

Forced Chaotic Pendulums Realized in ODE Architect and as a Result Transformed into Ideal Limit Cycles

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ABSTRACT

Last Century discoveries attracted attention to the following problem. The classical continuum mathematics appears to be inadequate for describing new phenomena such as chaos and complexity, media with microstructure and dislocations, space-time quantization, and so on. Some approaches that have been suggested for resolving the problem introduce special mathematical structures , e.g. finite Hilbert spaces, cellular automata, etc.

This presentation reviews the results of a simple method of transformation of chaotic motion represented by a model of forced chaotic pendulum into ideal limit cycles using ODE Architect as a simple computing software. **Keywords:** Forced Chaotic Pendulum Control, Chaos, Discrete Algorithms, ODE Architect, Limit Cycles.

INTRODUCTION [1]

Let us first consider forced chaotic pendulum (FCP) independently of ODE Architect (ODEArch):

$$y' = (c*q - f*z)*p - a*y + d*\sin(k*\sin(e*t))$$

$$z' = -b*(z + q), \quad p' = q*y, \quad q' = -p*y$$

where $p^2 + q^2 = \text{const}$ (typically 1)

In these equations we have 4 variables (y, z, p, q) and 7 parameters

(a, b, c, d, e, f, k) with characteristic examples of numerical values : a = 0.5, b = 0.4, c = 2, e = $\pi/2$, k = $\pi/16$ ($\pi = 3.141592653589\dots$). Parameters d and f have special importance. They will be considered below.

ODE ARCHITECT

In this work for computing the above system of differential equations the ODE Architect software was used. This package is a little known for professionals dealing with computations but it is a simple but still a powerful tool for solving differential equations. It does not demand from a user learning a special language to communicate but could be learned by simply 'playing' with it. At the end of this paper a screen shot of the ODEArch is shown (FIG) to get familiar with the program interface. In the book [2]

accompanying the software the definition of chaos is given but there are no methods describing of transformations the chaotic motions into the regular systems. The author of this presentation has developed simple and effective methods of using ODEArch for transformations of FCP into ideal limit cycles [3].

The algorithm of transforming a chaotic system into the regular one using ODEArch consists of three consecutive steps:

Step 1: A closed chaotic attractor (Fig.1) [4]

Step 2: Attractor contains narrow regions of trajectories (Fig. 2)

Decisive step 3: Transformation of indicated trajectories into limit cycles (Figs. 3,4) [5]. The corresponding realizations of the steps are given in 4 pictures below.

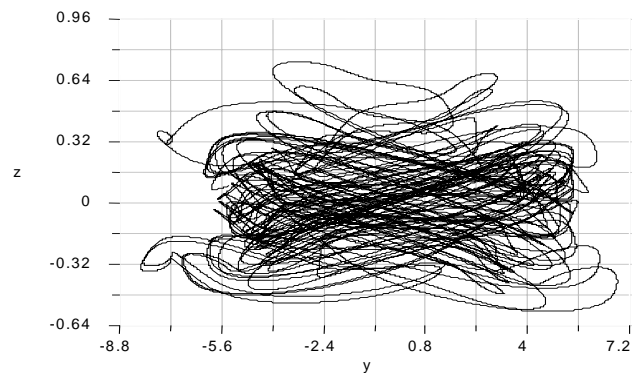


FIG. 1 (d=40, f=8)

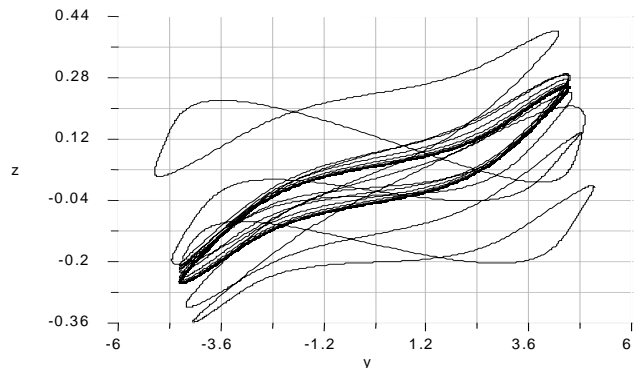


FIG. 2 (d=40, f=1)

