Dynamics of Spacecraft Guidance and Spin-up of the Gyrodine's Rotors at Pulse-width Control ⋆

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Abstract: Attitude motion of a flexible spacecraft at pulse-width control by the jet engines' thrust in process of a spatial attitude guidance and next spin-up rotors of six gyrodines, is considered. Results on analysis of the spacecraft guidance, nutation and flexible oscillations are presented. Guidelines according to a logic in sequence of the gyrodine's rotors spin-up for reduction of influence by the nutation oscillations, are represented.

Keywords: spacecraft. attitude control, jet engine, pulse-width modulation

1. INTRODUCTION

For large-scale spacecraft (SC) the structure oscillations can render an essential influence on its spatial motion. It is especially for initial damping mode, at the SC initial guidance on the Sun and on the Earth and next spin-up rotors of control moment gyros, single-gimbal control moment gyros – gyrodines (GDs) which contain the gyro moment clusters (GMCs). Pulse-width modulation (PWM) of the jet engine control in these modes have known advantages (for example, concerning fuel expenditure) and is applied onboard many spacecraft. The paper suggests new results on spatial guidance and the PWM attitude control of the a large-scale flexible spacecraft in initial modes, including a mode by spin-up of the gyrodine's rotors.

2. MATHEMATICAL MODELS

We introduce the inertial reference frame (IRF) $\mathbf{I}_{\oplus}$ ($\mathrm{O}_{\oplus}\mathrm{X}_{\mathrm{e}}^{\mathrm{I}}\mathrm{Y}_{\mathrm{e}}^{\mathrm{I}}\mathrm{Z}_{\mathrm{e}}^{\mathrm{I}}$), standard defined the body reference frame (BRF) $\mathbf{B}$ (Oxyz) with origin in the SC mass center O, the orbit reference frame (ORF) $\mathbf{O}$ ($\mathrm{O}x^{o}y^{o}z^{o}$). The BRF attitude with respect to the IRF is defined by quaternion $\boldsymbol{\Lambda}_{\mathrm{I}}^{b} \equiv \boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \lambda_3\}$, and with respect to the ORF – by vector-column $\boldsymbol{\phi} = \{\phi_i, i = 1 \div 3\}$ of Euler-Krylov angles ϕ_i. Let vectors $\boldsymbol{\omega}(t)$ and $\mathbf{r}(t)$ are standard denotations of the SC body vector angular rate and the SC mass center's position with respect to the IRF. Further the symbols $\langle\cdot,\cdot\rangle$, $\times$, $\{\,\cdot\,\}$, $[\,\cdot\,]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^{\mathrm{t}}$ for matrixes are conventional denotations.

The GMC's angular momentum (AM) vector $\boldsymbol{\mathcal{H}}$ have the form $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = \sum \mathrm{H}_p \mathbf{h}_p(\beta_p)$, where H_p is own AM value for each GD #p, $p = 1 \div m$ with the GD's AM unit $\mathbf{h}_p(\beta_p)$ and vector-column $\boldsymbol{\beta} = \{\beta_p\}$. Collinear pair of two GDs was named as *Scissored Pair Ensemble* (*SPE*) into original work *J.W. Crenshaw* (1973). Redundant multiply scheme, based on six gyrodines in the form of three collinear GD's pairs, was named as *3-SPE*. Fig. 1 presents a simplest arrangement of this scheme into a canonical orthogonal gyroscopic basis $\mathrm{O}x_{\mathrm{c}}^{\mathrm{g}}y_{\mathrm{c}}^{\mathrm{g}}z_{\mathrm{c}}^{\mathrm{g}}$. By a slope of the GD pairs suspension axes in this basis it is possible to change essentially a form of the AM variation domain $\mathbf{S}$ at any direction. Based on 4 GDs the minimal redundant scheme *2-SPE* is easily obtained from the *3-SPE* scheme – without third pair (GD #5 and GD #6). In park state of *3-SPE* scheme one can have the GMC's AM vector $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = \mathbf{0}$, see Fig. 2.

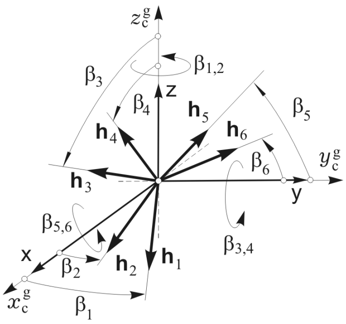

Fig. 1. The scheme *3-SPE*

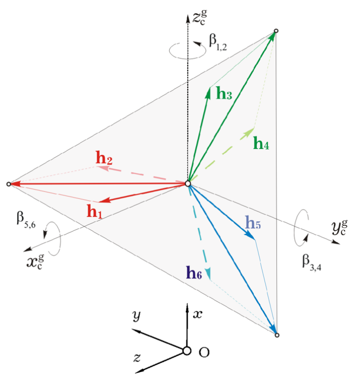

Fig. 2. Park of *3-SPE* scheme

Let column $\mathbf{H} = \{\mathrm{H}_p, p = 1 \div m\}$ presents the GD's own AM values. For a fixed position of the SC flexible solar array panels (SAPs) and the GD's fixed park angular positions $\boldsymbol{\beta} = \boldsymbol{\beta}^{\star}$ at the GMC, with some simplifying assumptions for $t \in \mathrm{T}_{t_0} = [t_0, +\infty)$ a SC angular motion model is appeared as follows:

