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SENSORLESS GENERALIZED $\mathcal{H}_\infty$ OPTIMAL CONTROL OF A MAGNETIC SUSPENSION SYSTEM

Dmitry Balandin
Department of Differential Equations
Lobachevsky State University
of Nizhny Novgorod
Russian
dbalandin@yandex.ru

Mark Kogan
Department of Mathematics
Architecture and Civil
Engineering State University
Russian
mkogan@nngasu.ru

Ruslan Biryukov
Department of Control Theory
Lobachevsky State University
of Nizhny Novgorod
Russian
ruslan.biryukov@itmm.unn.ru

Abstract
An optimal stabilization problem of a body in the electromagnetic suspension is studied. For the linearized system we synthesize the time-invariant output-feedback controller based on the measurement of the current in the solenoid circuit without measuring the position and velocity of the body. A generalized $\mathcal{H}_\infty$-norm of the linearized system is used as the optimality criterion. It characterizes the disturbance attenuation level for both exogenous signals and an uncertain initial state. The controller parameters are computed using linear matrix inequalities (LMIs). Numerical simulation carried out for the nonlinear mathematical model of the magnetic suspension system demonstrates some advantages of the generalized $\mathcal{H}_\infty$ controller over standard ones.

Key words
Generalized $\mathcal{H}_\infty$ control, magnetic suspension system, linear matrix inequalities

1 Introduction

The classical scheme of electromagnetic suspension presupposes the presence of a levitated body position sensor as a main element when forming feedback in the control loop [Schweitzer, 2009]. Improving the electromagnetic suspension design led to the idea of implementing levitation based only on measuring the current in the electromagnet circuit without measuring the position and velocity of the body, so-called sensorless suspension [Mukhopadhyay, 2005; Gruber et al., 2013]. The advantages of this implementation are provided by the greater compactness and reliability of the design and its lower cost compared with the traditional scheme. Usually there are two main approaches to exclude a displacement sensor. The first of them conditionally called algorithmic uses an observer which estimates the position and velocity of the body in real-time. The second one uses an additional high-frequency signal which allows to estimate the electric circuit parameters and calculate the position of the body [Gluck et al., 2010]. This paper is closely related to the first approach.

Optimal stabilization of the body in an electromagnetic suspension with various transient specifications was considered in recent papers [Kumar and Jerome, 2013; Yang Yifei and Zhu Huangqiu, 2013; Hutterer et al., 2014]. H_∞ and H_2 optimal output-feedback controllers were synthesized in [Davoodi, Sedgh and Amirifar, 2008]. An advantage of the H_∞ optimal control is the best disturbance attenuation of external deterministic signals with a bounded L_2-norm, whereas the H_2 optimal control provides the best transient processes for impulsive or random external signals. It would be desirable to combine these performance indices when synthesizing the control of a body in an electromagnetic suspension. Such an approach is used in this paper, where the performance measure of transient processes is the generalized H_∞ norm of the system which takes into account both exogenous disturbances and uncertain initial conditions [Khargonekar, Nagpal and Poolla, 1991; Balandin and Kogan, 2010]. LMI-based technique is used for synthesizing the feedback controller [Boyd et al., 1994; Gahinet and Apkarian, 1994; Iwasaki and Skelton, 1994].

2 Magnetic Suspension System

A schematic representation of the magnetic suspension system is given in Figure 1. It consists of a single electromagnet and the ferromagnetic object of mass m. By controlling the electric voltage U in the coil of the electromagnet, the magnetic field can be regulated such a way that the object will levitate in an equilibrium state when the electromagnetic force will be equal to the weight of the object. The dynamics of the simplest magnetic suspension system is described by the two equations:

$$\begin{aligned} m\ddot{s} &= F - mg, \\ \dot{\Psi} + RI &= U. \end{aligned} \tag{1}$$

The first equation expresses Newton's law and determines the change of the body's coordinate s due to the gravity force mg and the electromagnetic force F. The second equation represents Kirchhoff's law for the electric circuit of the electromagnet, where I is the current in the electromagnet coil, R is the coil's resistance, and Ψ is the flux linkage of the coil. If Φ is the magnetic flux passing through one turn, and n is the number of turns in the coil, then $\Psi = n\Phi$.