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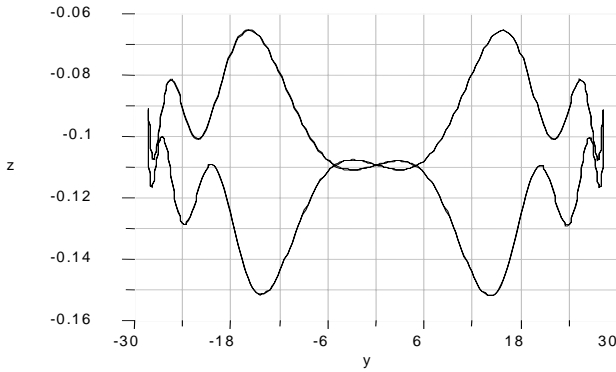


FIG. 3 (d=40, f=8)

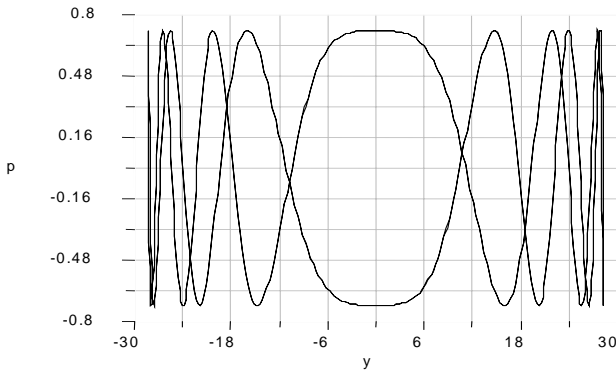


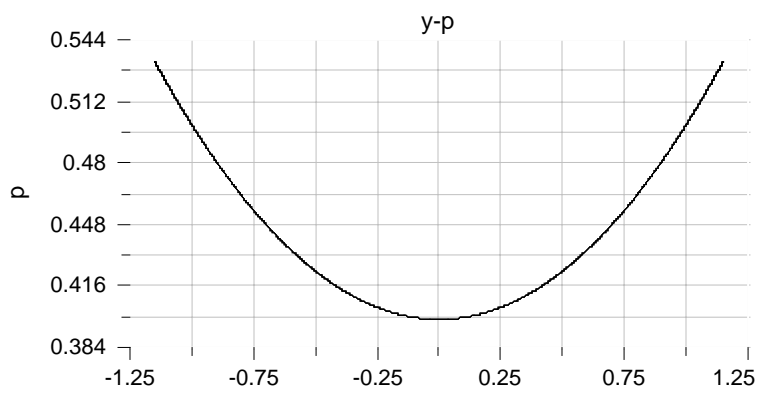
FIG. 4 (d=40, f=8)

LIMIT CYCLES AND PARAMETERS d, f

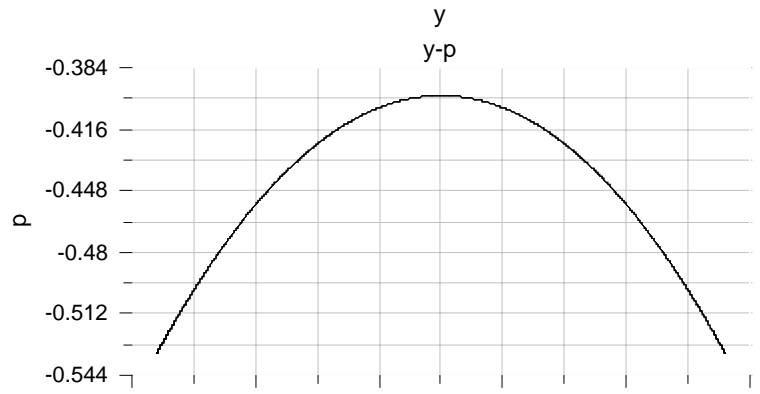
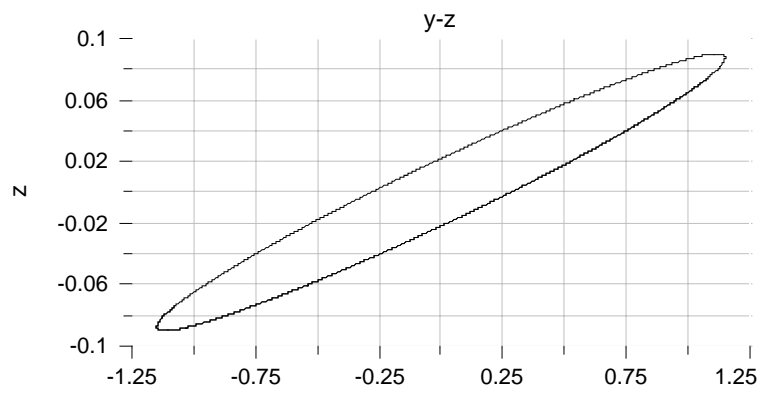
The author has been impressed very much by unexpected variety of limit cycles corresponding to different values of $d = \pm 8 * n$ where n are arbitrary (positive) integers. In addition f may have values ± 8 or ± 1 . The author has carried out a few hundred of experiments to justify the results. The variety of limit cycles obtained in experiments is demonstrated on Figures in the end of the paper. Notice that it would be very nice to prove the corresponding mathematical theorem as well as to carry out some additional experiments to find out the influence of other parameters of FCP on the transformation of this chaotic system