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} \circ \boldsymbol{\omega}/2; \;\; \mathbf{A}^{o}\,\{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}, \dot{\mathbf{H}}\} = \{\mathbf{F}^{\omega}, \mathbf{F}^{q}, \mathbf{F}^{r}\}. \qquad (1)$$

Here $\boldsymbol{\omega} = \{\omega_i, i = x, y, z \equiv 1 \div 3\}$, $\mathbf{q} = \{q_j, j = 1 \div n^q\}$;
$\mathbf{F}^{\omega} = -\boldsymbol{\omega} \times \mathbf{G} + \mathbf{M}^{\mathrm{e}} + \mathbf{M}_d^{o}(t, \boldsymbol{\Lambda}, \boldsymbol{\omega})$; $\mathbf{G} = \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(t) + \mathbf{D}_q \dot{\mathbf{q}}$;

$$\mathbf{F}^{q} = \{-((\delta^{q}/\pi)\Omega_j^{q}\dot{q}_j + (\Omega_j^{q})^2 q_j)\}; \qquad \mathbf{F}^{r} = \mathbf{M}^{\mathrm{r}} - \mathbf{M}_f^{\mathrm{r}};$$

$\boldsymbol{\mathcal{H}}(t) = \sum \mathrm{H}_p(t)\mathbf{h}_p(\beta_p^{\star}) = \mathbf{D}_r \mathbf{H}(t)$, vector-columns $\mathbf{M}_c^{\mathrm{r}}$ and $\mathbf{M}_f^{\mathrm{r}}$ present the control and the dry friction torques on

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the GD's rotor axes, vector $\mathbf{M}^{\mathrm{e}}$ is the orientation engine unit (OEU) torques, vector $\mathbf{M}_d^o(\cdot)$ is an external torque disturbance, the matrix

$$\mathbf{A}^o = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q & \mathbf{D}_r \\ \mathbf{D}_q^{\mathrm{t}} & \mathbf{I}_{n^q} & \mathbf{0} \\ J_r\mathbf{D}_r^{\mathrm{t}} & \mathbf{0} & \mathbf{I}_m \end{bmatrix},$$

$\mathbf{D}_r = [\mathbf{h}_p(\beta_p^\star)]$, and J_r is the GD rotor's moment of inertia.

The OEU based on six thermal-catalytic jet engines (JEs) with a pulse-width modulation (PWM) of the JE thrust. For the PWM of normalized command τ_r by the thrust inclusion $P^n(t,\tau_r^d) \in \{0,1\}$, $r \in \mathbb{N}_0 \equiv [0,1,2,\ldots)$ by each JE, namely

$$P^n(t,\tau_r^d) = \begin{cases} 1 & t \in [t_r, t_r + \tau_r^d); \\ 0 & t \in [t_k + \tau_r^d, t_{r+1}), \end{cases} \tag{2}$$

the modulation characteristic is described by the ratio

$$\tau_r^d = \begin{cases} 0 & \tau_r < \tau_{\mathrm{m}}; \\ \tau_r & \tau_{\mathrm{m}} \le \tau_r < \tau^{\mathrm{m}}; \\ \tau^{\mathrm{m}} & \tau^{\mathrm{m}} \le \tau_r < T_u^{\mathrm{e}}; \\ T_u^{\mathrm{e}} & \tau_r > T_u^{\mathrm{e}}. \end{cases} \tag{3}$$

Taking into account a transport delay T_{zu}^d dynamic processes on the normalized thrust $P_d^n(t)$ for each JE are presented by the differential equation

$$T^{\mathrm{d}}\,\dot P_d^n + P_d^n = P^n(t - T_{zu}^d, \tau_r^d)$$

with the initial condition $P_d^n(t_0) = 0$, where a time constant T^d accepts two values T_+^d or T_-^d according to the ratio:

$$if\, P^n = 1 \; then\; T^d = T_+^d \; else \; T^d = T_-^d.$$

For everyone j-th JE $D_j, j = 1 \div 6$ there is compared the vector $\mathbf{P}_j(t) = P^{\mathrm{m}}\, P_d^n(t)\, \mathbf{p}_j$ of the current jet thrust with fixed unit $\mathbf{p}_j$ beginning in a point $\mathrm{O}_j^{\mathrm{d}}$, where P^{m} is the current maximal thrust value, identical for all JEs. The point $\mathrm{O}_j^{\mathrm{d}}$ arrangement is defined by a radius-vector $\boldsymbol{\rho}_j$. The OEU control torques concerning axes Ox, Oy and Oz are created by JEs' pairs. Logic of the command τ_{jr} formation for inclusion everyone j-th JE takes into account a sign of a command signal v_{ir} on channel $i = x,y,z$ and is described by such algorithm: $\tau_{ir} = |\mathrm{v}_{ir}|$; $\mathrm{s}_{ir} = \mathrm{sign}\, \mathrm{v}_{ir}$; $i = x,y,z$ and then, for example for $i = x$:

$$if\; \mathrm{s}_{xr} > 0\, then\, (\tau_{1r} = \tau_{xr} \& \tau_{2r} = 0)\, else\, (\tau_{1r} = 0 \& \tau_{2r} = \tau_{xr}).$$

Formed by the OEU the control torque vector $\mathbf{M}^{\mathrm{e}}$ is calculated by formula $\mathbf{M}^{\mathrm{e}} \equiv \mathbf{M} = \sum \boldsymbol{\rho}_j^d \times \mathbf{P}_j$.

