MODELLING, ACTIVE VIBRATION CONTROL AND SIMULATION OF PIEZOELECTRIC LAYER STRUCTURES

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ABSTRACT

The lightweight structures are vulnerable to changeable surrounding. A perspective way to regulate their desired shape is to design them like layer composites and apply active vibration control. The paper deals with modeling, control design and numerical simulation of thin composite plates. The classical in bending is used based on the finite element modeling. The possible damages are introduced by equivalent reducing of the materials parameters. LQR and H_2 control schemes are considered for vibration suppression. Numerical simulations of the impact response of thin composite structures are carried out.

INTRODUTION

Recently, many structures like plates are made from lightweight materials. They are vulnerable to external disturbances and considerable attention is paid to the investigation of their vibration control. The passive damping technique is not relevant for the lightweight structures. Active vibration damping has the advantage of potential weight and volume savings that makes noise reduction possible at a reasonable cost.
This paper discusses modeling and investigation of the dynamical behaviour of damaged plates and active control for their vibration regulating. The choice of the control strategy for optimal performance and robustness against damage is important. The placement of the sensors and the control forces and its influence on the quality of the vibration regulating process governs the control process. Numerical simulations are carried out with the proposed controlled strategies.

PLATE MODELING

In the smart plate, the control actuators and the sensors are piezoelectric patches symmetrically bonded to the top and the bottom surfaces of the elastic host. The mechanical properties of the piezoelectric material, the viscose material and the host beam are independent in time. The linear theory of piezoelectricity is employed due to small structural vibrations. The equations are

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} - \begin{bmatrix} d_{31} \\ 0 \end{bmatrix} E_z \right) \tag{1}$$

$$D_z = Q_{11} d_{31} \varepsilon_x + \xi_{33} E_z \tag{2}$$

Equation (1) describes the inverse piezoelectric effect (actuator). Equation (2) describes the direct piezoelectric effect (sensor). The kinematical assumptions of the Euler-Bernoulli theory are used. A constant transverse electrical field is assumed for the piezoelectric layers.

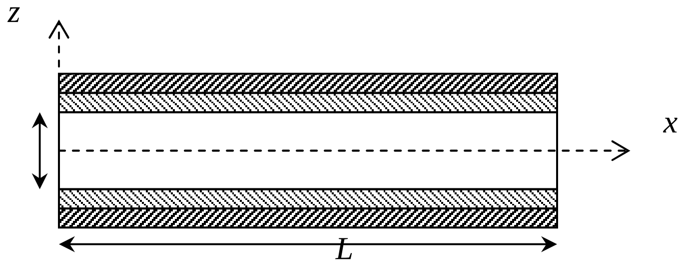


Figure 1. A laminate composite structure

The equation of motion for the plate is derived based on the classical Kirchhoff plate bending theory. One of the most important assumptions in this theory is that the transverse deformation is neglected.

Using Hamilton's principle for classical finite element approximation, the discrete dynamics system is derived

$$M\ddot{X} + D\dot{X} + KX = F_m + F_e \tag{3}$$

Defects of the plate are modelled in a smeared-crack sense by reducing the stiffness and mass matrices of the corresponding finite element.

For control purposes (3) is rewritten in the state space form

$$\dot{x} = Ax + B_1 w + B_2 u\ , \tag{4}$$

$$x = [X \quad \dot{X}]^T \qquad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$$

OPTIMAL CONTROL DESIGN

We consider a regulator type problem by means of a suitable feedback control law. Let the measured outputs $y(t)$ are linear combinations of the system states. The objective is to determine the active control forces $u(t)$ such that to reduce in an optimal way the response to the external excitations. The control is a linear combination of the outputs

$$u(t) = Ky(t) \tag{5}$$

The problem for vibration suppression for flexible plates is solved for both linear quadratic regulator (LQR) and H_2 optimal performance criteria.

LQR CONTROL

The following quadratic cost function is minimized

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \qquad \rightarrow \qquad \min \tag{6}$$

The control gain K_{LQR} is constant and has the form [4]

$$K_{LQR} = R^{-1} B_2{}^T P\ , \tag{7}$$

where P is the positive solution of ARE

This method ensures stability of the system and satisfactory suppression under proper choice of the design parameters. However, for limited number of available measurements and/or corrupted outputs, the most of LQR attractive properties are lost. In this case the more realistic H_2 control strategy can be used.

H_2 CONTROL

The measured output y is a linear combination of the states and is corrupted.

$$y = C_2 x + D_{21} w\ . \tag{8}$$

Let introduce a regulated output z that we are interested to control

$$z = C_1 x + D_{12} u\ . \tag{9}$$

The model (4),(6),(7) is suitable for H_2 optimal control problem. The exogenous signals are fixed or have fixed power spectrum. The controller is in form (5) and keeps the regulated outputs z as small as possible despite the exogenous inputs w. The resulting, closed-loop transfer function from w to z is denoted by T_{zw}.

