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SLOW OSCILLATIONS IN DRIVES OF MACHINE ASSEMBLIES

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Abstract

Machine assembly with additional degrees of freedom is considered. The method of direct separation of motions was used for research. It was shown that the braking vibration moment occurring by the reason of resonance effects in the driven object can lead to excitation of rotor oscillations of the assembly with a frequency lower than the rotation frequency. Slow rotor oscillations represent a transient process to the stationary motion mode, which is established when an additional load torque occurs. Moreover, the maximum oscillation amplitudes will be relatively large. By the example of a vibration machine with an inertial drive, the occurrence of slow oscillations of the vibration exciter rotor is demonstrated.

Key words

Machine assembly, resonance oscillations, drive dynamics, vibration moment, slow oscillations, inner pendulum.

1 Introduction

Oscillation of machine assemblies drives is an important engineering problem. Oscillations of drives are investigated in many publications on machine assemblies. First of all, we note the article [Kolovskii, 1985]. This paper investigates the dynamics of a machine assembly with an elastic transmission mechanism. It is shown by the method of a small parameter that when starting an assembly with a nonideal motor, a stationary mode of motion is possible, leading to the Sommerfeld effect. An expression is obtained for the additional average moment of resistance forces, driven to the motor rotor, caused by energy losses during oscillations. This moment is called vibrational in many works [Blekhman, 1971; Blekhman, 2018; Kolovskii, 1985].

A rigorous analytical study of the resonant dynamics of oscillatory systems with limited excitation was first performed by Ilya Izrailevich Blekhman [Blekhman, 1953]. Later, in his works devoted to this topic, a number of significant results were obtained, as well as new ideas for understanding the Sommerfeld effect were presented.

In the book, which has already become a classic, "Vibration Mechanics" [Blekhman, 2000], it is shown that the regularities that occur during the manifestation of the Sommerfeld effect are quite simply explained from the standpoint of vibration mechanics. It should be noted that a significant contribution to the development of the theory of systems with limited excitation was made by works [Kononenko, 1969; Alifov and Frolov, 1990; Balthazar et al., 2003].

The book [Blekhman, 2000] indicates the expediency of using the concept and apparatus of vibration mechanics to solve problems of the dynamics of machine assemblies. In particular, it is shown that the dependence of the moment of the resistance forces and the driven moment of inertia of machine assembly on the rotor rotation angle, as well as the elasticity of links and nonideality of the motor, can lead to manifestation of the Sommerfeld effect.

In the article [Blekhman and Kremer, 2017], the dynamics of a complex machine assembly with additional degrees of freedom is considered by the method of direct separation of motions. It is shown that these degrees of freedom modify the dynamics of the system with the basic degree of freedom; that when the frequency of the assembly rotor approaches one of the natural frequencies of the driven object, resonant braking of the rotor can take place; that the vibration moment arising from the resonance oscillations of the driven object can be significant. In addition, attention is drawn to the possibility of appearance in the systems under consideration, in addition to rapid oscillations of the rotor speed with the rotation frequency, also "semi-slow oscillations" (in some works called "slow oscillations") with a much lower frequency. However, these oscillations are not considered in the article.

The behavior of the so-called internal pendulum and its semi-slow motions is investigated using classical asymptotic methods of nonlinear mechanics in the works [Neishtadt, 1975; Pechenev, 1986; Fidlin, 2006]. Slow oscillations of inertial vibration exciters using the method of direct separation of motions are considered in the articles [Blekhman, 2008; Yaroshevich, 2020]. In article [Blekhman, 2008], for a system with inertial excitation of oscillations, the equation of motion of the internal pendulum ("equation of semi-slow oscillations") of an unbalanced rotor and an expression for the frequency of such oscillations were obtained, and a quantitative analysis of semi-slow oscillations was carried out. Consideration of the problem is also contained in works [Kremer, 2016; Yaroshevich, 2018; Filimonikhin, 2016; Yaroshevich, 2021].

It seems important that in over-resonant vibration machines with an inertial drive, semi-slow oscillations are used to facilitate the passage of an unbalanced rotor to the natural frequency zone [Tomchin and Fradkov, 2005; Tomchina and Gorlatov, 2021; Andrievskii, 2001].

This work is devoted to development of results of the article [Blekhman and Kremer, 2017].