ACKNOWLEDGEMENTS

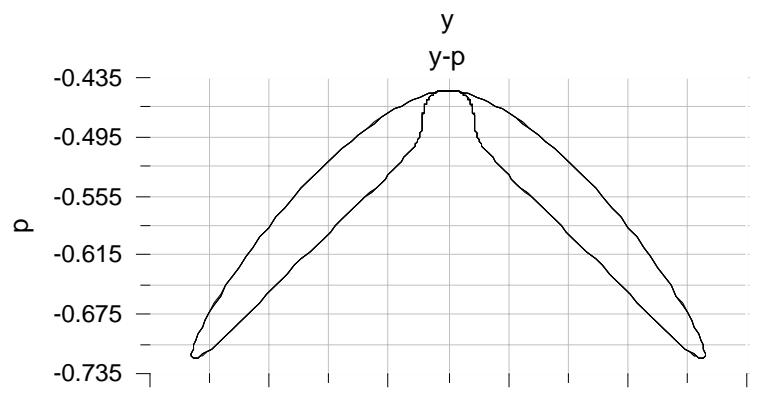
I.A. Kunin express deep gratitude to dear friend and collaborator Prof. Yu. Kuperin for joint works [6-8].



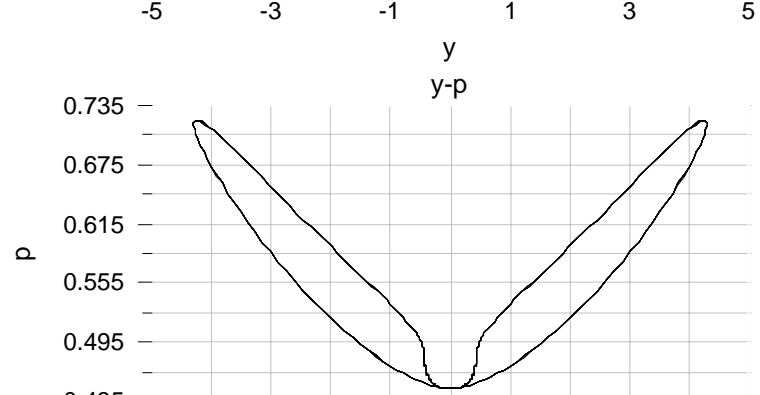
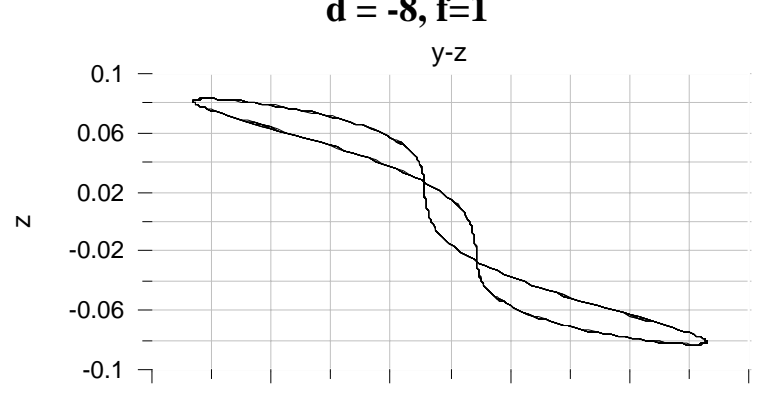
d = 8, f=1