At initial the SC damping, guidance on the Sun and on the Earth, signals of a block of angular rate sensors (ARSs) and the GPS/GLONASS navigation signals with period $T_q^{\mathrm{e}} = 1\,s$ are applied for forming the OEU width-pulse control with period $T_u^{\mathrm{e}} = 4\,s$. Because of the SC small measuring base at the SC attitude determination by the GPS/GLONASS navigation signals, the accuracy is poor, $\approx 0^\circ.5$. But that accuracy is enough for initial the SC guidance on the Sun and on the Earth and for next the SC attitude stabilization into the ORF by the OEU during the gyrodine rotors' spin-up and initial preparing the SC strapdown inertial system with astronomical correction.

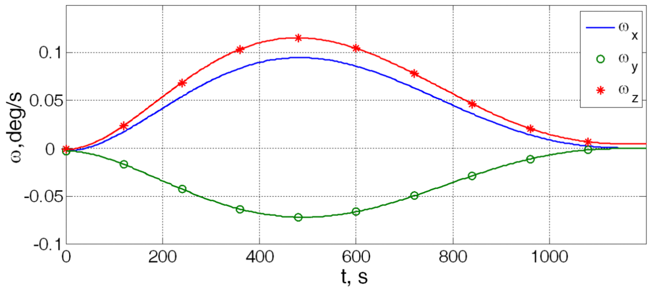

Fig. 3. Programmed values of the angular rate vector

3. THE PROBLEM STATEMENT

After separating a SC from buster and disclosing the SAPs at any time moment $t = t_0$ the angular rate vector accepts a value $\boldsymbol{\omega}(t_0) \in \mathbf{S}_\omega$ from the bounded convex domain $\mathbf{S}_\omega$. Let a constant command angular rate vector $\boldsymbol{\omega}^{\mathrm{c}} = \{\omega_i^{\mathrm{c}}\}$ is given. First problem consists in synthesis of the OEU width-pulse control so that components ω_i^{c} should be reached with given accuracy $|\omega_i(t) - \omega_i^{\mathrm{c}}| \le \delta_\omega\, \forall t \ge t_0 + T^{\mathrm{r}}$ for some acceptable duration T^{r} of damping mode.

For initial the SC guidance simultaneously on the Sun and on the Earth the SC's spatial rotation maneuver (SRM) is needed. Into the IRF the SC's SRM is described by kinematic relations

$$\dot{\boldsymbol{\Lambda}}(t) = \frac{1}{2}\boldsymbol{\Lambda}\circ\boldsymbol{\omega}(t);\; \dot{\boldsymbol{\omega}}(t) = \boldsymbol{\varepsilon}(t);\; \dot{\boldsymbol{\varepsilon}}(t) = \mathbf{v}, \tag{4}$$

where $\dot{\boldsymbol{\varepsilon}}(t) \equiv \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$, during given time interval T_p, e.g. $\forall t \in \mathrm{T}_p \equiv [t_{\mathrm{i}}^p, t_{\mathrm{f}}^p]$, $t_{\mathrm{f}}^p \equiv t_{\mathrm{i}}^p + T_p$. Second problem consists in determination of time functions $\boldsymbol{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ for the boundary conditions on left $(t = t_{\mathrm{i}}^p)$ and right $(t = t_{\mathrm{f}}^p)$ trajectory ends

$$\boldsymbol{\Lambda}(t_{\mathrm{i}}^p) = \boldsymbol{\Lambda}_{\mathrm{i}};\; \boldsymbol{\omega}(t_{\mathrm{i}}^p) = \boldsymbol{\omega}_{\mathrm{i}};\; \boldsymbol{\varepsilon}(t_{\mathrm{i}}^p) = \boldsymbol{\varepsilon}_{\mathrm{i}}; \tag{5}$$

$$\boldsymbol{\Lambda}(t_{\mathrm{f}}^p) = \boldsymbol{\Lambda}_{\mathrm{f}};\; \boldsymbol{\omega}(t_{\mathrm{f}}^p) = \boldsymbol{\omega}_{\mathrm{f}};\; \boldsymbol{\varepsilon}(t_{\mathrm{f}}^p) = \boldsymbol{\varepsilon}_{\mathrm{f}}. \tag{6}$$

Starting from the time t_{f}^p the SC body is stabilized into the ORF by the OEU with the PWM of the JE thrust.