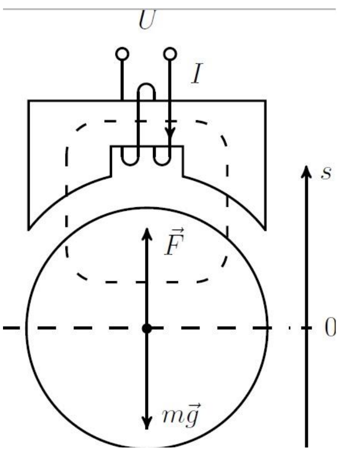

Figure 1. Schematic diagram of the magnetic suspension system.

The flux linkage Ψ and current I are determined by

$$\Psi = L(s)I, \quad L(s) = \frac{C_L}{\delta - s}, \quad C_L = \frac{\mu_0 n^2 A}{2}, \tag{2}$$

where $L(s)$ is the inductance of the electromagnet, C_L is a parameter and δ is the nominal gap between the electromagnet and the levitated body. If we denote the nominal inductance as $L_0 = L(0)$, then $C_L = L_0\delta$, and

$$L(y) = \frac{L_0}{1 - s/\delta}. \tag{3}$$

The magnetic force F can be written using the magnetic energy W as

$$F = \frac{\partial W}{\partial s} = \frac{C_L I^2}{2(\delta - s)^2}, \quad W = \frac{L(s)I^2}{2}. \tag{4}$$

Now substituting the expressions (2) and (4) into (1), we get the final nonlinear motion equation:

$$\begin{aligned} m\ddot{s} &= \frac{C_L I^2}{2(\delta - s)^2} - mg, \\ \frac{C_L}{\delta - s}\frac{dI}{dt} &+ \frac{C_L}{(\delta - s)^2} I\dot{s} + RI = U. \end{aligned} \tag{5}$$

When $U = U_0 = R\sqrt{2\delta^2 mg/C_L}$ the system (5) has a single equilibrium point $s = 0$, $I = I_0 = U_0/R$, which is unstable. Denote

$$I = I_0 + I_v, \qquad U = U_0 + U_v, \tag{6}$$

where I_v and U_v are time varying parts of I and U, respectively. The linearized equations of (5) are of the form

$$\begin{aligned} m\ddot{s} &= c_s s + h_i I_v, \\ L_0\frac{dI_v}{dt} &+ h_i\dot{s} + RI_v = U_v, \end{aligned} \tag{7}$$

where

$$c_s = \frac{C_L}{\delta^3} I_0^2 = \frac{L_0}{\delta^2} I_0^2, \quad h_i = \frac{C_L}{\delta^2} I_0 = \frac{L_0}{\delta} I_0. \tag{8}$$

We make the change of variables

$$t = Tt', \; s = \delta x_1, \; I_v = I_0 x_3, \; T = \sqrt{m/c_s} \tag{9}$$

to reduce the linearized system to the following dimensionless form omitting the prime symbol

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_1 + x_3 + w_1, \\ \dot{x}_3 &= -x_2 - ax_3 + u + w_2, \end{aligned} \tag{10}$$

where

$$a = \frac{RT}{L_0}, \qquad u = \frac{U_v T}{L_0 I_0}. \tag{11}$$

It should be noted that the physical meaning of the parameter a is the ratio of the characteristic time T of the "mechanical" part to the characteristic time L_0/R of the "electrical" part of the system. Typical values of the parameter are $a = 5 \div 10$. The exogenous disturbances w_1 and w_2 stand instead of nonlinear terms in the system model. The state of a system (10) is often not available for a measurement. Further, we will assume that the current in the solenoid circuit is available for measurement only, i.e.

$$y = x_3 + w_3, \tag{12}$$

where w_3 is the output measurement error.