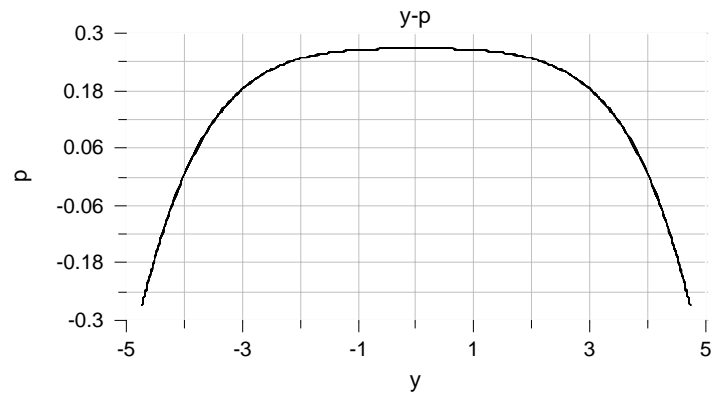
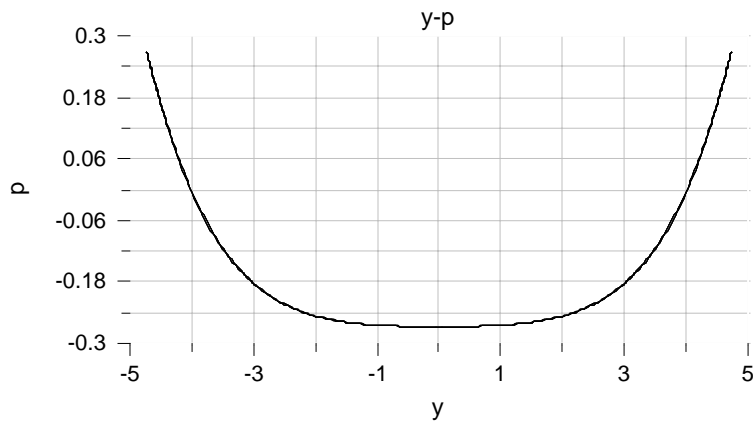
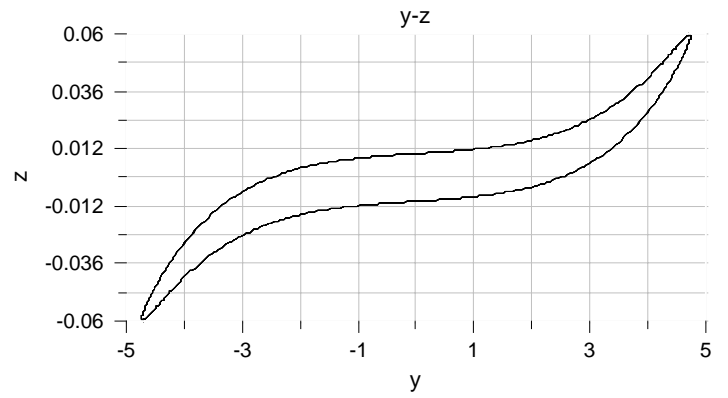
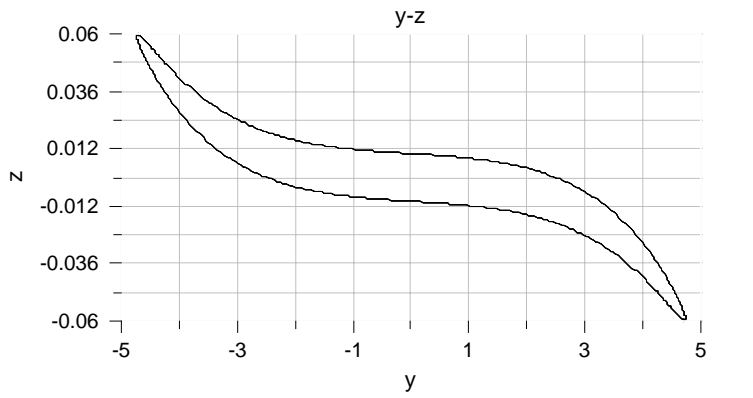
d = -8, f=1