2 System diagram and equations of motion

Let us consider a more general than in [Blekhman and Kremer, 2017)] dynamic model of a machine assembly, taking into account the elasticity of the most nonrigid links of a mechanical system (Fig. 1). The model consists of a rotary motor connected to a driven object (operating mechanism) by a transmission mechanism. The latter will be considered an elastic inertialess element; its stiffness and drag coefficients (viscous friction) are denoted by c_c and β_c respectively. As generalized coordinates, we choose the angle of rotation of the motor rotor φ_1 and the angle of rotation of the input link of the driven object φ_2, which is driven to the motor shaft; moreover, since the driven object has additional degrees of freedom, its position is determined by a k-dimensional vector $\mathbf{U}$. Such a machine assembly is called in [Blekhman and Kremer, 2017] a complex real machine assembly.

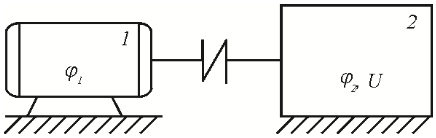

Figure 1. Machine assembly diagram

The equations of motion of the described system can be represented in the form

$$I_1\ddot{\varphi}_1 + \beta_c\dot{\varphi}_{12} + c_c\varphi_{12} = L(\dot{\varphi}_1),$$
$$I_2\ddot{\varphi}_2 - \beta_c\dot{\varphi}_{12} - c_c\varphi_{12} = -R(\dot{\varphi}_2) + M(\dot{\varphi}_2, \varphi_2, \dot{\mathbf{U}}, \mathbf{U}), \qquad (1)$$
$$D(\varphi_2, \mathbf{U}) = 0\,, \qquad (2)$$

where I_1, I_2 – driven moments of inertia of the rotors of motor and driven object; $\varphi_{12} = \varphi_1 - \varphi_2$; $L(\dot{\varphi}_1)$ – motor torque; $R(\dot{\varphi}_2)$ – moment of rotation resistance forces; M – moment representing the load from the driven object.

Let us assume that the given moments of inertia of rotors of the motor and the driven object are constant, since the variability of the moment of inertia does not have a significant effect on the studied dynamic processes, however, it significantly complicates the calculations.

Note that the system of equations (1) is the equations of the rotational motion of rotors of the motor and the driven object, and the equation (2) is a conditional record of the equations of motion of the driven object.

3 The equation of dynamics of a machine assembly by I.I. Blechman

For research we will use the approach of vibrational mechanics and the method of direct separation of movements [Blekhman, 2000]. In accordance with the basic premise of the method, we assume that the considered motions can be represented in the form

$$\dot{\varphi}_i = \omega(t) + \dot{\psi}_i(t, \tau), \;\; u = u(t, \tau), \; i = 1, 2 \qquad (3)$$

where t – «slow», and $\tau = \omega t$ «fast» time; $\omega(t)$ – slow, $\dot{\psi}_i$ and u – fast time functions, besides the latter are 2π - periodic in $\tau = \omega t$ and zero mean for the period.

Then, for coordinates φ_i, taking into account (3), we can take: $\varphi_i = \tau - \alpha_i(t) + \psi_i(t, \tau)$, where $\alpha_i(t)$ – is a some function t, which we will consider slow.

Applying the method used, we obtain the following system of equations for slow and fast motions

$$\begin{gathered} I_1\dot{\omega} + c_c\alpha_{12} = L(\omega), \\ I_2\dot{\omega} - c_c\alpha_{12} = -R(\omega) + \\ + \left\langle M(\omega + \dot{\psi}_2, \tau - \alpha_2(t) + \psi_2, \dot{\mathbf{U}}, \mathbf{U}) \right\rangle, \end{gathered} \qquad (4)$$

$$\begin{gathered} I_1\ddot{\psi}_1 + \beta_c\dot{\psi}_{12} + c_c\psi_{12} + k_1\dot{\psi}_1 = 0, \\ I_2\ddot{\psi}_2 - \beta_c\dot{\psi}_{12} - c_c\psi_{12} + k_2\dot{\psi}_2 = \Psi(\dot{\omega}, \omega, \dot{\psi}_2, \psi_2), \\ D(\tau - \alpha_2(t) + \psi_2, \mathbf{U})) = 0\,, \end{gathered} \qquad (5)$$

where $\alpha_{12} = \alpha_1 - \alpha_2$; $\psi_{12} = \psi_1 - \psi_2$; $\Psi(\dot{\omega}, \omega, \dot{\psi}_2, \psi_2) = M(\omega + \dot{\psi}_2, \tau - \alpha_2(t) + \psi_2, \dot{\mathbf{U}}, \mathbf{U})) - -\left\langle M(\omega + \dot{\psi}_2, \tau - \alpha_2(t) + \psi_2, \dot{\mathbf{U}}, \mathbf{U})) \right\rangle.$

