IMPROVEMENT IN TRANSIENT RESPONSE IN A MULTIRATE CONTROL SYSTEM

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Abstract

This paper discusses a design method for improvement in the transient response of a multirate control system, in which the sampling interval of the plant output is an integer multiple of the hold interval of the control input. A design method for a multirate control system has been proposed such that the intersample response is improved independent of the sample response. In the conventional method, a control system is designed to eliminate the steady-state intersample ripples. However, in this study, the transient response is also improved as well as the steady-state response.

Key words

multirate system, transient response, steady-state response, intersample response, sample response

1 Introduction

This paper discusses a sampled-data control system, in which a continuous-time plant is controlled using a discrete-time controller, and it is assumed that the dynamic characteristics in continuous time cannot be obtained, but those in discrete time are obtained. Further, the sampling interval of the plant output is an integer multiple of the hold interval of the control input, and such systems are referred to as multirate systems [Åström and Wittenmark, 1997; Araki and Hagiwara, 1986; Polyakov, 2006]. Hence, a multirate system is controlled using the discrete-time plant output instead of the continuous-time one.

An extension method for a multirate control system has been proposed [Sato, 2008]. In this method, the sample response in an existing control system is maintained, and a multirate control law is extended. As a result, the intersample response can be improved independent of the sample response. However, in the conventional method, design parameters of the extension method are selected such that only the steady-state ripples are eliminated. Hence, in this study, design parameters are selected to improve not only the steady-state response also the transient response.

In this study, z_1^{-1} denotes the one-step backward shift operator, $z_1^{-1}y[k] = y[k-1]$ and $z_j^{-1} = z_1^{-j}$. A polynomial is described as $A[z_l^{-1}]$, and a polynomial vector and a polynomial matrix are described as $\boldsymbol{A}[z_l^{-1}]$.

2 Multirate Control System

Consider a single-input single-output (SISO) single-rate control system given as:

$$A_s[z_1^{-1}]y[k] = B_s[z_1^{-1}]^T u[k-1] \tag{1}$$

where $y[k]$ and $u[k]$ are the plant output and the control input, respectively. In the case that the plant output can be sampled every step, this system is referred to a fast-rate single-rate (FRSR) system. However, it is assumed that the control input can be updated every step, but the sampling interval of the plant output is an integer multiple of the hold interval of the control input. Hence, the plant output is only measured every l steps. The measurable output signals and the unmeasurable output signals are summarized as follows.

$$\begin{aligned}
&\underbrace{y[k]}_{measured}, \underbrace{y[k+1], y[k+2], \cdots, y[k+l-1]}_{unmeasured},\\
&\underbrace{y[k+l]}_{measured}, \underbrace{y[k+l+1], \cdots, y[k+2l-1]}_{unmeasured},\\
&\underbrace{y[k+2l]}_{measured}, \underbrace{y[k+2l+1], \cdots, y[k+3l-1]}_{unmeasured},\\
&\underbrace{y[k+3l]}_{measured}, \quad \cdots
\end{aligned}$$

Because this system is a multirate system, a FRSR control system cannot be obtained. In this study, to design this multirate system as a single-rate system, this SISO

FRSR model is transformed into a multi-input single-output (MISO) slow-rate single-rate (SRSR) system using the lifting. [Chen and Francis, 1995; Lu et al., 1990; Ishitobi et al., 2002]. This paper discusses a design method for a MISO SRSR system given as:

$$\begin{aligned} A[z_l^{-1}]y[k] &= \boldsymbol{B}[z_l^{-1}]^T\boldsymbol{u}[k-l] \qquad (2)\\ A[z_l^{-1}] &= 1 + a_1 z_l^{-1} + \cdots + a_n z_l^{-n}\\ \boldsymbol{B}[z_l^{-1}] &= \left[B_1[z_l^{-1}]\ B_2[z_l^{-1}]\ \cdots\ B_l[z_l^{-1}]\right]^T\\ B_j[z_l^{-1}] &= b_{j,0} + b_{j,1}z_l^{-1} + \cdots + b_{j,n}z_l^{-n}\\ \boldsymbol{u}[k] &= [u[k]\ u[k+1]\ \cdots\ u[k+l-1]]^T\\ j &= 1, \cdots, l \end{aligned}$$