d = 16, f=1

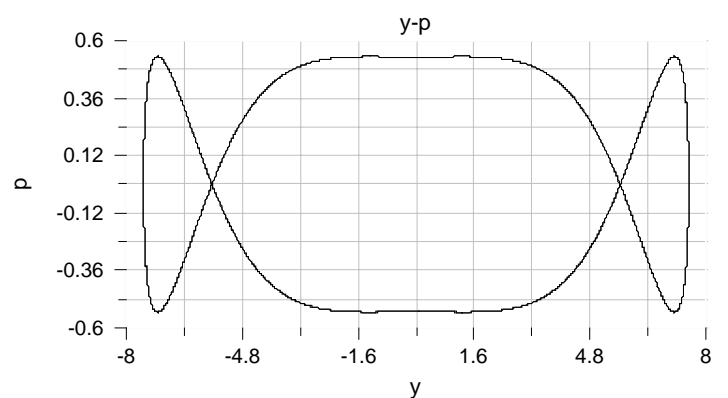
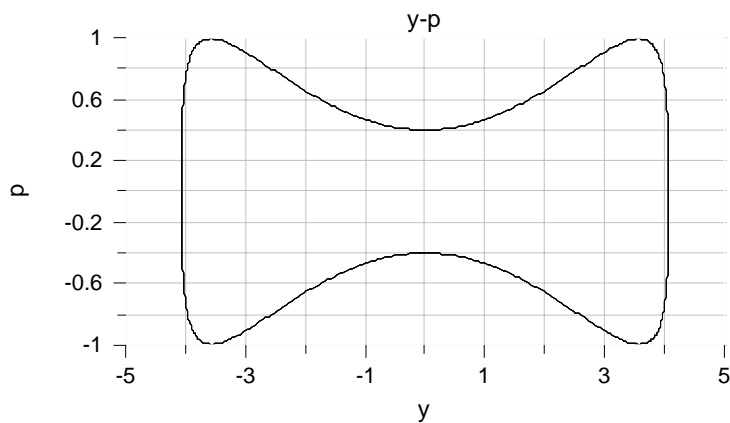
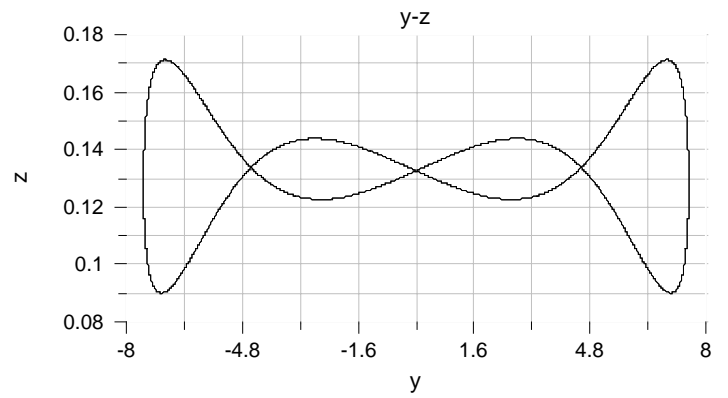
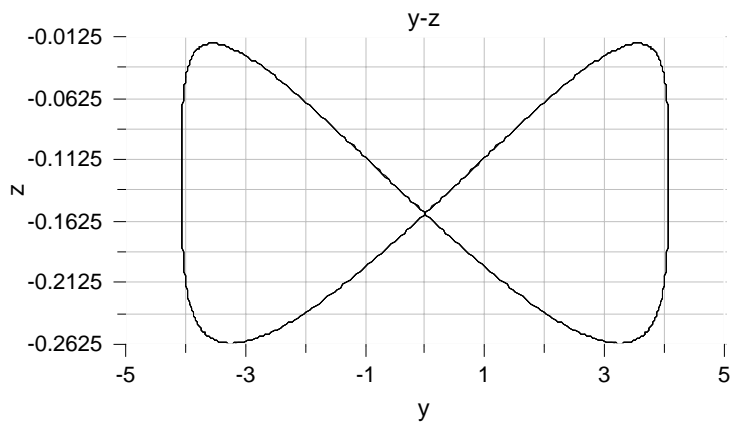