When the GD's rotors begin to spin-up, the OEU control torques successfully compensate the disturbing reaction torques, which appear by accelerations of the rotors. For preliminary research we will consider a case when GMC's AM vector $\boldsymbol{\mathcal{H}}$ is a constant vector parameter. Then linearized model of attitude motion for the SC as solid is represented in the form

$$\dot{\boldsymbol{\phi}} = \boldsymbol{\omega} - [\boldsymbol{\omega}^o\times]\boldsymbol{\phi};\; \mathbf{J}\dot{\boldsymbol{\omega}} = ([(\mathbf{J}\boldsymbol{\omega}^o + \boldsymbol{\mathcal{H}})\times] - [\boldsymbol{\omega}^o\times]\mathbf{J})\boldsymbol{\omega} + \mathbf{M}. \tag{7}$$

Here a constant vector $\boldsymbol{\omega}^o = \{0, 0, \omega_o^o\}$ represents an orbital angular rate of the SC mass' center into the ORF.

It is obvious that when sufficient energy onboard possibilities are present, rationally to use a simultaneous acceleration of rotors for all six GDs. Both the disturbing reaction torques and the SC nutation oscillations are not appeared at this case: $\boldsymbol{\mathcal{H}} \equiv \mathbf{0}$. Problem arises at absence such energy possibilities, when the sequential acceleration of the rotors is possible only for pairs of GDs. So, this problem consists in selection of logic by the sequential acceleration of the GDs' rotors in pairs with respect to the SC nutation oscillations, namely: to what values of own AM for each GD in the pair one should accelerate, e.g. the rotor angular rates directly derive to their nominal value or with certain step, consecutively on the pairs ?

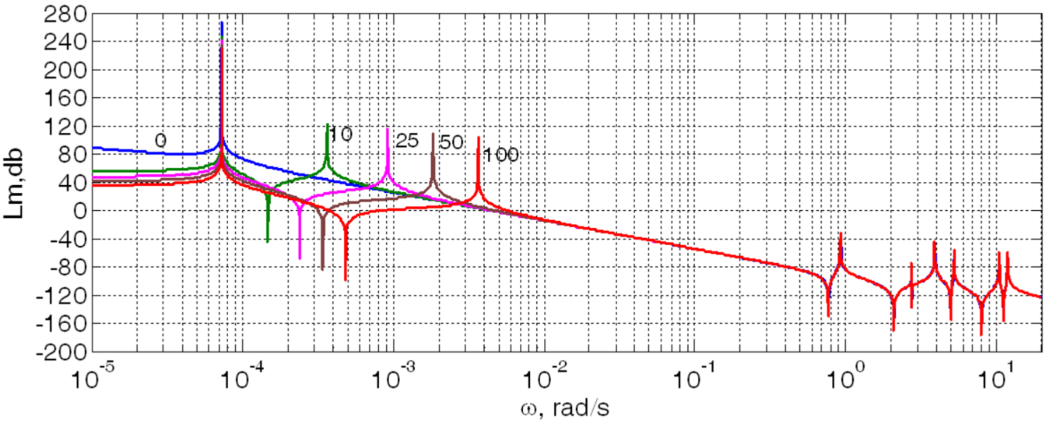

Fig. 4. Logarithmic amplitude frequency characteristics on yaw channel with different GD's AM values h1

