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CONTROLLED MOTION OF MECHANICAL SYSTEMS INDUCED BY VIBRATION AND DRY FRICTION

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Abstract.
The paper deals with non-traditional vibration-driven locomotion systems. In the first part, the motion of a chain of interconnected bodies (mass points) along a straight line on a rough surface is considered. The system is subjected to kinematic constraints modeling the excitation mode. It is assumed that there is dry (Coulomb's) friction acting between the plane and each body. The magnitude of the friction force depends on the direction of the motion. The expression for the average velocity of the steady-state motion of the system as a whole is found. In the second part, the motion of two bodies (mass points) connected by a linear spring is studied for the case, where the coefficient of friction is independent of the direction of the motion. The system is driven by two unbalanced rotors attached to the bodies. It is shown, that the direction of motion can be reversed without changing the direction of rotation of the rotors.

Key words
Dry friction, vibration, locomotion, control

1 Introduction

The motion of a system of rigid bodies connected in series by viscoelastic elements along a straight line on a rough plane was studied by a number of authors. Dry (Coulomb's) friction was assumed to act between the bodies and the plane, with the normal force exerted on the bodies by the plane remaining constant. The excitation of the motion of the system was due to the forces acting between the points and changing periodically in time. The friction was assumed to be asymmetric, i.e., the coefficient of friction between a body and the plane depended on the direction of motion of the body. The asymmetry can be provided, for example, by applying scales or spikes to the contact surface of the constituent bodies [Miller, 1988; Steigenberger, 1999].

In [Zimmermann et al., 2002; 2004a; 2007] and [Bolotnik et al., 2007], the dynamics of a system of two bodies joined by a linearly elastic element was studied. The motion was excited by a harmonic force acting between the bodies. In [Zimmermann et al., 2004b], a system with an elastic element made from a magnetizable polymer, the motion of which was excited by a magnetic field, was considered. In the case of small friction, an analytical expression for the average velocity of the steady-state velocity-periodic motion of the system was found. The motion of the system with this velocity was shown to be stable. A similar investigation was performed for a system of two bodies joined by a spring with a nonlinear (cubic) characteristic [Zimmermann et al., 2007; Bolotnik et al., 2007]. Algebraic equations for the average velocities of the steady-state motion were obtained. There exist up to three different steady-state modes of motion, of which at most two can be stable. The limiting case of non-symmetric friction was modeled by the kinematic condition that allowed the motion only in one direction.

In [Bolotnik et al., 2006], the rectilinear motion on a horizontal rough plane was considered for a vibration-driven system consisting of a main body (housing), which interacted directly with the plane, and of internal masses that performed harmonic oscillations relative to the housing. The vertical and horizontal oscillations of the internal masses had the same frequency but were shifted in phase. It was shown that by controlling the phase shift of the horizontal and vertical oscillations, it is possible to change the velocity of the steady-state motion of the housing, use scales or spikes to provide non-symmetric friction being unnecessary. In [Chernousko, 2005; Chernousko, 2006], the rectilinear motion of a body with a movable internal mass moving along a straight line parallel to the line of the motion of the body on a rough plane was investigated. A periodic control mode was constructed for the relative motion of the internal mass to provide a velocity-periodic motion for the main body with maximal average speed.

In the first part of the present paper, we consider the rectilinear motion along a rough plane of a chain of identical point masses. The motion of the system is excited kinematically, precisely, the time history of the distance between the neighboring points is prescribed. Friction between the plane and the mass points is assumed to be dry friction modelled by Coulomb's law. The coefficient of friction depends on the direction of motion. In the second part, a system of two bodies connected by a linear spring is considered. The motion is excited by two unbalance vibration exciters located on the bodies. The rotors of the vibration exciters rotate synchronously in the same direction. The coefficient of dry friction between the bodies and the supporting plane is independent of the direction of the motion.