3 Problem Formulation

Rewrite the system (10), (12) as follows

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \qquad x(0) = x_0, \\ y &= C_2 x + D_2 w, \end{aligned} \tag{13}$$

where $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ is the state of the system, $y \in \mathbb{R}^{n_y}$ is the measurable output, $w = (w_1,\, w_2,\, w_3) \in \mathbb{R}^{n_v}$ is the exogenous input, and $u \in \mathbb{R}^1$ is the control input. It is assumed that the exogenous disturbance $w(t) \in L_2[0, +\infty)$ and that the plant initial state x_0 is unknown. If the current in the coil of the electromagnet can be measured, then $n_y = 1$ and the matrices in (13) have the following form

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & -a \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0\,0\,0 \\ 1\,0\,0 \\ 0\,1\,0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} 0\;0\;1 \end{pmatrix}, \qquad D_2 = \begin{pmatrix} 0\;0\;1 \end{pmatrix}.$$

Consider the controlled output $z \in \mathbb{R}^{n_z}$ to describe transient processes occurring in the system:

$$z = C_1 x + D_1 u, \tag{14}$$

where

$$C_1 = \begin{pmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \\ 0\,0\,0 \end{pmatrix}, \qquad D_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Therefore the square of the L_2-norm of the controlled output is a quadratic cost of the state components and control

$$\|z\|^2 = \int_0^\infty \left(x_1^2 + x_2^2 + x_3^2 + u^2\right) dt.$$

Now suppose that the initial state of the system (13), (14) is zero. To evaluate the response of the object to the disturbance w, we define the worst case performance measure J_1 by the formula:

$$J_1(u) = \sup_{w \neq 0} \frac{\|z\|}{\|w\|}. \tag{15}$$

Note that for a fixed control law u this cost coincides with $\mathcal{H}_\infty$-norm of the system (13), (14).

However, there exist situations when exogenous disturbance does not act to the system and its initial state is possibly nonzero and unknown. The nonzero initial state causes an additional unknown so-called initial disturbance. Define the worst case performance measure J_2 as follows [Balandin and Kogan, 2008, 2009]:

$$J_2(u) = \sup_{x_0 \neq 0} \frac{\|z\|}{|x_0|}. \tag{16}$$

Finally, if both types of the disturbances act to the system (13), (14), we define the performance measure as the worst-case norm of the controlled output over all admissible exogenous disturbances and initial states:

$$J(u) = \sup_{\|w\|^2 + \rho^2 |x_0|^2 \neq 0} \frac{\|z\|}{\sqrt{\|w\|^2 + \rho^2 |x_0|^2}}, \tag{17}$$

where $\rho \geqslant 0$ is a given weighting coefficient [Balandin and Kogan, 2010]. Obviously, when $x_0 = 0$ the cost J is reduced to the cost (15), and for $w = 0$ we obtain $J = J_2(u)/\rho$. In addition, the following properties were proved in [Balandin and Kogan, 2010]:

$$\lim_{\rho \to 0} \rho\, J(u) = J_2(u), \qquad \lim_{\rho \to \infty} J(u) = J_1(u).$$

Therefore, the parameter ρ is a measure of the importance of the unknown initial conditions x_0 over exogenous disturbance w. For this reason the cost (17) is a trade-off between performance measures J_1 and J_2 [Khargonekar, Nagpal and Poolla, 1991; Balandin and Kogan, 2010].

Now we are ready to formulate two control problems associated with the cost (17). The first is a state-feedback problem.

Problem 1. For the system (13), (14), find a stabilizing state-feedback control law

$$u = \Theta x, \qquad \Theta = \begin{pmatrix} \theta_1\; \theta_2\; \theta_3 \end{pmatrix}, \tag{18}$$

minimizing the cost (17). If the initial state of the object is zero, then the formulated problem is the $\mathcal{H}_\infty$-optimal control problem. In the opposite case, when an external disturbance w does not act on the object, the problem is equivalent to finding the minimal γ_0 for which the following inequality holds:

$$\inf_u \|z\|^2 = \inf_u \int_0^\infty \left(x^\top C_1^\top C_1 x + u^\top D_1^\top D_1 u\right) \leq$$
$$\leq \gamma_0^2 \rho^2 |x_0|^2, \quad \forall x_0 \neq 0.$$

This minimization problem is the classical linear-quadratic control problem. It is known that its optimal value depends on the initial conditions x_0 and is written as $x_0^\top X x_0$, where the matrix X is a stabilizing solution of the Riccati equation

$$A^\top X + XA - XB_2B_2^\top X + C_1^\top C_1 = 0.$$

At the same time, the value of the cost J_2 does not depend on x_0, i.e.

$$\frac{x_0^\top X x_0}{|x_0|^2} \leqslant \gamma_0^2 \rho^2, \quad \forall x_0 \neq 0.$$

Such a reformulation of the linear-quadratic control problem in terms of the attenuation level of the initial disturbances is important in the case of the output-feedback controller design.