An appropriate performance criterion for this purpose is H_2 norm of T_{zw} that must be minimized over all controllers internally stabilizing the plant. The cost functional is

$$\|T_{zw}\|_2 = \left(\frac{1}{2}\int_{-\infty}^{+\infty} trace[T_{zw}{}^*(j\omega) \ \ T_{zw}(j\omega)]d\omega \right)^{1/2} \qquad \rightarrow \qquad \min\ . \tag{10}$$

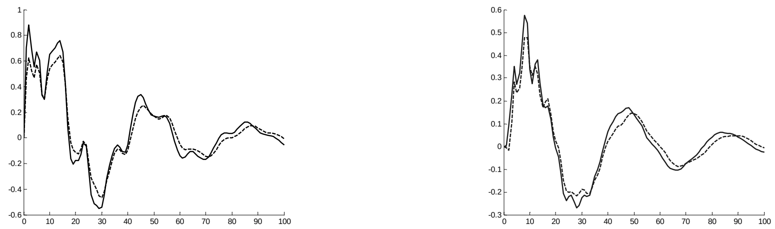
If X and Y are positive solutions of two ARE associated with the control problem the unique controller K_2 satisfying the criterion (10) is given with the formula [5].

$$K_2 = \left[\begin{array}{c|c} A - B_2 B_2^T X - Y C_2^T C_2 & Y C_2^T \\ \hline - B_2^T X & 0 \end{array}\right] \tag{11}$$

The H_2 control design technique provides robustness with respect to the disturbances.

NUMERICAL INVESTIGATION

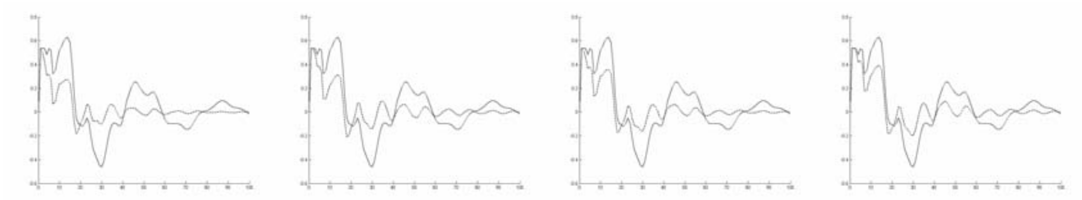
For numerical investigations GUI within MATLAB has been prepared [6]. A square plate four with fixed foundaries is considered for numerical investigations. The ambient vibrations are excited by a concentrated instantaneous force applied in the vertical direction to the plate centre. Figure 2. shows the response of the centre and close to the damaged place.



a. b.

Figure 2: Vibration of two points of the plate with crack (solid) and without crack (dashed).

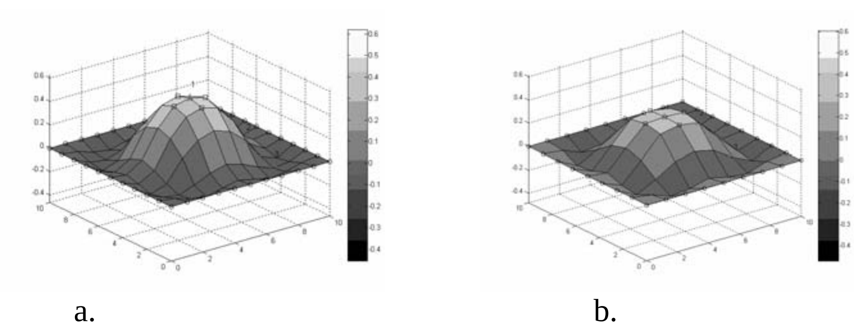
The number and the position of the actuators are investigated. It is better to use less number of controllers if it is possible. The position of a restricted number of actuators must be suitably chosen. Numerical investigations are implemented under the assumption, that all states are available for measuring. The case when only a part of the whole state vector can be measured is as well considered.
The two control strategies are applied for vibration suppression. The control forces act in vertical direction reducing the effect of the adverse vibrations. The response of the closed loop system is compared with the response of the open loop system with respect to the reduction of the maximum magnitude of the vertical displacement. Figure 3 displaces the results.



a. b. c. d

Figure 3: Free vibration (solid) and LQR controlled response (dashed) of the centre of the plate.

Figure 4 depicts 3D plot of the response of the uncontrolled and controlled plate calculated with H_2 control strategy.



a. b.

Figure 4: Response of the uncontrolled a. and controlled b. plate in the same time

Figure 5 shows the response of the plate center under white noise loading.

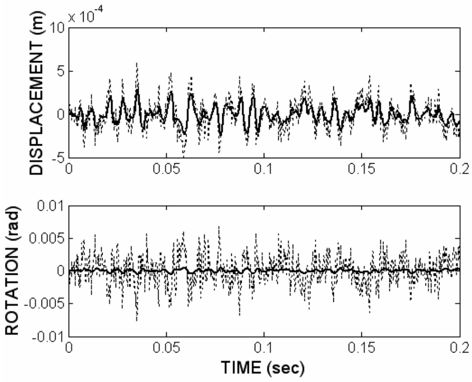


Figure 5: Deflections of the plate center due white noise: uncontrolled (dot) and H_2 (solid) controlled

Experiments made for the two controlled strategies with different groups of measured states show that if not all states are available for measurement better effectiveness is achieved if the transverse deflections are measured.

CONCLUSIONS

Computational methods for modeling and active regulating of composite plates are investigated. It illustrates optimal LQR and robust H_2 control design techniques for a thin plate with piezoelectric and voscoelastic layers. The work presents the mathematical formulation and the computational model for the active vibration control of a composite plate in bending. Damages are modelled by reducing the stiffness and mass matrices of the defected element. The problem of active vibration control is studied using LQR and robust H_2 optimal approaches. The influence of the number and placement of the controllers and sensors on the control design is investigated numerically. Computer simulations are given to demonstrate the research experience.

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