PERIODIC STEADY STATE RESPONSE OF A LARGE SCALE CITY BUS MODEL WITH NONLINEAR CHARACTERISTICS

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Abstract
A computationally efficient methodology is presented for capturing periodic steady state response of a periodically excited city bus model. First, the equations of motion for each of the components of the bus are set up by applying the finite element method. As a consequence of the geometric complexity, the number of the resulting equations is quite high. In addition, the composite model possesses strongly nonlinear characteristics. Therefore, a suitable method is applied originally in order to reduce the dimension of the system. This then allows the application of appropriate numerical methodologies for predicting steady state response of the nonlinear models examined to periodic road excitation. As a result, selected response quantities are evaluated and presented for characteristic combinations of the bus suspension stiffness and damping parameters.

Key words
Periodic steady state, Nonlinear dynamic systems, Finite element modeling, Vehicle dynamics.

1 Introduction

Urban buses are widely used vehicles to transfer passengers throughout the world. It is therefore important to develop and study mechanical models leading to an accurate and fast determination of their dynamic response. In addition, the ability to do this provides the basis for performing many other direct or indirect analyses. Accurate and fast determination of the dynamics of large scale mechanical models has become more tractable and feasible, especially in the last three decades (e.g., [Fey, 1996; Chen, 1998; Papalukopoulos, 2007]). However, there is still plenty of room for improvements when complex mechanical systems are examined. This is especially true for urban buses, where the previous research studies are either limited to simplified models or study specific aspects of the bus ride and handling response only [Rakheja, 2001; Yu, 2002; Cunha, 2001].

The main objective of the present work is to develop a systematic methodology leading to a direct determination of steady state response of periodically excited complex mechanical models of a city bus. Here, the term complex refers to both the large number of degrees of freedom and the nonlinearities of the system. The basic idea is to first reduce the dimension of the system examined by applying an appropriate coordinate transformation, based on an automatic multi-level substructuring of its components [Papalukopoulos, 2007; Bennighof, 2000]. This methodology is coupled with an appropriate numerical procedure leading to a direct determination of periodic steady state motions of the bus model chosen, resulting in response to periodic road excitation [Doedel, 1986].

The organization of this paper is as follows. First, the mechanical model examined is briefly presented in the following section. Then, the basic steps of the methodology employed, including both the coordinate reduction and the steady state determination parts, are summarised in the third and fourth section, respectively. Next, the dynamic response of the bus models subjected to specified periodic road excitation is investigated. Emphasis is placed on capturing periodic steady state motions for bus velocities, which are appropriate for ride studies of the vehicle models examined. The work is completed by summarizing the highlights in the last section.

2 Mechanical Model of the Bus

The complete mechanical model of the vehicle examined is shown in Fig. 1. This vehicle is a low floor urban bus with two axles, designed primarily for inner city operation. Besides the detailed modeling of the bus upper body structure (or superstructure) and chassis frame, it was considered as equally important to model in as good a manner as possible several important subsystems, like the front and rear axle, including the steering system and the tires, the transmission system, the differential, the power unit and the brakes.

Among the vehicle components, the chassis frame and the body superstructure play a dominant role in its overall performance. The main parts of the chassis frame were geometrically discretized by a relatively large number of shell finite elements, leading to a model with 337,260 degrees of freedom. On the other hand, the finite element discretization of the vehicle superstructure led to a model possessing 955,866 degrees of freedom. In addition to the structural parts, special added mass elements were also employed in modeling systems like the air-condition unit, the fuel tanks, the bus floor including the passenger seats and the baggage store compartment.

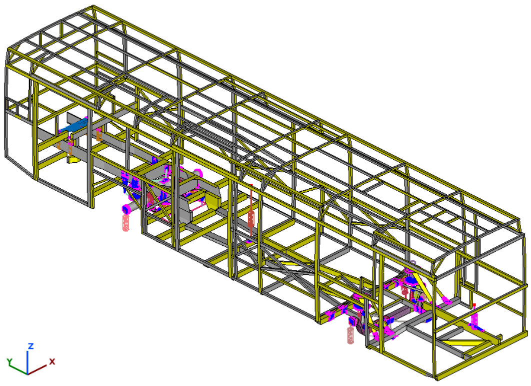


Figure 1. Complete bus model.