Problem 2. For the system (13), (14), find a stabilizing full order dynamic output-feedback controller described by the equations

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r y, \quad x_r(0) = 0, \\ u &= C_r x_r + D_r y. \end{aligned} \tag{19}$$

The generalized $\mathcal{H}_\infty$-suboptimal controller is defined by inequality $J(u) < \gamma$ for a given $\gamma > 0$. Respectively we say that the control law u^* is the generalized $\mathcal{H}_\infty$-optimal if

$$u^* = \arg\inf_u J(u), \qquad \gamma^* = J(u^*).$$

Similarly, using the attenuation level of the initial disturbances $J_2(u)$ and the attenuation level of the exogenous disturbances $J_1(u)$, we define γ_0- and $\mathcal{H}_\infty$-suboptimal controllers.

4 Control System Design

In this section, we present some results which will be applied for solving the problems indicated in the previous section. More details can be found in [Balandin and Kogan, 2010].

Consider the linear system defined by (13), (14). The design of linear state-feedback control law of the form (18), such that the value $J(u)$ of the closed-loop system is less than a given $\gamma > 0$, is based on the theorem.

Theorem 1 ([Balandin and Kogan, 2010]). *The generalized $\mathcal{H}_\infty$ state-feedback controller exists for a given γ if and only if the LMIs*

$$\begin{bmatrix} YA^\top + AY + B_2Z + Z^\top B_2^\top & \star & \star \\ B_1^\top & -\gamma^2 I & \star \\ C_1Y + D_1Z & 0 & -I \end{bmatrix} < 0, \tag{20a}$$

$$\begin{bmatrix} Y & \star \\ I & \gamma^2\rho^2 I \end{bmatrix} > 0 \tag{20b}$$

are feasible for $(n_u \times n_x)$-matrix Z, and a symmetric positive definite $(n_x \times n_x)$-matrix Y. When such a pair of matrices Z and Y are found, a gain matrix Θ can be computed as $\Theta = ZY^{-1}$.

Note that the optimal value of the cost (17) can be found as a minimal value of γ for which LMIs (20) are feasible in variables Z, $Y = Y^\top > 0$, and $\gamma^2 > 0$. This may be done using standard LMI solvers in Matlab.

Also note that $\mathcal{H}_\infty$-optimal state-feedback controller is computed as solution of (20a), while γ_0-optimal state-feedback controller is found as solution of (20a) where the second row and column are deleted, and (20b) with $\rho = 1$.

Now we turn to design the optimal output-feedback dynamic controller of the form (19). The necessary and sufficient conditions for solvability of the generalized $\mathcal{H}_\infty$ problem are given by following theorem.

Theorem 2 ([Balandin and Kogan, 2010]). *The full order generalized $\mathcal{H}_\infty$ output-feedback controller exists for a given $\gamma > 0$ if and only if the LMIs*

$$M_1^\top \begin{bmatrix} A^\top X_{11} + X_{11}A & \star & \star \\ B_1^\top X_{11} & -\gamma^2 I & \star \\ C_1 & 0 & -I \end{bmatrix} M_1 < 0, \tag{21a}$$

$$M_2^\top \begin{bmatrix} Y_{11}A^\top + AY_{11} & \star & \star \\ C_1Y_{11} & -I & \star \\ B_1^\top & 0 & -\gamma^2 I \end{bmatrix} M_2 < 0, \tag{21b}$$

$$\begin{bmatrix} X_{11} & I \\ I & Y_{11} \end{bmatrix} \geq 0, \tag{21c}$$

$$X_{11} < \gamma^2\rho^2 I \tag{21d}$$

are feasible in $(n_x \times n_x)$-matrices $X_{11} = X_{11}^\top > 0$ and $Y_{11} = Y_{11}^\top > 0$, where

$$M_1 = \begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix}, \quad M_2 = \begin{bmatrix} N_2 & 0 \\ 0 & I \end{bmatrix}$$

and the columns of the matrices N_1 and N_2 form bases of kernels of matrices $(C_2 \;\; D_2)$ and $(B_2^\top \;\; D_1^\top)$, respectively.

