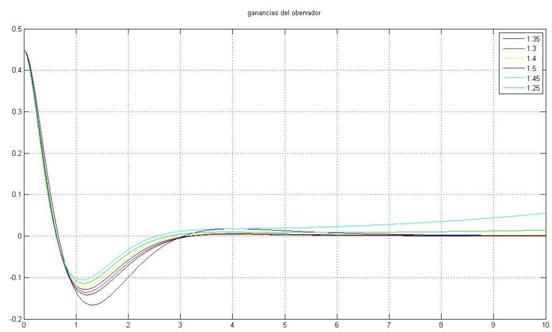

Figure 4. Observer gains in the 1.0 to 1.5 interval

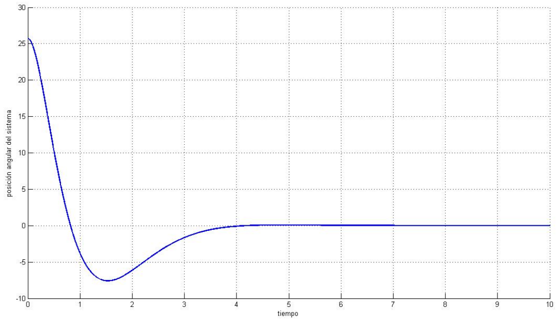

Figure 5. Closed-loop system angular position θ_1

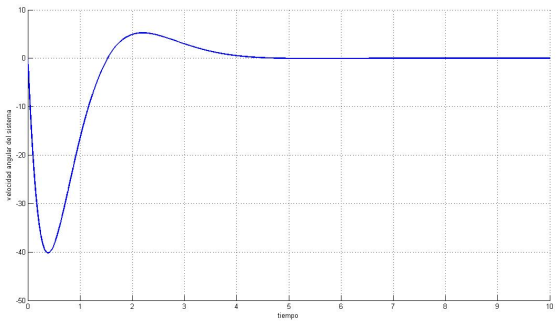

Figure 6. Closed-loop system angular velocity θ_2

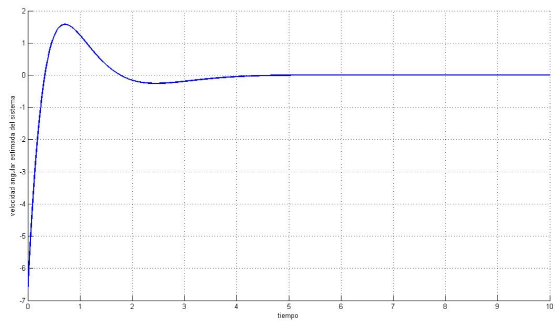

Figure 7. Closed-loop system estimed angular velocity $\hat{\theta}_2$

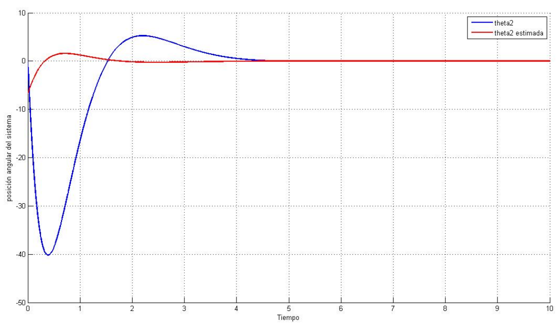

Figure 8. θ_2 and $\hat{\theta}_2$

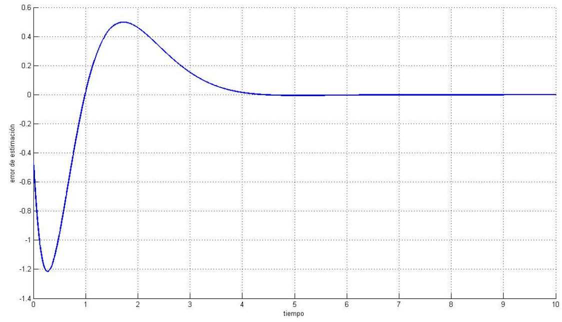

Figure 9. Closed-loop system error

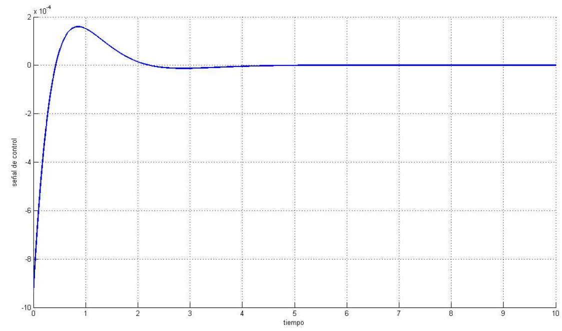