The flexible parts of the rear suspension were modeled with shell finite elements. However, some parts of the front suspension were modeled with solid finite elements. In addition, rigid body elements were employed for modeling the action of the interconnections and supports in both the front and the rear suspension subsystems. Likewise, the seat-passenger subsystems as well as the wheel subsystems were represented by appropriate sets of discrete mass, stiffness and damping elements. Finally, the stiffness and damping properties of the suspension units were modeled according to the graphs included in Fig. 2. In the former case, the nonlinearity is caused by the presence of suspension bump stops and rebound stops. In the latter case, the rebound (extension) equivalent damping coefficients are much higher than the corresponding jounce (compression) damping coefficients, in accordance to common practice [Gillespie, 1992]. Moreover, the equivalent damping coefficients are lower at higher velocities.

An attractive feature of the system examined is that its nonlinearities appear mainly at connections of the suspensions with the wheels and the body. This makes possible the application of special techniques, which reduce significantly the number of degrees of freedom by removing the dynamics from some of the high frequency modes [Fey, 1996; Chen, 1998; Papalukopoulos, 2007]. This facilitates the subsequent application of a numerical methodology leading to a direct determination of periodic steady state response. More details on the methodology developed are presented in the following two sections.

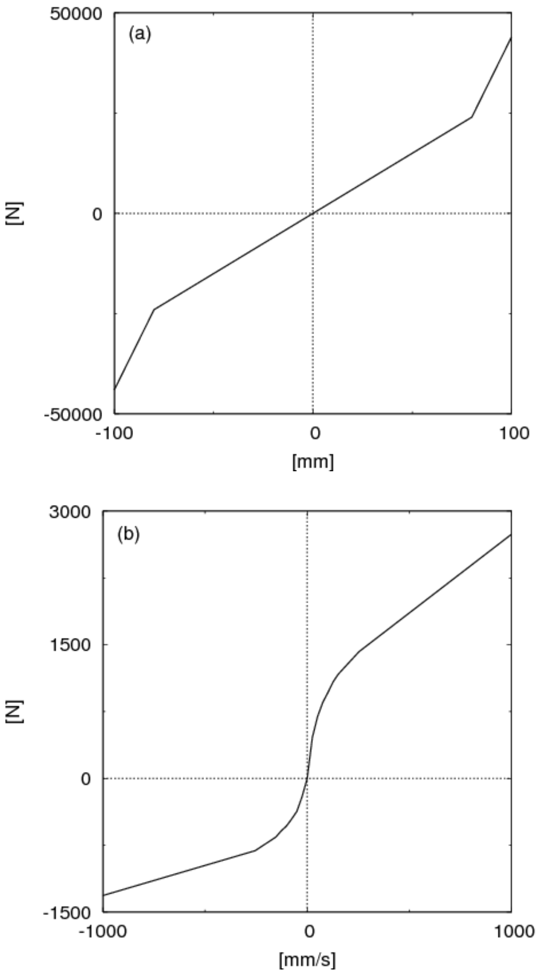


Figure 2. (a) Force-displacement characteristics of the suspension springs. (b) Force-velocity properties of the suspension dampers.

3 Substructuring Method

The equations of motion of the class of dynamical systems examined in the present study can be cast in the form of a system of nonlinear ordinary differential equations, as follows

$$M\,\ddot{\underline{x}} + C\,\dot{\underline{x}} + K\,\underline{x} + \underline{h}(\underline{x},\dot{\underline{x}}) - \underline{f}(t)\,. \qquad (1)$$

All the unknowns are included in the vector

$$\underline{x}(t) = \begin{pmatrix} x_1 & x_2 & \dots & x_n \end{pmatrix}^T,$$

while M, C and K are the mass, damping and stiffness matrix of the system, respectively. Moreover, the elements of the vector $\underline{h}(\underline{x},\dot{\underline{x}})$ include the nonlinear terms arising from the action of the coupled dynamical system, while the vector $\underline{f}(t)$ represents the terms arising from the external forcing.