In order to synthesize the controller we use the technique described in [Gahinet and Apkarian, 1994; Iwasaki and Skelton, 1994; Balandin and Kogan, 2010]. Suppose that we have solved LMIs (21) for X_{11} and Y_{11}. Define a positive definite matrix

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^\top & X_{22} \end{bmatrix},$$

where the blocks X_{12} and X_{22} can be chosen, for example, as $X_{12} = X_{22} = X_{11} - Y_{11}^{-1}$. After that we collect all controller parameters into the single matrix variable

$$\Theta = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix}$$

and introduce the matrices

$$\Psi = \begin{bmatrix} A_0^\top X + XA_0 & XB_0 & C_0^\top \\ B_0^\top X & -\gamma^2 I & 0 \\ C_0 & 0 & -I \end{bmatrix},$$

$$P = \begin{bmatrix} \mathcal{C} & \mathcal{D}_2 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} \mathcal{B}^\top X & 0 & \mathcal{D}_1^\top \end{bmatrix},$$

where

$$A_0 = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} C_1 & 0 \end{bmatrix},$$

$$\mathcal{B} = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix},$$

$$\mathcal{D}_1 = \begin{bmatrix} 0 & D_1 \end{bmatrix}, \quad \mathcal{D}_2 = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}.$$

Then the required controller can be found as a solution Θ of the LMI

$$\Psi + P^\top \Theta^\top Q + Q^\top \Theta P < 0. \tag{22}$$

Note that $\mathcal{H}_\infty$ output-feedback controller can be computed as solution of (21a) – (21c), while γ_0 output-feedback controller is found as solution of the linear matrix inequalities

$$\begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix}^\top \begin{bmatrix} A^\top X_{11} + X_{11}A & \star \\ C_1 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix} < 0, \tag{23a}$$

$$N_2^\top \begin{bmatrix} Y_{11}A^\top + AY_{11} & \star \\ C_1 Y_{11} & -\gamma^2 I \end{bmatrix} N_2 < 0, \tag{23b}$$

and (21c), (21d) where $\rho = 1$. The columns of the matrix N_1 forms bases of the kernel of the matrix C_2.

5 Simulation Results

We present the results of numerical solution of problems 1 and 2. Using the variables (9), we transform the original nonlinear object (5) to the dimensionless form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{2}\left[\frac{(1+x_3)^2}{(1-x_1)^2} - 1\right], \\ \dot{x}_3 &= -\frac{1+x_3}{1-x_1}x_2 - a(1-x_1)x_3 + (1-x_1)u, \end{aligned} \tag{24}$$

where a and u are defined by (11). Then we substitute the found optimal controllers to the nonlinear system (5). In numerical simulation the parameter a is fixed and equal to 7.5.

Firstly, we consider the solution of the problem 1. To find the optimal feedback gain matrix, we set $\rho^2 = 0.05$ and apply Theorem 1:

1. $\Theta_\infty = \big(-585.3589 \;\; -552.2473 \;\; -64.9617 \big)$;
2. $\Theta_0 = \big(-17.0923 \;\; -16.1215 \;\; -1.9601 \big)$;
3. $\Theta_{\infty,0} = \big(-21.3529 \;\; -20.1411 \;\; -2.4324 \big)$.

Here Θ_∞ is $\mathcal{H}_\infty$-controller gain, Θ_0 is γ_0-controller gain, and $\Theta_{\infty,0}$ is the generalized $\mathcal{H}_\infty$-controller gain. It is seen from the given data that the feedback coefficients for the $\mathcal{H}_\infty$-controller are substantially greater then those of the γ_0-controller. The plots of transient processes in the nonlinear closed-loop system are shown in Figs. 2 and 3. It can be seen that the $\mathcal{H}_\infty$-optimal controller attenuates initial disturbances much worse than other controllers. This is expected, since the $\mathcal{H}_\infty$-optimal controller is designed to attenuate only external disturbances. At the same time, the generalized $\mathcal{H}_\infty$-optimal controller (solid line) provides a trade-off between the $\mathcal{H}_\infty$-optimal and γ_0-optimal controllers.