Figure 10. Closed loop system control signal

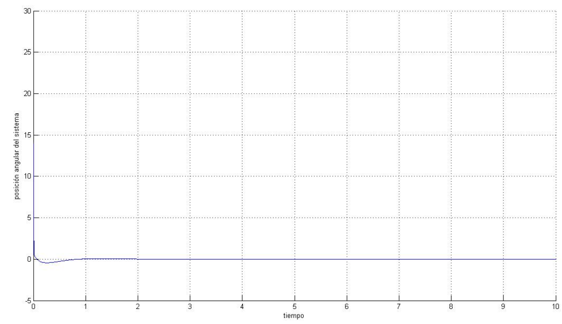

Figure 11. Closed loop system angular position θ_1

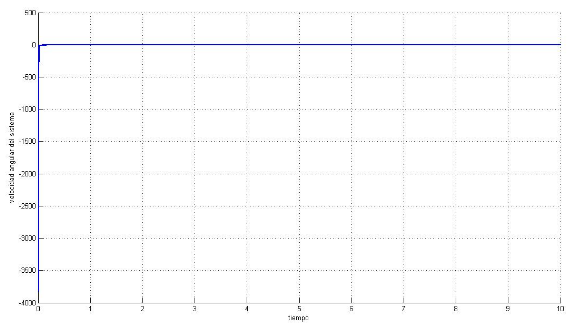

Figure 12. Closed loop system angular speed θ_2

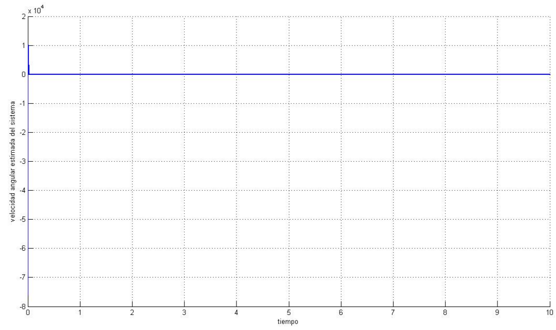

Figure 13. Closed loop system estimed angular velocity $\hat{\theta_2}$

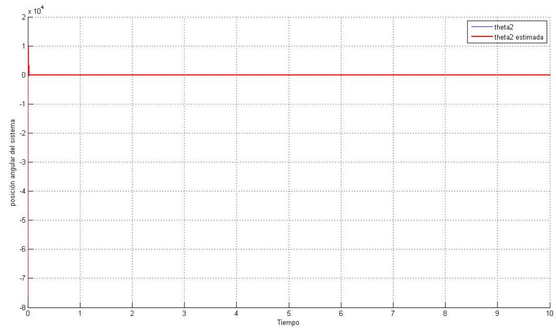

Figure 14. θ_2 and $\hat{\theta_2}$

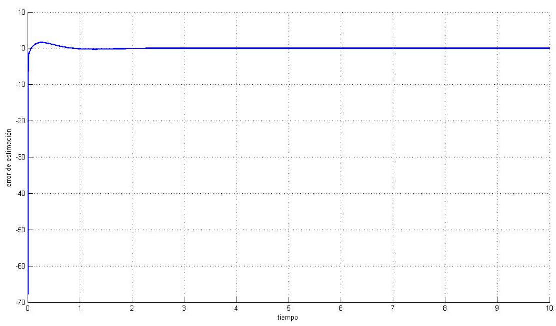

Figure 15. Closed loop system error

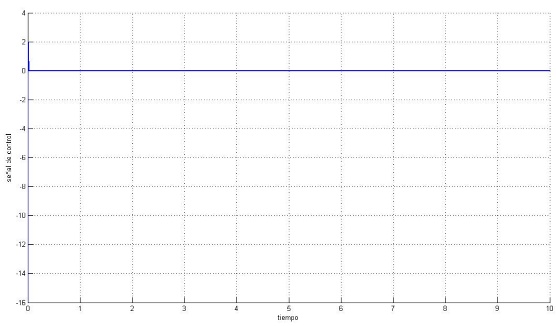

Figure 16. Closed loop system signal control