It is assumed that (2) is stably controlled using a multirate control law given as:

$$\begin{aligned} \boldsymbol{Y}[z_l^{-1}]\boldsymbol{u}[k] &= \boldsymbol{K}[z_l^{-1}]w[k] - \boldsymbol{X}[z_l^{-1}]y[k] \qquad (3)\\ \boldsymbol{Y}[z_l^{-1}] &= \mathrm{diag}\{Y_1[z_l^{-1}],\ \cdots,\ Y_l[z_l^{-1}]\}\\ \boldsymbol{K}[z_l^{-1}] &= \left[\,K_1[z_l^{-1}]\cdots K_l[z_l^{-1}]\,\right]^T\\ \boldsymbol{X}[z_l^{-1}] &= \left[\,X_1[z_l^{-1}]\cdots X_l[z_l^{-1}]\,\right]^T \end{aligned}$$

where $\boldsymbol{Y}[z_l^{-1}]$ is non-singular.

Using the multirate control law, the closed-loop system is calculated as:

$$\begin{aligned} y[k] &= \frac{z_l^{-1}\boldsymbol{Y}_B[z_l^{-1}]^T\boldsymbol{K}[z_l^{-1}]}{T[z_l^{-1}]}w[k] \qquad (4)\\ T[z_l^{-1}] &= Y_p[z_l^{-1}]A[z_l^{-1}] + z_l^{-1}\boldsymbol{Y}_B[z_l^{-1}]^T\boldsymbol{X}[z_l^{-1}]\\ Y_p[z_l^{-1}] &= \prod_{i=1}^{l} Y_i[z_l^{-1}]\\ \boldsymbol{Y}_B[z_l^{-1}] &= \left[\,Y_{B1}[z_l^{-1}]\cdots Y_{Bl}[z_l^{-1}]\,\right]^T\\ Y_{Bi}[z_l^{-1}] &= B_i[z_l^{-1}]\prod_{j=1}^{l,j\neq i} Y_j[z_l^{-1}] \end{aligned}$$

The derivation of (4) is shown in [Sato, 2008].

In the previous work [Sato, 2008], in order to eliminate the steady-state intersample ripples independent of the closed-loop system (4), the multirate control law (3) is extended as follows.

$$\begin{aligned} \boldsymbol{Y}_e[z_l^{-1}]\boldsymbol{u}[k] &= \boldsymbol{K}[z_l^{-1}]w[k] - \boldsymbol{X}_e[z_l^{-1}]y[k] \qquad (5)\\ \boldsymbol{Y}_e[z_l^{-1}] &= \boldsymbol{Y}[z_l^{-1}] - z_l^{-1}\boldsymbol{U}_u[z_l^{-1}]\boldsymbol{B}[z_l^{-1}]^T\\ \boldsymbol{X}_e[z_l^{-1}] &= \boldsymbol{X}[z_l^{-1}] + \boldsymbol{U}_y[z_l^{-1}]A[z_l^{-1}]\\ \boldsymbol{U}_u[z_l^{-1}] &= \left[\,U_{u,1}[z_l^{-1}]\cdots U_{u,l}[z_l^{-1}]\,\right]^T\\ \boldsymbol{U}_y[z_l^{-1}] &= \left[\,U_{y,1}[z_l^{-1}]\cdots U_{y,l}[z_l^{-1}]\,\right]^T\\ \boldsymbol{X}_e[z_l^{-1}] &= \begin{bmatrix} X_1[z_l^{-1}] + U_{y,1}[z_l^{-1}]A[z_l^{-1}]\\ \vdots\\ X_l[z_l^{-1}] + U_{y,l}[z_l^{-1}]A[z_l^{-1}] \end{bmatrix}\\ \boldsymbol{Y}_e[z_l^{-1}] &= \begin{bmatrix} Y_{e_{1,1}}[z_l^{-1}] & \cdots & Y_{e_{1,l}}[z_l^{-1}]\\ \vdots & & \vdots\\ Y_{e_{l,1}}[z_l^{-1}] & \cdots & Y_{e_{l,l}}[z_l^{-1}] \end{bmatrix}\\ Y_{e_{i,j}}[z_l^{-1}] &= \\ &\begin{cases} Y_i[z_l^{-1}] - z_l^{-1}U_{u,i}[z_l^{-1}]B_i[z_l^{-1}] & (i=j)\\ -z_l^{-1}U_{u,i}[z_l^{-1}]B_j[z_l^{-1}] & (i\neq j) \end{cases} \end{aligned}$$

$\boldsymbol{Y}_e[z_l^{-1}]$ must be non-singular.