2 The motion of a chain of mass points along a straight line on a rough plane

Consider a chain of n identical point bodies of mass m arranged in a line (Figure 1). The system is allowed to move on a horizontal plane along a straight line designated by the coordinate x and coinciding with the line of the chain. Let x_i $(i = 1,\ldots,n)$ denote the coordinate of the i-th body relative to a fixed reference frame. The motion of the system is excited kinematically, precisely, the distance between the neighboring points is prescribed as an explicit function of time t: $x_{i+1} - x_i = L_i(t), (i = 1,2,...,n-1)$. We assume non-symmetric (anisotropic) dry Coulomb's friction force $F(\dot{x}_i)$ to be exerted on the points by the supporting plane. Let k_+ and k_- ($k_+ \le k_-$) denote the coefficients of friction resisting the forward and backward motion, respectively.

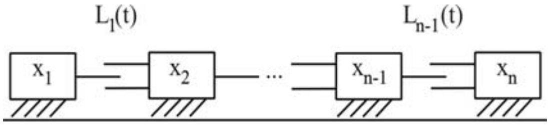

Figure 1. The chain with n mass points.

Let v denote the velocity of the center of mass of the system, i.e.,

$$v = (\dot{x}_1 + \ldots + \dot{x}_n)/n$$

Newton's second law for the center of mass of the chain is represented by the equation

$$n \cdot m\dot{v} = F(\dot{x}_1) + \ldots + F(\dot{x}_n) \qquad (1)$$

with $\dot{x}_1 + \ldots + \dot{x}_n = n v$.

We take into account $n-1$ kinematic constraints

$$x_2 - x_1 = L_1(t), \ldots, x_n - x_{n-1} = L_{n-1}(t) \qquad (2)$$

We assume that all $L_i(t) = L(t) = L_0 + b \cdot \sin \omega t$ and proceed to the dimensionless variables (labeled by the asterisk)

$$x_i = \frac{x_i^*}{b}, \quad F(\dot{x}_i) = \frac{F^*(\dot{x}_i)}{mgk_-}, \quad F_0 = \frac{F_0^*}{mgk_-}, \quad t = t^*\omega \quad ,$$

$$V = \frac{V^* \cdot \omega}{b}, \quad L = \frac{L^*}{b},\ i = 1,\ldots,n-1, \quad \mu = \frac{k_+}{k_-} \le 1, \quad \varepsilon = \frac{g \cdot k_-}{b \cdot \omega^2}$$

where g is the acceleration due to gravity. To investigate the motion of the system under consideration we will use the asymptotic technique. To that end we assume the parameter ε to be small. This means that for each mass point, the maximum acceleration due to friction is much lower than the maximum acceleration due to the prescribed relative motion of the points. With reference to (2), equation (1) can be reduced to the form

$$\dot{v} = \varepsilon \cdot \frac{1}{n} \sum_{i=1}^{n} F\left(v - \left(\frac{n+1}{2} - i \right) \cos t \right) \qquad (3)$$

Average the right-hand side of this equation with respect to t to obtain the equation of the first approximation according to the method of averaging [Bogoljubow and Mitropolski; 1965]. We will be interested in the steady-state solution of the averaged equation. To find the steady-state values of the velocity v one should calculate the roots of the averaged right-hand side of equation (3). The steady-state velocity of the averaged equation can be regarded as an approximation to the average velocity of the center of mass of the chain. For the chains comprising from 2 to 5 mass points, the velocity corresponding to the steady-state solution of the averaged equation is expressed as follows:

$$n=2\,,\; v=\begin{cases} v_s \ge \dfrac{1}{2}\,, & \mu=0 \\ \dfrac{1}{2}\sin\left(\dfrac{\pi}{2}\cdot\dfrac{1-\mu}{1+\mu}\right), & 0<\mu\le 1 \end{cases}$$