d = -16, f=1



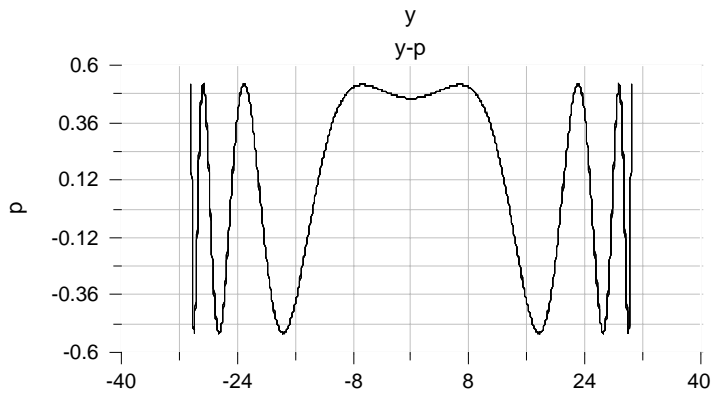
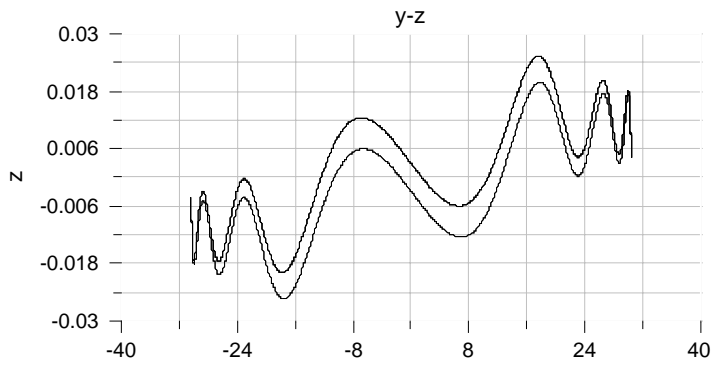
d = 40, f=1

d = -40, f=1

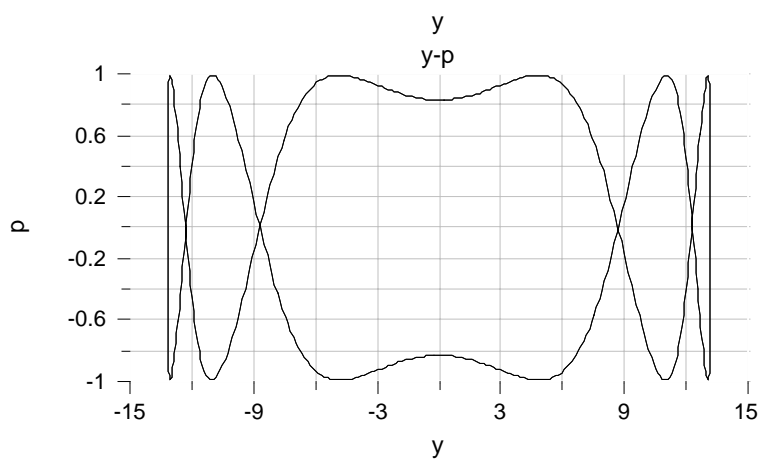
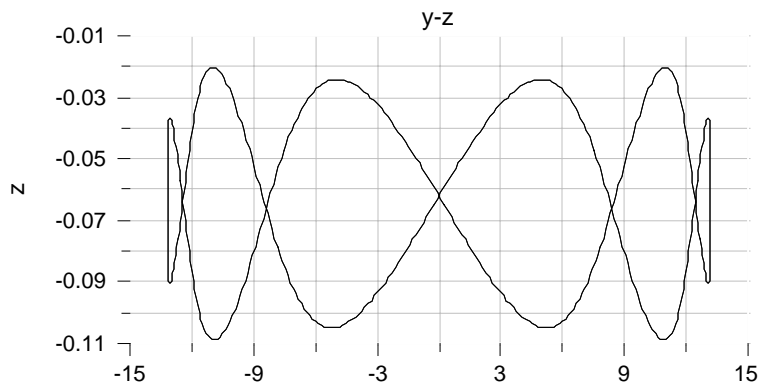