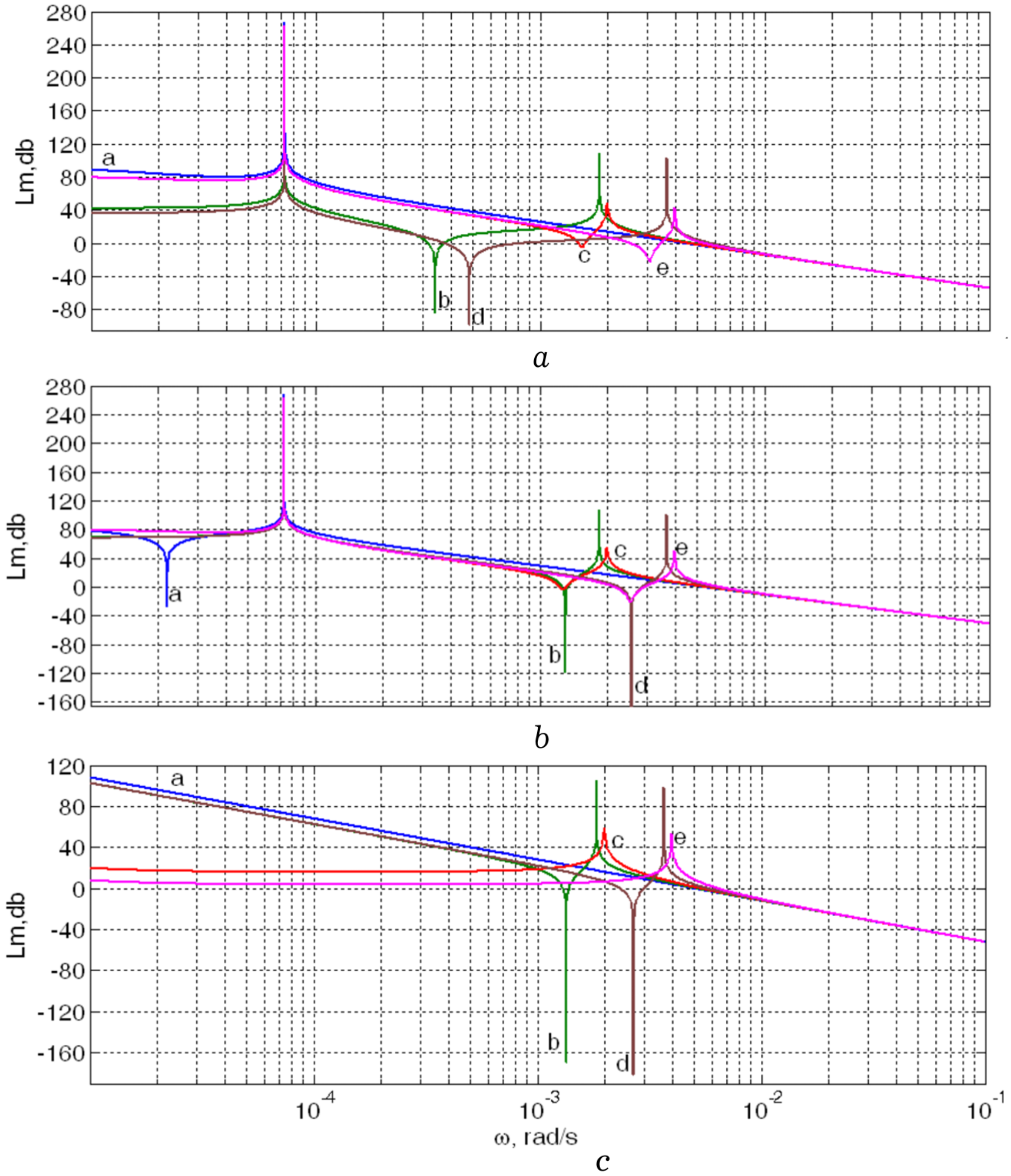

Fig. 5. Logarithmic amplitude frequency characteristics with different GD's AM values h1, h2 and h3: *a* — on yaw channel; *b* — on roll channel; *c* — on pith channel.

4. A SPATIAL ROTATION MANEUVER

Developed analytical approach is based on necessary and sufficient condition for solvability of Darboux problem. At general case the solution is presented as result of composition by three ($k = 1 \div 3$) simultaneously derived elementary rotations of embedded bases $\mathbf{E}_k$ about units $\mathbf{e}_k$ of Euler axes, which positions are defined from the boundary conditions (5) and (6) for initial spatial problem. For all 3 elementary rotations with respect to units $\mathbf{e}_k$ the boundary conditions are analytically assigned. Into the IRF $\mathbf{I}_\oplus$ the quaternion $\mathbf{\Lambda}(t)$ is defined by the production

$$\mathbf{\Lambda}(t) = \mathbf{\Lambda}_{\rm i} \circ \mathbf{\Lambda}_1(t) \circ \mathbf{\Lambda}_2(t) \circ \mathbf{\Lambda}_3(t), \tag{8}$$

where $\mathbf{\Lambda}_k(t) = (C(\varphi_k(t)/2), S(\varphi_k(t)/2)\mathbf{e}_k)$, $C(\alpha) \equiv \cos\alpha$, $S(\alpha) \equiv \sin\alpha$, and functions $\varphi_k(t)$ present the elementary rotation angles in analytical form.

Let the quaternion $\mathbf{\Lambda}^* \equiv (\lambda_0^*, \boldsymbol{\lambda}^*) = \tilde{\mathbf{\Lambda}}_{\rm i} \circ \mathbf{\Lambda}_{\rm f} \neq \mathbf{1}$ have the Euler axis unit $\mathbf{e}_3 = \boldsymbol{\lambda}^*/S(\varphi^*/2)$ by 3-rd elementary rotation where angle $\varphi^* = 2\arccos(\lambda_0^*)$. For the rotations there are applied next the boundary quaternion values:

$$\begin{aligned} &\mathbf{\Lambda}_1(t_{\rm i}^p) = \mathbf{\Lambda}_1(t_{\rm f}^p) = \mathbf{\Lambda}_2(t_{\rm i}^p) = \mathbf{\Lambda}_2(t_{\rm f}^p) = \mathbf{1};\\ &\mathbf{\Lambda}_3(t_{\rm i}^p) = \mathbf{1};\ \mathbf{\Lambda}_3(t_{\rm f}^p) = (C(\varphi_3^f/2), \mathbf{e}_3 S(\varphi_3^f/2)), \end{aligned} \tag{9}$$

where $\varphi_3^f = \varphi^*$ and $\mathbf{1}$ is a single quaternion. Unit $\mathbf{e}_1$ of 1-st elementary rotation's on Euler's axis is selected by simple algorithm, then unit $\mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1$ is defined. All vectors $\boldsymbol{\omega}_k(t) = \dot{\varphi}_k(t)\mathbf{e}_k$, $\boldsymbol{\varepsilon}_k(t) = \ddot{\varphi}_k(t)\mathbf{e}_k$ and $\dot{\boldsymbol{\varepsilon}}_k(t) = \dddot{\varphi}_k(t)\mathbf{e}_k$ have analytic presentations.