A complex mechanical system includes contributions from several subsystems. For instance, consider a system composed of two components. If both of these components possess linear characteristics and the damping effects are negligible, the equations of motion for component 1 alone are first derived in the following form

$$M_1 \underline{\ddot{x}}_1 + K_1 \underline{x}_1 = \underline{f}_1(t) . \qquad (2)$$

Then, through a coordinate transformation of the form

$$\underline{x}_1(t) = T_1\, \underline{q}_1(t) , \qquad (3)$$

the original set of equations is replaced by a considerably smaller set, expressed in terms of the generalized coordinates $\underline{q}_1$. More specifically, application of the Ritz transformation (3) into the set of equations (2) yields the smaller dimension set

$$\hat{M}_1 \underline{\ddot{q}}_1 + \hat{K}_1 \underline{q}_1 = \hat{\underline{f}}_1(t) , \qquad (4)$$

where

$$\hat{M}_1 = T_1^T M_1 T_1 , \quad \hat{K}_1 = T_1^T K_1 T_1 , \quad \hat{\underline{f}}_1 = T_1^T \underline{f}_1 .$$

The most intensive numerical computations encountered in setting up equation (4) are those associated with the evaluation of the columns of the transformation matrix T_1. For example, in the classical Craig-Bampton method, this matrix includes a number of fixed interface normal modes, complemented by constraint modes [Nayfeh, 1995].

Repeating the same process for component 2 leads to another reduced set of equations similar to (4). Then, combining these equations leads eventually to the equations of motion of the composite system in the standard form

$$M \underline{\ddot{q}} + K \underline{q} = \underline{f}(t) .$$

The stiffness and mass matrices of the composite system are finally derived in the form

$$K = \begin{bmatrix} \Lambda_1 & 0 & 0 \\ & \Lambda_2 & 0 \\ sym. & & K_3 \end{bmatrix}$$

and

$$M = \begin{bmatrix} I_1 & 0 & \hat{\mu}_{1,3} \\ & I_2 & \hat{\mu}_{2,3} \\ sym. & & M_3 \end{bmatrix},$$

respectively, where the submatrices Λ_i and I_i are diagonal. Also, the mass matrix includes nonzero elements only at places where there is coupling between the involved degrees of freedom. Finally, when damping effects are present, the transformed damping matrix has a form similar to the transformed mass matrix.

The contribution of more components is treated in a similar manner. However, when the dimension of the system components is relatively large, the computations associated with the numerical evaluation of the transformations required in each step become excessive. For this reason, a new class of methods has been developed recently, which overcomes these difficulties in an efficient manner. These methods have their origin in a method called automated multi-level substructuring [Bennighof, 2000], which is performed within each structural component in an automatic way. Since the individual transformations are performed on many small dimensional systems instead of one larger dimensional system, a drastic reduction in the computation time is achieved. Besides, this approach leads to other important numerical benefits, since it is associated with a much smaller volume of data transfer and causes a tremendous reduction in the computer memory required for the execution of the overall computations [Papalukopoulos, 2007, Bennighof, 2000]. As a consequence of the applied transformations, the order of the final set of the equations of motion is reduced substantially, while maintaining their accuracy up to a suitably selected forcing frequency level.

4 Periodic Steady State Motions

In developing a methodology, leading to direct determination of periodic steady state response of the dynamical systems examined, the equations of motion (1) are first rewritten as

$$\underline{g}(\underline{\ddot{x}}, \underline{\dot{x}}, \underline{x}, t) \equiv M\underline{\ddot{x}} + C\underline{\dot{x}} + K\underline{x} + \underline{h}(\underline{x}, \underline{\dot{x}}) - \underline{f}(t) = \underline{0}$$