In the case that the extended multirate control law (5) is employed instead of the original multirate control law (3), the closed-loop system is extended as follows.

$$y[k] = \frac{z_l^{-1}\boldsymbol{Y}_B[z_l^{-1}]^T\boldsymbol{K}[z_l^{-1}]}{T_e[z_l^{-1}]}w[k] \qquad (6)$$

$$T_e[z_l^{-1}] = T[z_l^{-1}] + \bar{T}_e[z_l^{-1}] \qquad (7)$$

$$\bar{T}_e[z_l^{-1}] = z_l^{-1}\boldsymbol{Y}_B[z_l^{-1}]^T(\boldsymbol{U}_y[z_l^{-1}] - \boldsymbol{U}_u[z_l^{-1}])A[z_l^{-1}] \qquad (8)$$

where $\boldsymbol{U}_u[z_l^{-1}]$ and $\boldsymbol{U}_y[z_l^{-1}]$ are set as follows.

$$\begin{aligned} U_{u,i}[z_l^{-1}] &= U_i[z_l^{-1}]B_{i+1}[z_l^{-1}]Y_i[z_l^{-1}] \ \ (i\neq l)\\ U_{u,l}[z_l^{-1}] &= U_l[z_l^{-1}]B_1[z_l^{-1}]Y_l[z_l^{-1}]\\ U_{y,i}[z_l^{-1}] &= U_{i-1}[z_l^{-1}]B_{i-1}[z_l^{-1}]Y_i[z_l^{-1}] \ \ (i\neq 1)\\ U_{y,1}[z_l^{-1}] &= U_l[z_l^{-1}]B_l[z_l^{-1}]Y_1[z_l^{-1}] \end{aligned}$$

where $U_i[z_l^{-1}]$ $(i = 1, \cdots, l)$ are design polynomials, and then, $\bar{T}_e[z_l^{-1}] = 0$. Therefore, the new closed-loop system is equal to the original one, and $U_i[z_l^{-1}]$ can be selected independent of the original closed-loop system (4). In the previous work, new design polynomials $U_i[z_l^{-1}]$ $(i = 1, \cdots, l)$ are designed such that the steady-state gains from the reference input to the control inputs are to be equivalent. As a result, the plant output converges to the step-wise reference input without intersample ripples in steady state.

In this study, it is shown that the transient response can be also improved independent of the sample response. In the conventional method, the design polynomials are set to scalars to have the steady-state gains be equivalent. However, in this study, to improve both the steady-state and the transient responses these design parameters are set as polynomials, and its effectiveness is demonstrated in the next section.

3 Numerical Example

Consider a continuous-time model given as:

$$G(s) = \frac{0.7}{(1.9s+1)^2} \tag{9}$$

This is a model of a hot-air tunnel, and its simplified figure is illustrated as Fig. 1, in which the air flow inside the tunnel is controlled by a ventilator [Matušů and Prokop, 2008]. The plant output is the airflow speed measured by a vane flowmeter, and the control input is the ventilator voltage.

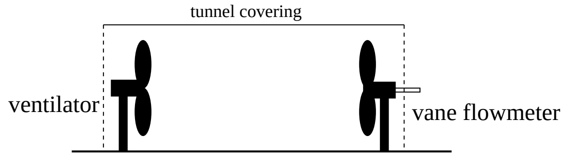

Figure 1. Model of a hot-air tunnel

It is assumed that the dynamic characteristics in continuous time are not obtained, but the discrete-time model can be obtained. Further, the plant output is sampled at intervals of 0.5[s], and the control input can be updated at intervals of $0.5/2$[s]. In this case, a control law is designed using a two-input single-output SRSR system given as:

$$\begin{aligned}&(1 - 1.2z_2^{-1} + 0.35z_2^{-2})y[k] = \\ &\left[0.048 + 0.010z_2^{-1} \quad 0.020 + 0.038z_2^{-1}\right] \boldsymbol{u}[k-2]\end{aligned} \tag{10}$$

A multirate control law is designed such that the closed-loop poles are set to 0.1, 0.2 and 0.4, and its control result using this control law is shown in Fig. 2 and Fig. 3. These figures show that the sampled output can converge to the unit step reference input, but the control input oscillates and does not converge to a constant, and the intersample output oscillates.