$$n=3\,,\; v=\begin{cases} v_s \ge 1\,, & \mu=0 \\ \sin\left(\dfrac{\pi}{2}\cdot\dfrac{1-2\mu}{1+\mu}\right), & 0<\mu<\dfrac{1}{2} \\ 0\,, & \dfrac{1}{2}\le\mu\le 1 \end{cases}$$

$$n=4\,,\; v=\begin{cases} v_s \ge \dfrac{3}{2}, & \mu=0 \\ \dfrac{3}{2}\sin\left(\dfrac{\pi}{2}\cdot\dfrac{1-3\mu}{1+\mu}\right), & 0<\mu<\vartheta \\ \dfrac{3}{2}\cdot\dfrac{\sin\left(\pi\cdot\dfrac{1-\mu}{1+\mu}\right)}{\sqrt{10+6\cos\left(\pi\cdot\dfrac{1-\mu}{1+\mu}\right)}}\,, & \vartheta\le\mu\le 1 \end{cases}$$

$$n=5\,,\; v=\begin{cases} v_s \ge 2\,, & \mu=0 \\ 2\sin\left(\dfrac{\pi}{2}\cdot\dfrac{1-4\mu}{1+\mu}\right), & 0<\mu<\dfrac{2}{13} \\ 2\cdot\dfrac{\sin\left(\dfrac{\pi}{2}\cdot\dfrac{2-3\mu}{1+\mu}\right)}{\sqrt{5+4\cos\left(\dfrac{\pi}{2}\cdot\dfrac{2-3\mu}{1+\mu}\right)}}\,, & \dfrac{2}{13}\le\mu<\dfrac{2}{3} \\ 0\,, & \dfrac{2}{3}\le\mu\le 1 \end{cases}$$

where v_s is an arbitrary constant; $\vartheta=\left(\pi-2\arcsin(1/3)\right)/\left(3\pi+2\arcsin(1/3)\right)$.

To investigate the dependence of the steady-state velocity v on the parameter μ, characterizing the degree of anisotropy of friction, calculate the derivative of v with respect to μ as a derivative of an implicit function. For fixed n we thus obtain

$$v'_\mu=-\frac{\dfrac{2}{\pi}\displaystyle\sum_{i=1}^{p}\arcsin\frac{2v}{n+1-2i}+n-p}{\dfrac{4(\mu+1)}{\pi}\displaystyle\sum_{i=1}^{p}\frac{1}{n+1-2i}\cdot\frac{1}{\sqrt{1-\left(2v/(n+1-2i)\right)^2}}}<0$$

where $p=[n/2]$ is the integer part of $n/2$. Therefore, the steady-state velocity v decreases as μ increases, i.e., the less the relative difference between the coefficients of friction k_- and k_+, the lower the velocity v.

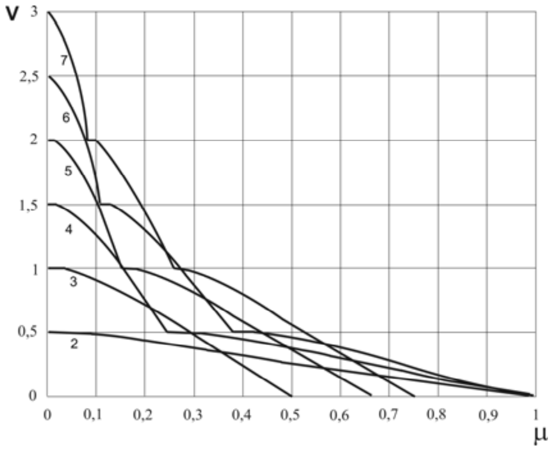

Figure 2. Velocity v versus μ for various n.

Figure 2 plots the dimensionless steady-state velocity v versus the ratio $\mu=k_+/k_-\le 1$ for chains of various lengths. The corresponding number of mass points n labels the respective curve.