d = 32, f= 8

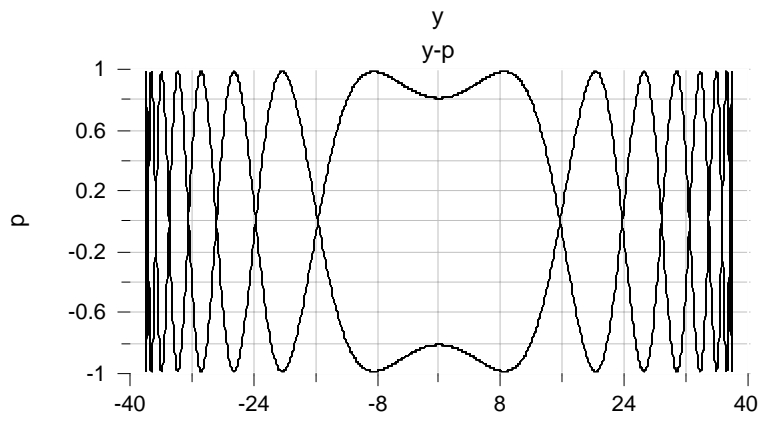
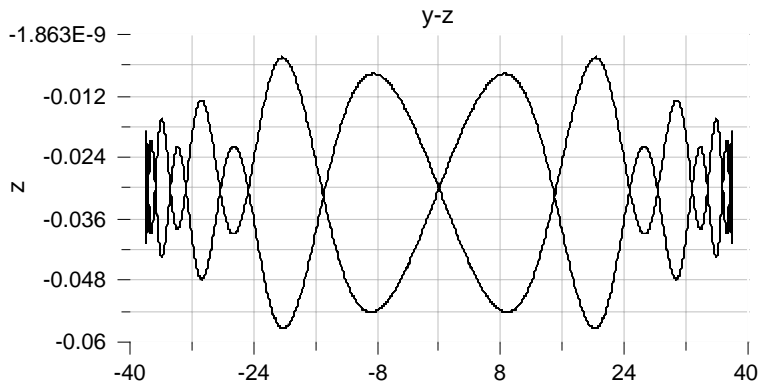
d=64, f=1



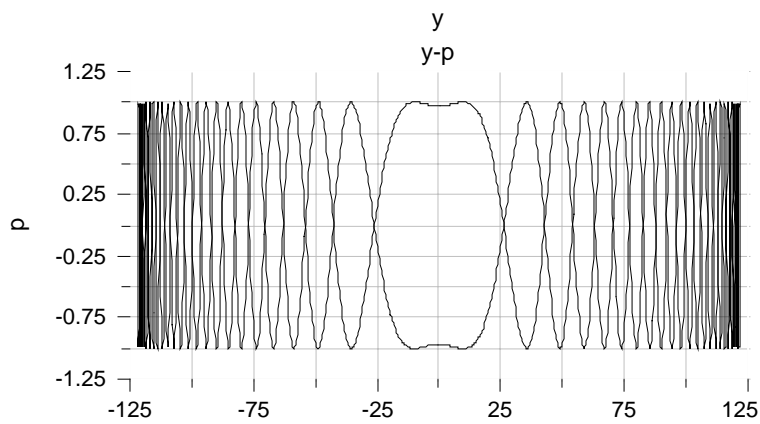
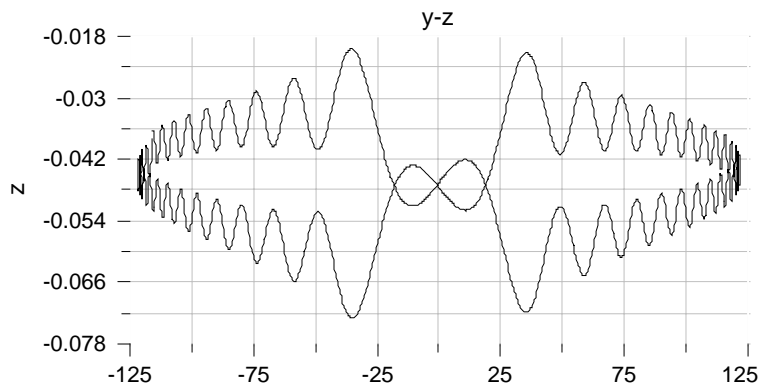
$d = 256, f=8$