However, the sample response is ideal, and it should not be changed because the closed-loop poles of the discrete-time control system are assigned to the desired values, although the intersample output oscillates. The sample response is desired and, it is assumed not to be changed because the closed-loop poles of the discrete-time control system are assigned to the desired values, although the intersample output oscillates. Therefore, the objective of the extension (5) is to improve intersample response without changing the sample response. First, the new design parameters are designed to eliminate the steady-state ripples, and next, those parameters are designed such that the transient response is also improved. Using $U_2[z_2^{-1}] = 0$, in the design method for improvement in the steady-state intersample response [Sato, 2008], the steady-state ripples can be eliminated if the following equation is satisfied.

$$\begin{aligned}&U_1[1] = \bar{U}_1 \\ &\bar{U}_1 = 6.1 \times 10^2\end{aligned}$$

In the conventional method [Sato, 2008], $U_1[z_2^{-1}] = \bar{U}_1$ is employed, and hence, only the steady-state ripples are eliminated. Its control result is shown in Fig. 4 and Fig. 5. From this result, the plant output converges to the reference input without intersample ripples, but the transient response is deteriorated compared with that of the original multirate control system shown as Fig. 4.

Next, to improve both the steady-state and the transient responses, the design parameter $U_1[z_2^{-1}]$ is designed as:

$$U_1[z_2^{-1}] = \frac{U_{tr}[z_2^{-1}]}{U_{tr}[1]}\bar{U}_1$$

where $U_{tr}[z_2^{-1}]$ is set to $U_{tr1}[z_2^{-1}]$ and $U_{tr2}[z_2^{-1}]$, respectively, and simulations are conducted.

$$U_{tr1}[z_2^{-1}] = 1 + 0.9z_2^{-1} \tag{11}$$

$$\begin{aligned}U_{tr2}[z_2^{-1}] &= 1 + 0.9z_2^{-1} + 0.7z_2^{-2} + 0.6z_2^{-3} \\ &+ 0.5z_2^{-4} + 0.4z_2^{-5} + 0.3z_2^{-6} + 0.2z_2^{-7} + 0.1z_2^{-8}\end{aligned} \tag{12}$$

The simulation result using $U_{tr1}[z_2^{-1}]$ (11) is shown in Fig. 6 and Fig. 7. The transient response is slightly improved, but it is not sufficiently improved. Hence, $U_{tr2}[z_2^{-1}]$ (12) is employed instead of (11), and the plant is controlled. Fig. 8 shows that both the overshoot and the under-shoot are reduced between the sampled outputs, and the steady-state response is not deteriorated, although the plant output slightly vibrates. The control input converges to the constant value (Fig. 9). For comparison, all the control results are shown in Fig. 10 and Fig. 11, and it can be seen that the sample response can be maintained.

4 Conclusion

This paper discussed a design method for a multirate control system, in which the sampling interval of the plant output is longer than the hold interval of the control input. In this study, a design method [Sato, 2008] for a multirate control system was applied to a hot-air tunnel [Matušů and Prokop, 2008], and it is shown that the extension method of a multirate control system can improve not only the steady-state response but also the

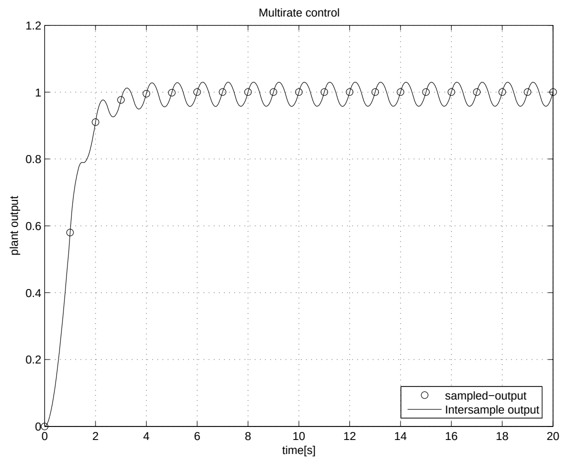

Figure 2. Output result (original control system)