3 Two-body system with unbalance vibration exciters

Consider the rectilinear motion along a rough plane of two identical bodies of mass M connected by a spring of stiffness c. To each of the bodies an unbalance vibration exciter is attached. The exciter is designed as a rigid rotor the axis of which is fixed to the body and is perpendicular to the vertical plane passing through the line of motion of the system. The center of mass of the rotor does not lie on the axis of rotation. Let x designate the coordinate along the line of motion of the system. Both rotors have the same mass m and the same distance l between the center of mass and the axis of rotation. Let x_1 and x_2 denote the coordinates measuring the displacements of the constituent bodies of the system. We assume isotropic Coulomb's dry friction with coefficient k to act between the supporting plane and the bodies. The system is driven by the vibration exciters that rotate synchronously at the same angular velocity ω in the same direction with a phase shift of φ_0. The system under consideration is depicted in Figure 3.

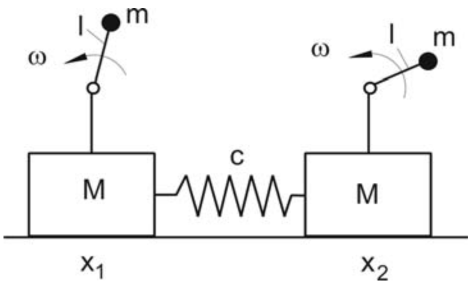

Figure 3. System with two unbalanced rotors.

Introduce the dimensionless variables and parameters (labeled with the asterisk)

$$(x_1, x_2) = \frac{\left(x_1^{\,*}, x_2^{\,*}\right)}{2a}, \qquad t = t^* \omega_0 ,$$

$$\omega_0^{\,2} = \frac{c}{(m+M)}, \qquad \nu = \frac{\omega}{\omega_0},$$

$$\varepsilon = \frac{(m+M)k\,g}{c\cdot 2a}, \qquad \alpha = \frac{m\,c\,l}{(m+M)^2 g}, \qquad \beta = \frac{\alpha}{k}$$

Here $(m+M)k\,g$ is the maximal value of the friction force acting on each of the bodies, the parameter $2a$ characterizes the maximal elongation of the spring and, accordingly, $c\,2a$ characterizes the maximal value of the spring force. We assume that $\varepsilon << 1$. The value of the characteristic (steady-state) amplitude is unknown beforehand. For this reason, we can set the value $2a$ arbitrarily, for example, $2a = l$. After determining the steady-state amplitude, it should be verified that the elastic force is in fact much larger than the friction force (to justify the hypothesis that $\varepsilon << 1$).

The dimensionless equations of motion have the form

$$\begin{aligned} &\ddot{x}_1 + x_1 - x_2 = \varepsilon\,\beta\,\nu^2 \cos\nu\,t + r_1 , \\ &\ddot{x}_2 + x_2 - x_1 = \varepsilon\,\beta\,\nu^2 \cos\left(\nu\,t + \varphi_0\right) + r_2 \end{aligned} \tag{4}$$

where

$$r_i = \begin{cases} -\varepsilon\,n_i \operatorname{sgn}\dot{x}_i , & \text{if } \dot{x}_i \neq 0, \\ -f_i , & \text{if } \dot{x}_i = 0 \text{ and } |f_i| \le \varepsilon\,n_i , \\ -\varepsilon\,n_i \operatorname{sgn} f_i , & \text{if } \dot{x}_i = 0 \text{ and } |f_i| > \varepsilon\,n_i \end{cases}$$

with

$$\begin{aligned} &f_1 = \varepsilon\,\beta\,\nu^2 \cos\nu\,t - x_1 + x_2 , \\ &n_1 = 1 - \alpha\,\nu^2 \sin\nu\,t , \\ &f_2 = \varepsilon\,\beta\,\nu^2 \cos\left(\nu\,t + \varphi_0\right) + x_1 - x_2 , \\ &n_2 = 1 - \alpha\,\nu^2 \sin\left(\nu\,t + \varphi_0\right), \quad i = 1, 2 \end{aligned}$$

The quantities n_i represent the normal pressure forces exerted on the body by the supporting plane. Since the plane resists penetration but does not resists separation of the bodies, these quantities must be nonnegative. To guarantee this condition we assume $\alpha\,\nu^2 \le 1$.