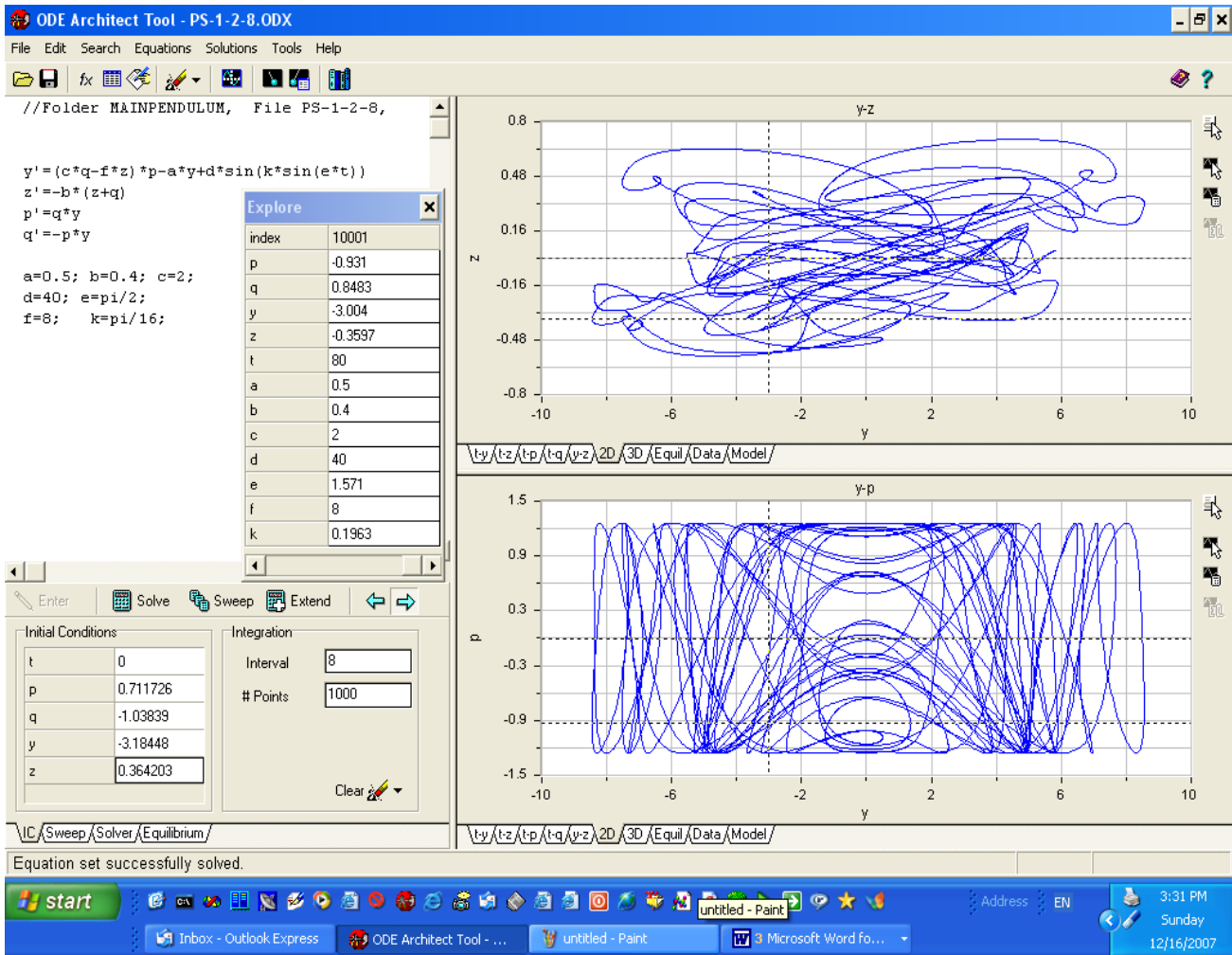
$d=112, f=8$



$d = 320, f = 8$



$d = 1024, f = 8$



The screen short of the interface of ODE Architect