Fig. 3 presents programmed values of the angular rate vector during time $t \in \mathrm{T}_p = [0, T_p]$ with $T_p = 1200$ s and next boundary conditions:

$$\begin{aligned} \mathbf{\Lambda}_{\rm i} &= (0.31654115192627, 0.68415660285153,\\ &\quad -0.43738057165327, -0.49033629016431);\\ \mathbf{\Lambda}_{\rm f} &= (0.04035668828809, 0.19121562479099,\\ &\quad -0.95957861048741; -0.20252607941019);\\ \boldsymbol{\omega}_{\rm i} &= \{-0.00283, -0.0028, -0.00107\}^\circ/s;\\ \boldsymbol{\omega}_{\rm f} &= \{0., 0., 0.00417\}^\circ/s;\\ \boldsymbol{\varepsilon}_{\rm i} &= \{0., 0., 0.\}^\circ/s^2; \quad \boldsymbol{\varepsilon}_{\rm f} = \{0., 0., 0.\}^\circ/s^2. \end{aligned}$$

5. A FREQUENCY ANALYSIS

Frequencies of the SC nutational oscillations depend on the relationship between the SC moments of the inertia along the channels and on the value of a constant vector parameter $\boldsymbol{\mathcal{H}}$, i.e. on module and direction of the GMC AM vector in the BRF. In general case the nutation oscillations seize immediately all three channels of the SC orbital stabilization. For model (7) with the SC diagonal tensor $\mathbf{J}$, it is simple to obtain explicit analytical relations for calculating the frequencies of the SC nutation oscillations.

At traditional period $T_u^{\rm e} = 4$s of the PWM thrust control for the practical relationships between the SC moments of inertia and the module of vector $\boldsymbol{\mathcal{H}}$ the modulation frequency of control (0.25 Hz) and frequency of the SC nutation oscillations are distinguished not less than to 3 orders. Therefore for preliminary frequency analysis of the SC nutation oscillations it is completely acceptable to examine the continuous forming the control vector $\mathbf{M}$ in (1) and (7). For this purpose these models are presented in standard continuous form, which makes their possible for obtaining the logarithmic amplitude frequency characteristics for any combination of components of the control vector $\mathbf{M} = \{\mathrm{M}_x, \mathrm{M}_y, \mathrm{M}_z\}$ and components of the output column-vector $\boldsymbol{\phi} = \{\phi_1, \phi_2, \phi_3\}$, comprised of the yaw, roll and pitch angels, respectively. Numerical results were obtained by software system *MatLab*.

We will use symbols h1, h2 and h3 for the fixed values of own AM for each GD in the first, the second and in third pair, respectively. For certainty we will consider that nominal AM value for each of six GDs is equal $h_g = 100$ Hms, moreover values h1, h2 and h3 can increase with steps of 10, 25 or 50 Hms up to the value $h_g = 100$ Hms.

Let normed to h_g the SC tensor $\mathbf{J} = \mathrm{diag}\{521, 353, 400\}$ and flexibility of the SC structure is considered by 11 tones of oscillations. Logarithmic amplitude frequency characteristics on the yaw channel with different GD's AM values h1 in 1st pair and fixed AM h2=0 and h3=0 in 2nd and 3rd pair for flexible SC model are presented in Fig. 4. Fig. 5 represent such frequency characteristics on all 3 channels for next combination of the GD AM values:

a	h1 = 0;	h2 = 0;	h3 = 0;
b	h1 = 50;	h2 = 0;	h3 = 0;
c	h1 = 50;	h2 = 50;	h3 = 0;
d	h1 = 100;	h2 = 0;	h3 = 0;
e	h1 = 100;	h2 = 100;	h3 = 0.

The straight comparison of numerical results along all channels make it possible to obtain the following qualitative conclusions:

(1) resonance frequency of oscillations, which corresponds to orbital angular rate, remains practically constant;
(2) at an autonomous increasing own AM into each GD pair as the parameter, the resonance frequencies of the SC nutation oscillations also increase, which can create some problems on the SC orbital stabilization taking into account the flexibility of its structure;
(3) frequency characteristics of the SC models as rigid and as flexible bodies are distinguished only in the region of sufficiently high frequencies.

The sequential "accelerations" of the GD pairs in two combinations were carried out:

1) the GD rotor acceleration of 1st pair, further the GD rotor acceleration of 2nd pair and at last the GD rotor acceleration of 3rd pair;

2) the GD rotor acceleration of the 3rd pair, further the GD rotor acceleration of the 2nd pair and at last the GD rotor acceleration of of 1st pair.

These combination are close, insignificant quantitative difference is caused principally by the fact that for the beginning 2nd sequence of "accelerations" from 3rd GD pair there appears the AM projection on negative direction of the pitch axis.