Introduce new variables: the velocity of the center of mass, $V = \left(\dot{x}_1 + \dot{x}_2\right)/2$, and the deviation of the bodies from their common center of mass, $z = \left(x_2 - x_1\right)/2 = a\cos\varphi$. To investigate the motion of the system in the neighborhood of the main resonance, we assume $\nu = \sqrt{2} + \varepsilon\,\Delta$, where Δ is a constant quantity that has an order of unity. After transforming system (4) to the standard form by introducing the slow variable $\psi = \nu\,t + \varphi_0/2 - \varphi$ and applying the procedure of averaging with respect to the fast variable φ we obtain

$$\dot{V} = \begin{cases} -\varepsilon, & V < -a\sqrt{2}, \\ -\dfrac{2\varepsilon}{\pi}\left[\arcsin\dfrac{V}{a\sqrt{2}} - \right. & \\ \left. -2\alpha\sin\dfrac{\varphi_0}{2}\sin\psi\sqrt{1-\dfrac{V^2}{2a^2}}\right], & |V| \le a\sqrt{2}, \\ \varepsilon, & V > a\sqrt{2} \end{cases}$$

$$\dot{a} = \begin{cases} \dfrac{\varepsilon}{\sqrt{2}}\sin\dfrac{\varphi_0}{2}\left(\beta\cos\psi + \alpha\sin\psi\right), & |V| < -a\sqrt{2}, \\ -\dfrac{\varepsilon}{\sqrt{2}\,\pi}\left[2\sqrt{1-\dfrac{V^2}{2a^2}} - \pi\beta\sin\dfrac{\varphi_0}{2}\cos\psi - 2\alpha\sin\dfrac{\varphi_0}{2}\sin\psi\,\cdot\right. & \\ \left.\left(\arcsin\dfrac{V}{a\sqrt{2}} - \dfrac{V}{a\sqrt{2}}\sqrt{1-\dfrac{V^2}{2a^2}}\right)\right], & |V| \le a\sqrt{2}, \\ -\dfrac{\varepsilon}{\sqrt{2}}\sin\dfrac{\varphi_0}{2}\left(-\beta\cos\psi + \alpha\sin\psi\right), & |V| > a\sqrt{2} \end{cases}$$

$$\dot{\psi}=\begin{cases}\dfrac{\varepsilon}{a\sqrt{2}}\sin\dfrac{\varphi_0}{2}\left(\beta\cos\psi+\alpha\sin\psi\right)+\varepsilon\Delta, & |V|<-a\sqrt{2},\\ -\dfrac{\varepsilon}{a\sqrt{2}\,\pi}\sin\dfrac{\varphi_0}{2}\left[\pi\beta\cos\psi-2\sin\psi\left(\arcsin\dfrac{V}{a\sqrt{2}}+\right.\right. & \\ \left.\left.+\dfrac{V}{a\sqrt{2}}\sqrt{1-\dfrac{V^2}{2a^2}}\right)\right]+\varepsilon\Delta, & |V|\le a\sqrt{2},\\ -\dfrac{\varepsilon}{a\sqrt{2}}\sin\dfrac{\varphi_0}{2}\left(-\beta\cos\psi+\alpha\sin\psi\right)+\varepsilon\Delta, & |V|>a\sqrt{2}\end{cases}$$