When the forcing is periodic, that is

$$\underline{f}(t + T_E) = \underline{f}(t) ,$$

the long term response of the system may also reach a periodic steady state, with

$$\underline{x}(t + T) = \underline{x}(t) .$$

The period T of the response is in general a commensurate multiple of the forcing period T_E, with the most common case being $T = mT_E$, corresponding to harmonic ($m = 1$) or subharmonic ($m = 2,3,...$) response. Then

$$\underline{g}(\underline{\ddot{x}}, \underline{\dot{x}}, \underline{x}, t + T) = \underline{g}(\underline{\ddot{x}}, \underline{\dot{x}}, \underline{x}, t) ,$$

with

$$\underline{x}(T) = \underline{x}(0) \text{ and } \underline{\dot{x}}(T) = \underline{\dot{x}}(0) .$$

In the present study, a shooting method is selected for the temporal discretization of the last set of equations [Nayfeh, 1995]. This leads eventually to a system of algebraic equations having the form

$$\underline{g}(\underline{y}) = \underline{0} , \qquad (5)$$

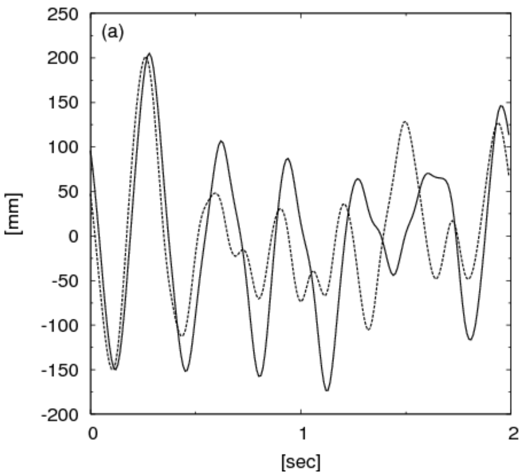
with unknowns included in the vector $\underline{y}$. In general, this system is nonlinear and an appropriate Newton-Raphson type methodology is applied for its numerical solution. This subsequently leads to a direct location of periodic motions of the original equations of motion (1).

5 Numerical Results

In a typical situation, periodic steady state motions of a dynamical system with nonlinear characteristics subjected to periodic forcing may be captured by performing direct integration of the equations of motion. However, this is a time consuming process with an unpredictable outcome [Nayfeh, 1995]. The main contribution of the present study is the development of a systematic method leading to direct determination of such motions. In the remainder of this section, some typical numerical results are presented along this direction.

One of the objectives was to explore the effect of some important technical parameters on the vehicle ride dynamics. To achieve this, the following sequence of figures shows results corresponding to periodic steady state motions developed due to base excitation caused by passage of the bus over a typical road. In fact, the road profile was chosen to be relatively rough, in order to excite the structural nonlinearities. Moreover, the road profile was assumed to be repeated, with a wavelength of 100 m, while the bus runs with a constant horizontal velocity $v_0 = 50$ km/h.

First, Fig. 3 depicts the history of the vertical displacement of the driver and a selected passenger, over a response period. The continuous curves represent results obtained by considering the fully nonlinear model, while the broken lines were determined for the linearized model arising around the corresponding static equilibrium position of the bus. As expected, there are significant deviations between the predictions of the two models. This illustrates the need to include the nonlinear terms in the equations of motion. In addition, it is obvious that the maximum value of a response quantity is in general different than its minimum value. This is expected to occur, due to the asymmetries in both the connection elements and the mechanical elements of the suspension shock absorber and spring units.



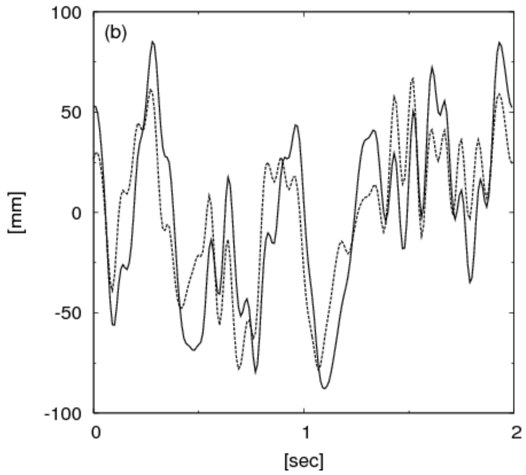


Figure 3. Periodic steady state vertical displacement of: (a) the driver and (b) a selected passenger.