Now we consider the output-feedback optimal control. We assume that only the current in the coil of the electromagnet is measured. In this case, the output-feedback controller is described by (19) and defined by 16 parameters, unlike the state-feedback controller depending on three parameters. Using the technique described in the previous section, we obtained the following output-feedback gain-matrix for the generalized $\mathcal{H}_\infty$-optimal controller:

$$\Theta_{\infty,0} = \left(\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array} \right) =$$

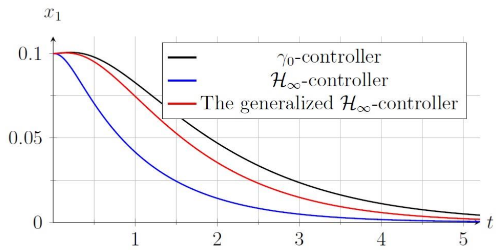

Figure 2. Time-history of position of the body when using a state-feedback γ_0-controller, $\mathcal{H}_\infty$-controller, and generalized $\mathcal{H}_\infty$-controller.

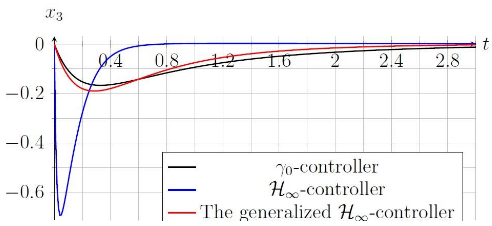

Figure 3. Time-history of current in the coil when using a state-feedback γ_0-controller, $\mathcal{H}_\infty$-controller, and generalized $\mathcal{H}_\infty$-controller.

$$= \left(\begin{array}{ccc|c} -1.437 & -0.355 & 51.027 & 51.222 \\ -0.386 & -1.307 & 49.626 & 48.814 \\ -17.026 & -17.058 & -107.690 & -98.242 \\ \hline 17.365 & 16.378 & 93.639 & 91.644 \end{array}\right).$$

The plots of transient processes in the nonlinear closed-loop system are shown in Figs. 4 and 5. The initial state is chosen as $\big(-0.067,\ 0.0,\ 0.0\big)$. It can be seen from Fig. 5 that the $\mathcal{H}_\infty$-optimal controller leads to rather intensive oscillatory processes, whereas the generalized $\mathcal{H}_\infty$-optimal regulator provides a very good quality of the transient process. Moreover, the Figs. 4 also show some advantage of the generalized $\mathcal{H}_\infty$-optimal controller over γ_0-optimal one.

The quality of the transient processes provided by each controller can be estimated by the ratio $\eta = |Re(\lambda)|_{\max}/|Re(\lambda)|_{\min}$. The smaller this ratio the better the quality of the transients is achieved. For the generalized $\mathcal{H}_\infty$-optimal regulator, we have $\eta_{\infty,0} = 14.3$, for γ_0-optimal controller is $\eta_0 = 93.4$, and for $\mathcal{H}_\infty$-optimal regulator is $\eta_\infty = 166.4$.

6 Conclusion

We have applied a concept of the generalized $\mathcal{H}_\infty$ control to synthesize a sensorless magnetic suspension system. The approach is based on the generalized $\mathcal{H}_\infty$-norm of the linear system and its LMI characterization. The results of numerical simulation have showed the advantages of the generalized $\mathcal{H}_\infty$ controller over the classical $\mathcal{H}_\infty$ and $\mathcal{H}_2$ optimal controllers.

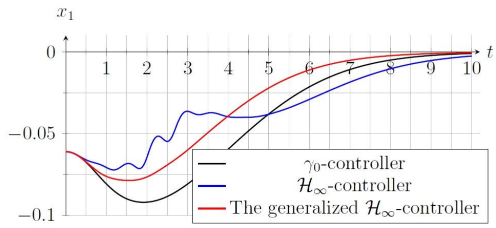

Figure 4. Time-history of position of the body when using a output-feedback γ_0-controller, $\mathcal{H}_\infty$-controller, and generalized $\mathcal{H}_\infty$-controller.