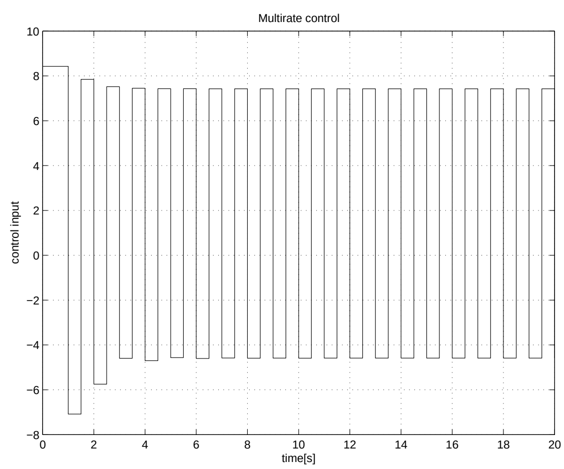

Figure 3. Input result (original control system)

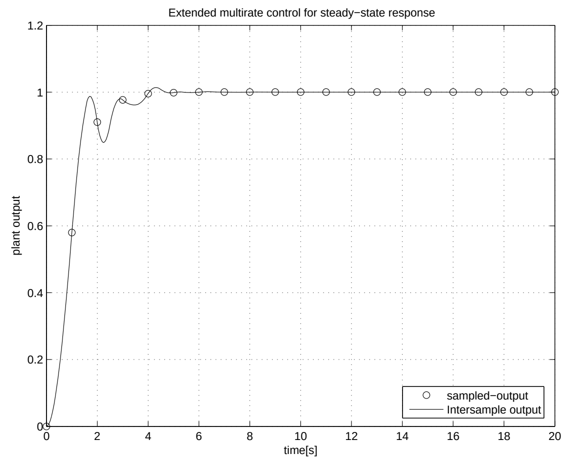

Figure 4. Output result (extension for steady-state response)

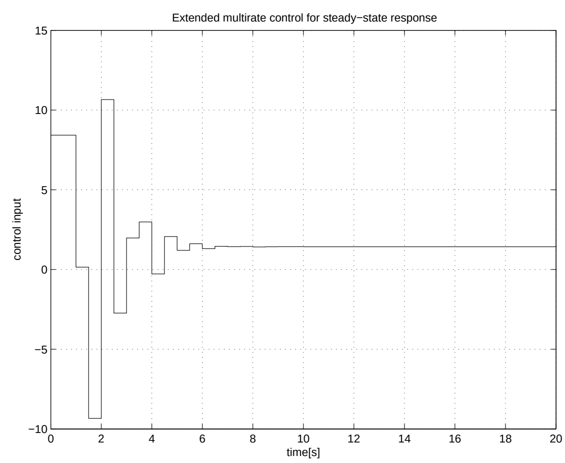

Figure 5. Input result (extension for steady-state response)

transient response. However, an optimal design method for design parameters should be made clear, although its effectiveness was demonstrated through numerical examples.

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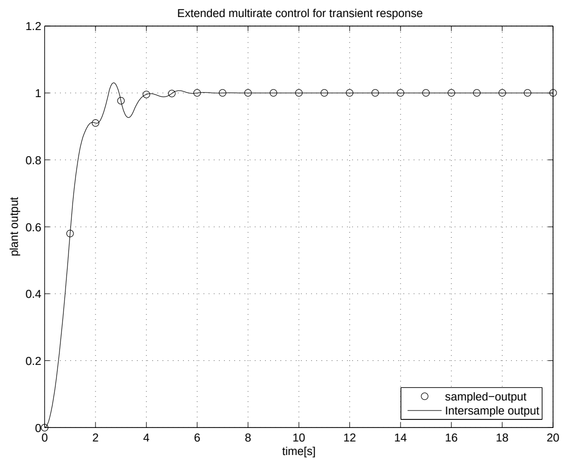

Figure 6. Output result (extension for transient response 1)

