Phase transitions, chaos and dynamical anomalies in long-range models of coupled oscillators

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Long-range interacting systems have been intensively studied in the last years and new methodologies have been developed in the attempt to understand their intriguing features. One of the most promising directions is the combination of statistical mechanics tools and methods adopted in dynamical systems. In particular, phase transitions have been extensively explored in both conservative and dissipative long-range systems. The Hamiltonian mean-field (HMF) model [1] and the Kuramoto model [2] represent two paradigmatic toy models, the former conservative and the latter dissipative, for many real systems with long-range forces and have several interesting applications. Both models share the same order parameter and display a spontaneous phase transition from a homogeneous/incoherent phase to a magnetized/synchronized one.

In this paper we address the chaotic behavior of the synchronization phase transition in the Kuramoto model and we discuss the relationship with analogous features found in the Hamiltonian mean-field (HMF) model. Our results support the connection between the two models, which can be considered as limiting cases (dissipative and conservative, respectively) of a more general dynamical system of damped/driven coupled pendula [3]. We also discuss the similarities of the dynamical anomalies found in the out-of-equilibrium regime of the two models and address the possible application of generalized Tsallis statistics [4] to explain this anomalous behavior [5].

In the following as an example of the various similarities we have found, we show the behavior of the Largest Lyapunov Exponent (LLE) for the HMF model as a function of the energy per particle U (see Fig.1) and for the Kuramoto model as a function of the coupling parameter (see Fig.2). In both cases the LLE has a peak just before the phase transition which is of second order type.

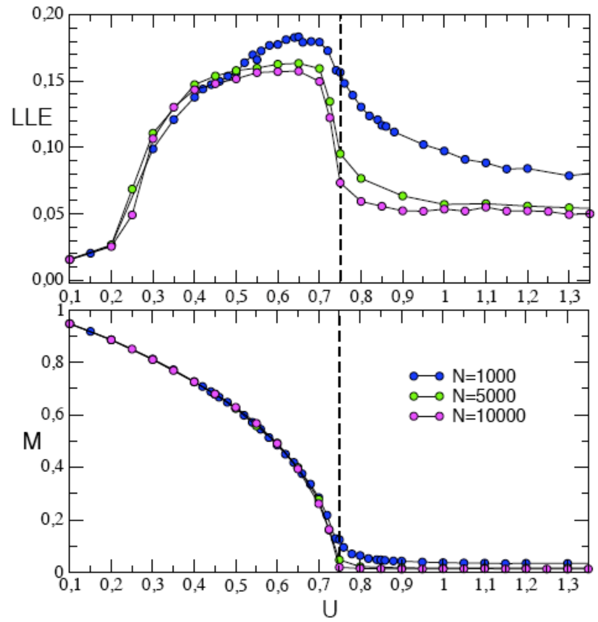

Fig. 1: (Lower panel) The modulus of the order parameter the magnetization M of the HMF model is plotted as a function of the energy density U for various system sizes at equilibrium: N = 1000, 5000, 10000;
(Upper panel) Numerical calculation of the equilibrium largest Lyapunov exponent (LLE) as a function of U for the same system sizes.

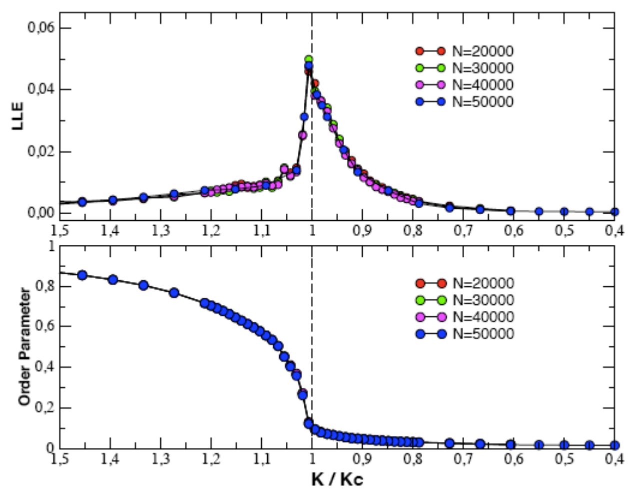

Fig. 2: (Lower panel) The asymptotic order parameter r of the Kuramoto model is plotted as a function of the ratio K/ K_C for several system sizes (N = 20000, 30000, 40000 and 50000) and for a Gaussian distribution of the natural frequencies. Decreasing values of K/K_C are used in order to compare the data with the HMF ones. A 2nd- order-like dynamic al transition from the homogeneous phase to a synchronized one, similar to that one observed in the HMF model (Fig.1), is clearly visible. (Upper panel) The Largest Lyapunov Exponent (LLE) is plotted as function of K/K_C. A well defined peak around the phase transition indicates a microscopic chaotic activity which is maximal at the critical point.

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