We are interested in the velocity-periodic steady-state motion of the system as a whole. The steady-state solution of the averaged equations of motion, corresponding to constant V, can serve as an acceptable approximate model for the steady-state motion of the basic system. Accordingly, the velocity corresponding to this solution can be used as an approximation to the average velocity of the basic system. If the velocity V according to the averaged equation is constant, the amplitude a is also constant. Introduce the variables $u=\dfrac{V}{a\sqrt{2}}$ and $\gamma=\sin\dfrac{\varphi_0}{2}$ and eliminate ψ from the averaged equations to obtain the system of algebraic equations for the steady-state solution

$$\frac{\pi^2\arcsin^2 u}{4\pi^2\left(1-u^2\right)}+\frac{4k^2\left[2\left(1-u^2\right)-\arcsin u\left(\arcsin u-u\sqrt{1-u^2}\right)\right]^2}{4\pi^2\left(1-u^2\right)}=\left(\gamma\alpha\right)^2$$

$$a=\frac{1}{\Delta}\cdot\frac{1}{\pi\sqrt{2\left(1-u^2\right)}}\left\{\frac{\pi}{2k}\arcsin u-\frac{2k}{\pi}\cdot\left[\left(2+u^2\right)\left(1-u^2\right)\arcsin u+2u\left(1-u^2\right)\sqrt{1-u^2}-\arcsin^3 u\right]\right\} \tag{5}$$

Although the detailed analysis of the nonlinear system of equations (5) is complicated, one can draw an important conclusion. Let $\left(u_0,a_0\right)$ be a solution of the system of equations (5) for a given set of parameters $\left(\gamma,\alpha,k,\Delta\right)$. Then $\left(-u_0,a_0\right)$ is a solution for the set of parameters $\left(\gamma,\alpha,k,-\Delta\right)$. Hence, one can control the direction of motion of the system by changing the resonant detuning Δ in sign. We performed numerical calculations for the experimental model of the vibration-driven system shown in Figure 4.

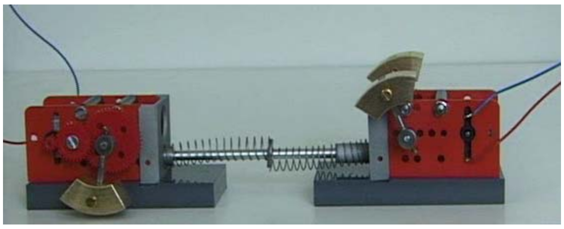

Figure 4. The prototype of the vibration driven system.

This model has the following parameters: $M=0.12\,kg$, $m=0.03\,kg$, $l=0.015\,m$, $c=130\,N/m$, $k=0.1$. Natural frequency of the system $\omega_n=\sqrt{2}\,\omega_0=44.4\,s^{-1}$, excitation frequency $\omega=50\,s^{-1}$, the dimensionless parameters $\alpha=0.17$, $\varepsilon=0.066$, $\Delta=2.7$. For $\varphi_0=\pi$ the dimensionless $V=\sqrt{2}\,u\,a$ found from system (5) is approximately equal to 0.17. From the numerical solution of the exact equation (4) it follows that the average velocity of the steady-state motion of the system is close 0.2, which demonstrates good agreement with the value $V\approx 0.17$. The corresponding dimensional value is $V\approx 0.08\,m/s$. The velocity corresponding to $\varphi_0=\pi$ is maximal.

4 Conclusion

For the chain of interconnected bodies, the steady-state velocity is studied as a function of the number of bodies in the chain and the relative difference of the coefficients of friction resisting the forward and backward motion. If the constituent bodies of the chain harmonically oscillate relative to each other, the average velocity of motion of the system as a whole decreases as the relative difference of the friction coefficients decreases.

For the system of two bodies connected by a linear spring and excited by unbalanced rotors, the direction of motion can be reversed by changing the difference between the natural frequency of the system and the angular velocities of the rotors in sign. The change in the direction of rotation of the rotors is not required. The magnitude of the velocity of the motion can be controlled by changing the phase shift between the rotations of the rotors.

An experimental model of the vibration-driven system with unbalance vibration exciters was designed and constructed.

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