In the second example, the stiffness coefficients of the main springs in all the suspension units were reduced to half of their default values. The results presented in Fig. 4 refer to the same quantities presented in the diagrams of the previous figure. Obviously, direct comparison demonstrates a drastic change in the system response. Such changes in the response are useful to record and investigate in order to perform damage detection studies for the bus suspension parameters in a systematic way.

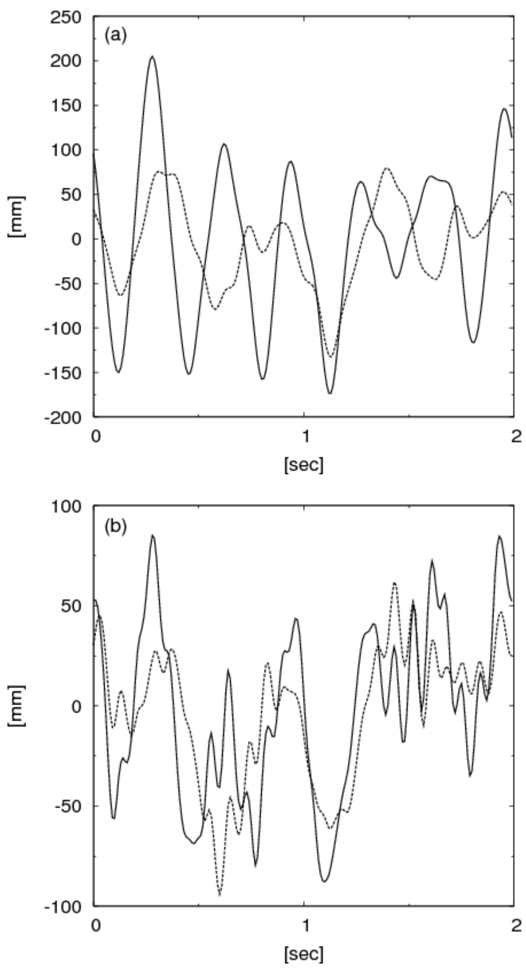


Figure 4. Periodic vertical displacement history of: (a) the driver and (b) a selected passenger.

Finally, similar changes in the response were detected by changing other, non-structural parameters. For instance, in Fig. 5 are presented and compared similar results, obtained by reducing the horizontal velocity of the bus to 20 km/h. More specifically, the results obtained for the lower velocity are represented by the broken curves. Again, significant changes appear in the periodic motions detected, which are mainly due to the differences induced in the frequency content of the loading.

In closing, it is noted that it is required frequently to locate complete branches of periodic motions of a mechanical system, as an important parameter of the system is varied. For instance, in periodic excitation of dynamical systems, a typical such parameter is the fundamental forcing frequency. In such cases, a proper continuation technique is applied in order to locate complete branches of response spectra, which provide useful information on the effect of the parameters on the response [Nayfeh, 1995]. Such results are presented in Fig. 6.