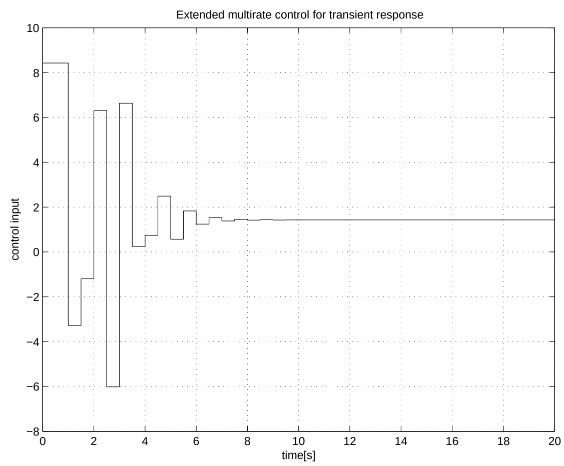

Figure 7. Input result (extension for transient response 1)

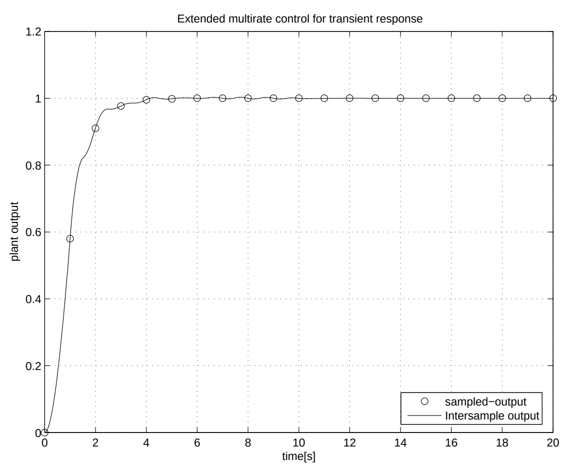

Figure 8. Output result (extension for transient response 2)

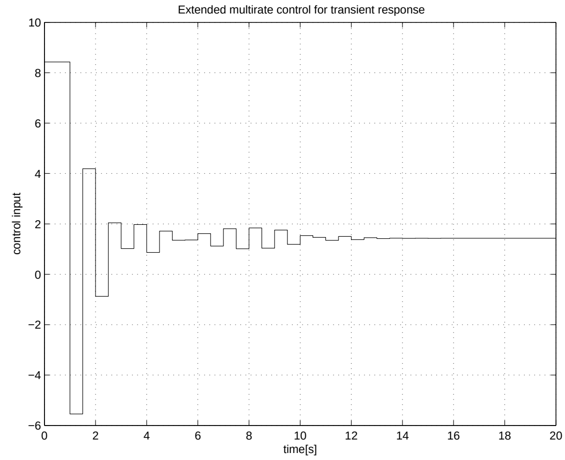

Figure 9. Input result (extension for transient response 2)

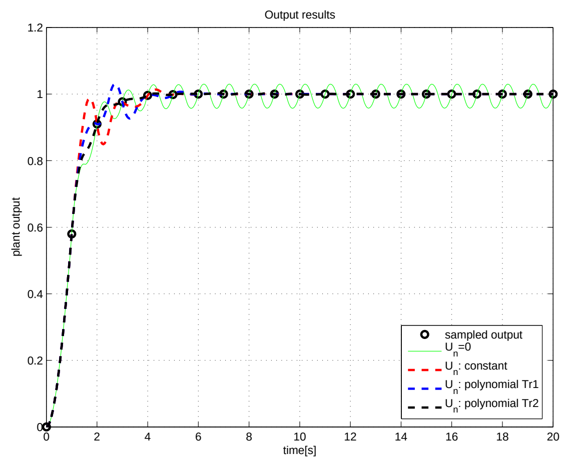

Figure 10. All output results

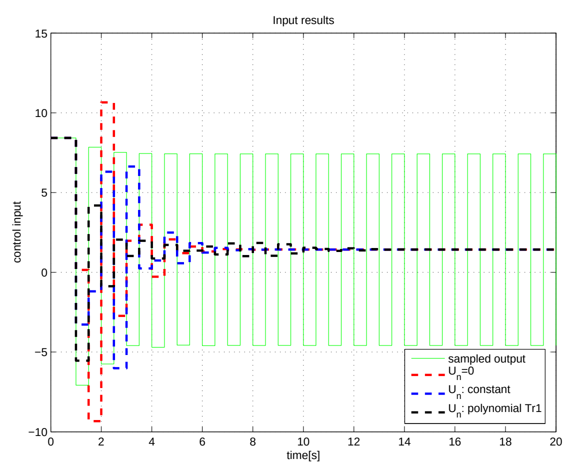

Figure 11. All input results