6. FILTERING AND CONTROL

For continuous forming the control torque $\mathbf{M}(t)$ at $\boldsymbol{\mathcal{H}} = \mathbf{0}$ and the SC model as a free rigid body the simplified object is such:

$$\dot{\mathbf{\Lambda}} = \mathbf{\Lambda} \circ \boldsymbol{\omega}/2; \quad \mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]\mathbf{J}\boldsymbol{\omega} = \mathbf{M}. \tag{10}$$

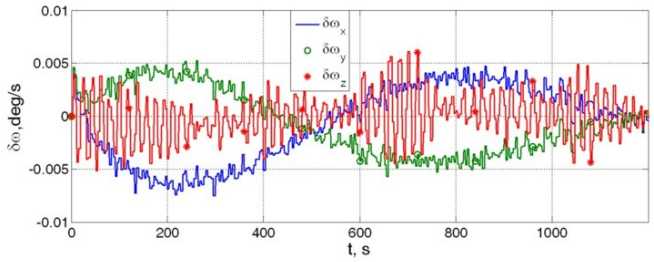

Fig. 6. Errors at realization of programmed angular rate

Let functions $\mathbf{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$ and $\boldsymbol{\varepsilon}^p(t) = \dot{\boldsymbol{\omega}}^p(t)$ represent the SC angular programmed motion. The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\mathbf{\Lambda}}^p(t) \circ \mathbf{\Lambda}$, Euler parameters' vector is $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e$, where $\mathbf{Q}_e \equiv \mathbf{Q}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 e_0 + [\mathbf{e}\times]$ with $\det(\mathbf{Q}_e) = e_0 \neq 0$. If error in the rate vector is defined as $\delta\boldsymbol{\omega} \equiv \tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \mathbf{C}_e \boldsymbol{\omega}^p(t)$, and required control torque vector $\mathbf{M}$ is formed as

$$\mathbf{M} = [\boldsymbol{\omega}\times]\mathbf{J}\boldsymbol{\omega} + \mathbf{J}(\mathbf{C}_e \dot{\boldsymbol{\omega}}^p(t) - [\boldsymbol{\omega}\times]\mathbf{C}_e \boldsymbol{\omega}^p(t) + \tilde{\mathbf{m}}), \quad (11)$$

then the simplest nonlinear model of the SC's attitude error is as follows:

$$\dot{e}_0 = -\langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle /2; \quad \dot{\mathbf{e}} = \mathbf{Q}_e \tilde{\boldsymbol{\omega}}/2; \quad \dot{\tilde{\boldsymbol{\omega}}} = \tilde{\mathbf{m}}. \quad (12)$$

For model (12) a *non-local nonlinear* coordinate transformation is defined and applied at analytical synthesis by the exact feedback linearization technique. This results in the nonlinear continuous control law

$$\tilde{\mathbf{m}}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}) = -\mathbf{A}_0 \; \mathbf{e} \; \mathrm{sgn}(e_0) - \mathbf{A}_1 \; \tilde{\boldsymbol{\omega}}, \quad (13)$$

where $\mathbf{A}_0 = ((2a_0^* - \tilde{\omega}^2/2)/e_0)\mathbf{I}_3$; $\mathbf{A}_1 = a_1^* \mathbf{I}_3 - \mathbf{R}_{e\omega}$, $\mathrm{sgn}(e_0) = (1, \text{ if } e_0 \geq 0) \vee (-1, \text{ if } e_0 < 0)$, matrix $\mathbf{R}_{e\omega} = \langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle \mathbf{Q}_e^{\mathrm{t}}[\mathbf{e}\times]/(2e_0)$, and parameters a_0^*, a_1^* are analytically calculated on spectrum $\mathrm{S}_{ci}^* = -\alpha_c \pm j\omega_c$. Simultaneously the Lyapunov function $\boldsymbol{v}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}})$ is analytically constructed for close-loop continuous system (12) and (13).

6.1 Filtering of Discrete Measurements

At given digital control period $T_u = T_u^{\mathrm{e}}$ discrete frequency characteristics are computed via absolute pseudo-frequency

$$\lambda = j \, \frac{2}{T_u} \frac{1 - \exp(j\omega T_u)}{1 + \exp(j\omega T_u)} = \frac{2}{T_u} \mathrm{tg}(\omega T_u/2),$$

where $j \equiv \sqrt{-1}$. For period's multiple n_{q} and a filtering period $T_q = T_u/\mathrm{n_q}$ applied filter have the discrete transfer function $\mathrm{W_f(z_q)} = (1 + \mathrm{b}_1^{\mathrm{f}})/(1 + \mathrm{b}_1^{\mathrm{f}} \mathrm{z_q}^{-1})$, where $\mathrm{b}_1^{\mathrm{f}} \equiv -\exp(-T_q/T_{\mathrm{f}})$ and $\mathrm{z_q} \equiv \exp(\mathrm{s}T_q)$. Measured error quaternion and Euler parameters' vector are $\mathbf{E}_s = (e_{0s}, \mathbf{e}_s) = \tilde{\mathbf{\Lambda}}^p(t_s) \circ \mathbf{\Lambda}_s^{\mathrm{m}}$ and $\boldsymbol{\mathcal{E}}_s = \{e_{0s}, \mathbf{e}_s\}$, and the error filtering is executed by the relations

$$\tilde{\mathbf{x}}_{s+1} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{B}}\mathbf{e}_s; \quad \mathbf{e}_s^{\mathrm{f}} = \tilde{\mathbf{C}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{D}}\mathbf{e}_s, \quad (14)$$

where matrices $\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ have diagonal form with $\tilde{a}_i = -\mathrm{b}_1^{\mathrm{f}}$; $\tilde{b}_i = \mathrm{b}_1^{\mathrm{f}}$; $\tilde{c}_i = -(1 + \mathrm{b}_1^{\mathrm{f}})$ and $\tilde{d}_i = 1 + \mathrm{b}_1^{\mathrm{f}}$.