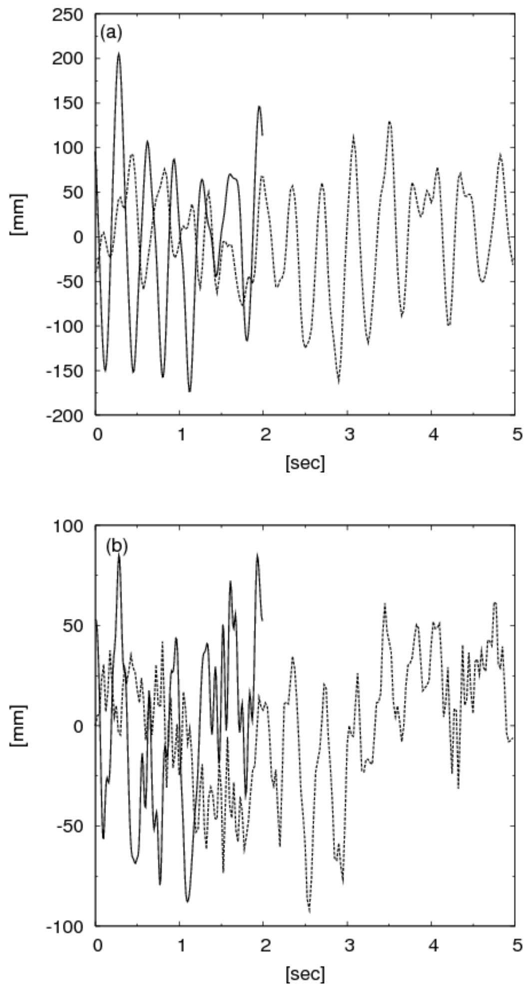


Figure 5. Periodic vertical displacement history of: (a) the driver and (b) a selected passenger.

In particular, Fig. 6 displays frequency-response diagrams obtained at two specific points of the bus considered. The first refers to the point of the bus frame where the driver seat is mounted, while the second is a selected point at the bus roof. More specifically, the bus was excited by a vertical harmonic forcing applied at the front left wheel of the bus. The root mean square value of the acceleration history is presented within the forcing frequency interval 0-30 Hz, which is typical for ride studies referring to ground vehicles [Gillespie, 1992].

As usual, the information of Fig. 6 is useful in assessing the forcing frequency ranges where the response quantity examined exhibits high level vibrations. For comparison purposes, the broken curves represent similar results, obtained by running the model resulted by linearizing the equations of motion around the static equilibrium position of the bus. The deviations observed between the predictions of the nonlinear and the linearized model are amplified as the forcing amplitude is increased.

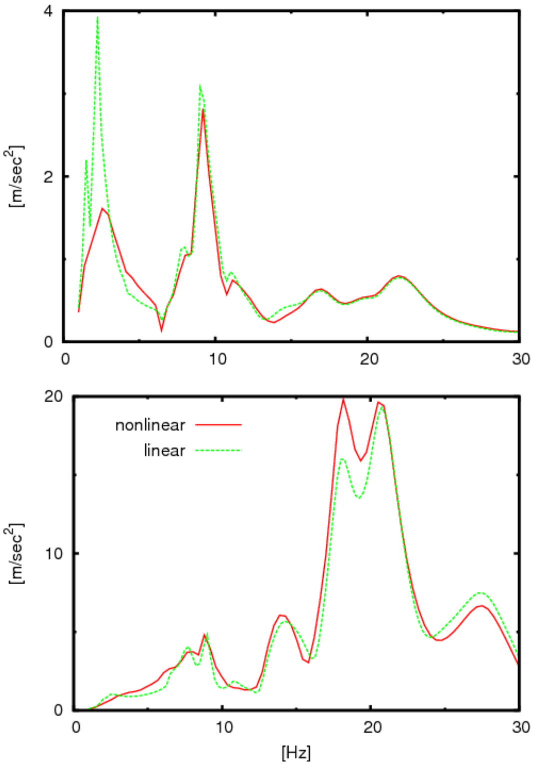


Figure 6. Frequency-response diagrams: (a) at the driver seat position and (b) at a selected point of the bus roof.

6 Synopsis

A systematic methodology was applied for determining steady state ride response of a periodically excited large order bus model in a computationally efficient way. The model examined belongs to a class of systems resulting from a quite detailed finite element discretization and possessing elements with strongly nonlinear properties. The basic idea was to first apply a substructuring method, so that the reduced model is sufficiently accurate up to a prespecified level of forcing frequencies. The analysis was then completed by a method leading to a direct determination of steady state response to periodic excitation. The validity of the methodology was illustrated by presenting numerical results obtained under periodic road excitation. Namely, response histories of quantities related to vehicle ride performance were constructed for steady state motions resulting from selected periodic road excitation. Among other things, some emphasis was put in noting deviations arising between predictions of nonlinear and corresponding linearized models.

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