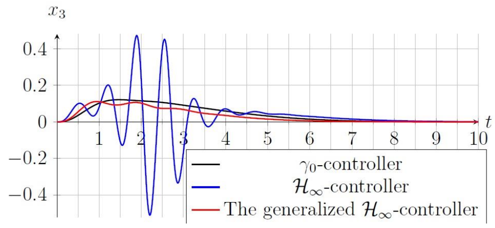

Figure 5. Time-history of current in the coil when using a output-feedback γ_0-controller, $\mathcal{H}_\infty$-controller, and generalized $\mathcal{H}_\infty$-controller.

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References

Schweitzer, G. (2009) *Magnetic Bearings Theory, Design, and Applications to Rotating Machinery*. Berlin: Springer.

Mukhopadhyay, S. C. (2005). Do We Really Need Sensors? A Sensorless Magnetic Bearing Perspective. *1st Intern. Conf. on Sensing Technology.* Palmerston North, New Zealand. pp. 425–431.

Gruber, W., Pichler, M., Rothbock, M. and Amrhein, W. (2013). Self-Sensing Active Magnetic Bearing Using 2-Level PWM Current Ripple Demodulation. *Proc. 7th Intern. Conf. on Sensing Technology.* Wellington, New Zealand. pp. 591–595.

Gluck, T., Kemmetmuller, W., Tump, C. and Kugi, A. (2010). Resistance Estimation Algorithm for Self-Sensing Magnetic Levitation Systems. *Proc. 5th IFAC Symp. on Mechatronic Systems.* Boston, USA. pp. 32-37.

Kumar, V. and Jerome, J. (2013). LQR Based Optimal Tuning of PID Controller for Trajectory Tracking of Magnetic Levitation System. *Procedia Engineering.* **64**. pp. 254-264.

Yang Yifei and Zhu Huangqiu (2013). Optimal Control and $\mathcal{H}_\infty$ Output Feedback Design Options for Active Magnetic Bearing Spindle Position Regulation. *J. Networks.*, **8**. pp. 1624–1631.

Hutterer, M., Hofer, M., Nenning, T. and Schrodl, M.

(2014). LQG Control of an Active Magnetic Bearing with a Special Method to Consider the Gyroscopic Effect. *14th Intern. Symp. on Magnetic Bearings.* Linz, Austria. pp. 54–59.

Davoodi, M., Sedgh, P. K. and Amirifar, R. (2008). $\mathcal{H}_2$- and $\mathcal{H}_\infty$- Dynamic Output Feedback Control of a Magnetic Bearing System via LMIs. *Proc. American Control Conf.* Washington, USA. pp. 2522–2527.

Khargonekar, P. P., Nagpal, K. M. and Poolla, K. R. (1991). $\mathcal{H}_\infty$-Control with Transients. *SIAM J. Control Optim.*, **29** (6), pp. 1373–1393.

Balandin, D.V. and Kogan, M.M. (2010). LMI-Based $\mathcal{H}_\infty$-Optimal Control with Transients. *Intern. J. Control*, **83**(8), pp. 1664–1673.

Boyd, S., El Ghaoui, L., Feron, E. and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, SIAM.

Gahinet, P. and Apkarian, P. (1994). A Linear Matrix Inequality Approach to $\mathcal{H}_\infty$ Control. *Intern. J. Robust and Nonlinear Control*, **4**, pp. 421–448.

Iwasaki T., Skelton R. E. (1994). All Controllers for the General $\mathcal{H}_\infty$ Control Problem: LMI Existence Conditions and State Space Formulas. *Automatica*, **30**, pp. 1307–1317.

Balandin, D. V. and Kogan, M. M. (2008). LMI based output-feedback controllers: γ-optimal versus linear quadratic. *Proc. of 17th World IFAC Congress, Seoul, Korea*, pp. 9905–9909.

Balandin, D. V. and Kogan, M. M. (2009). Revisited LQ Output-Feedback Control: Minimax Controller for a Set of Initial States. *Int. J. Control*, **82**(11), pp. 2051–2058.