6.2 Pulse-width Control Law

For initial modes the OEU pulse-width control is applied with period $T_u = T_u^{\mathrm{e}} = 4\,s$ and filtering period $T_q = T_q^{\mathrm{e}} = 1\,s$. At initial damping a forming the discrete command signals v_{ir} for the OEU width-pulse control on channels

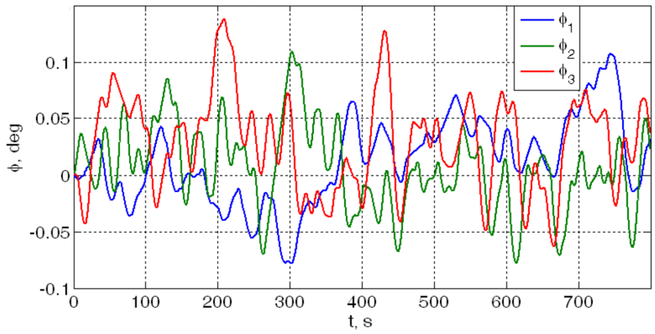

Fig. 7. Attitude errors of the SC orbital stabilization

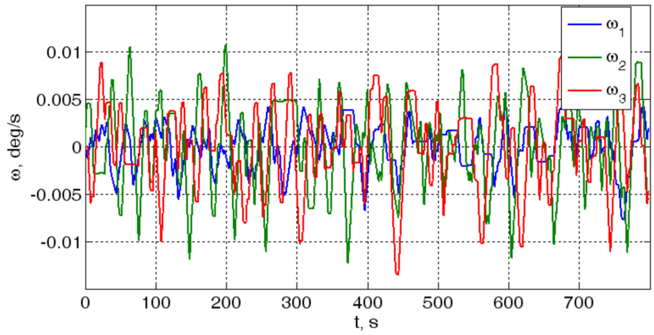

Fig. 8. Rate errors of the SC orbital stabilization

is very simple: $\mathrm{v}_{ir} = \mathrm{k}_i^{\omega}(\omega_i^{\mathrm{c}} - \omega_{ir}^{\mathrm{f}})$. Here ω_{ir}^{f} are filtered measurements of angular rate components and k_i^{ω} are constant gain factors.

At initial the SC guidance simultaneously on the Sun and on the Earth the SC attitude filtered error vector $\mathbf{e}_r^{\mathrm{f}}$ is also applied for forming the OEU width-pulse control $\mathbf{v}_r$. In first, here the stabilizing vector component $\tilde{\mathbf{m}}_r$ is calculated by the relation $\tilde{\mathbf{m}}_r = \mathbf{K}^p \mathbf{e}_r^{\mathrm{f}} + \mathbf{K}^{\omega} \tilde{\boldsymbol{\omega}}_r^{\mathrm{f}}$ with constant diagonal matrixes $\mathbf{K}^p$ and $\mathbf{K}^{\omega}$. Than preliminary vector $\tilde{\mathbf{v}}_r = \{\tilde{\mathrm{v}}_{ir}\}$ is evaluated for forming a required control torque $\mathbf{M}_r(t)$ for $t \in [t_r, t_r + T_u^{\mathrm{e}})$. At last, the command vector $\mathbf{v}_r = \{\mathrm{v}_{ir}\}$ is calculated by next simple algorithm:

$$\mathrm{q}_r = \max|\tilde{\mathrm{v}}_{ir}|, i = 1 \div 3; if \; \mathrm{q}_r > 0 \, then \; \mathrm{v}_{ir} = T_u^{\mathrm{e}} \tilde{\mathrm{v}}_{ir}/\mathrm{q}_r.$$

7. COMPUTER SIMULATION

Some results on nonlinear dynamics of pulse-width attitude control by a flexible spacecraft are presented in Fig. 6 (rate errors at the SC spatial guidance), in Fig. 7 and Fig. 8 — errors by the SC attitude orbital stabilization at the "accelerations" of 1st and 2nd GD pairs.

8. CONCLUSION

Results on the SC spatial guidance and analysis of the SC nutation oscillations at spin-up of the GD rotors to their nominal values in the form of several repetitive cycles, are represented. Each cycle is a sequence of the successive "accelerations" of three GD pairs with a certain step on its own AM by each GD. That approach makes possible to reduce the SC nutation frequency without any problem on stability of the SC orbital angular stabilization taking into account flexibility of its structure.