In the equations of systems (4), (5) and below, the angle brackets $\langle\ldots\rangle$ indicate the averaging of the expressions they contain over a period $2\pi/\omega$ in fast time τ; expressions for the moments $L(\dot{\varphi}_1)$ and $R(\dot{\varphi}_2)$ are linearized, as in [Blekhman, 2008], near the stationary values $\dot{\varphi}_i = \omega$ according to the formulas $L(\dot{\varphi}_1) = L(\omega) - k_1\dot{\psi}_1$, $R(\dot{\varphi}_2) = R(\omega) + k_2\dot{\psi}_2$; k_1, k_2 – electrical and mechanical damping coefficients.

Following the method used, we will find approximate solutions of the equations of fast motions (5), with "frozen" slow variables t, $\omega(t)$, $\alpha_i(t)$.

At first, considering rotation of the rotors, which occurs with an angular velocity close to equable motion $\dot{\varphi}_i = \omega(t)$, we assume $\Psi(\dot{\omega},\omega,\dot{\psi}_2,\psi_2) = \mu\Psi_2(\dot{\omega},\omega,\dot{\psi}_2,\psi_2)$, where μ – is a small parameter. Then, in the initial approximation ($\mu = 0, \psi_i = 0$), the first two and the last equations of system (4), (5) can be represented in the form obtained in [Blekhman, 2000]

$$I\dot{\omega} = L(\omega) - R(\omega) + V(\omega),$$
$$D(\tau - \alpha_2(t), \mathbf{U}_0) = 0 \tag{6}$$

where $I = I_1 + I_2$; $\mathbf{U}_0$ – 2π periodic solution of the last equation of the system (6); $V(\omega) = \left\langle M(\omega, \tau - \alpha_2(t) + \psi_2, \dot{\mathbf{U}}_0, \mathbf{U}_0)\right\rangle$ – vibration moment.

Note that the last equation of system (5) is actually the equations of forced (parametric) oscillations of the driven object; in the case of nonlinearity of system (5), there can be several solutions to this equation, and each solution will have its own expression for $V(\omega)$. Note that work [Blekhman and Kremer, 2017] provides a procedure for obtaining an expression for the vibration moment $V(\omega)$.

As you can see the first equation (6) exactly coincides with the one obtained by I.I. Blekhman in [Blekhman, 2000; Blekhman and Kremer, 2017], which shows the universality of the equation proposed by him for a complex real machine assembly.

Consequently: the results given in the article [Blekhman and Kremer, 2017] concerning stationary modes of motion $\omega = \omega_0 = const$ and their stability are also valid for the considered dynamic model of the machine assembly; the availability of an elastic-damping transmission mechanism in the drive of the assembly does not affect the slow motions of the rotors.

Thus, the resonance oscillations occurring in the driven object lead to the appearance of an additional moment of resistance forces $V(\omega)$ loading the motor. In [Blekhman and Kremer, 2017] it is emphasized that the value of the vibration moment can be significant in comparison with the traditionally taken into account the moment of resistance forces $R(\dot{\varphi}_2)$. It is important that in this case, the vibration moment can have a significant effect on the mode of motion of machine assembly; moreover, it also can change its operating mode. So, when starting, the motor may not reach the specified rotation mode, and in the operating mode, a significant decrease in its average angular velocity is possible. In addition, as will be shown below, important effects are found out at the level of rapid movements.

According to (4), in case of resonance oscillations in the driven object, the static deformation of the drive (average deformation) can significantly exceed the common static deformation.

4 The equation of fast motions of machine assembly

Having regard to the dynamic nature of vibration moment, it can be expected to have a noticeable effect on the fast motions of the rotors.

Let us consider the corresponding equations taking into account the following approximation ($\mu \neq 0$, $\dot{\varphi}_i = \omega + \dot{\psi}_i$). In this case, at first, we restrict ourselves, as in article [Blekhman and Kremer, 2017], to the case of a dynamic model of a rigid machine assembly.

Then the equations of fast motions of rotor of the assembly can be represented in the form

$$I\ddot{\psi} + k\dot{\psi} = M_\nu\left(\omega + \dot{\psi}, \tau + \psi, \dot{\mathbf{U}}_0, \mathbf{U}_0\right), \tag{7}$$

where $k = k_1 + k_2$; $M_\nu\left(\omega + \dot{\psi}, \tau + \psi, \dot{\mathbf{U}}_0, \mathbf{U}_0\right) = M\left(\omega + \dot{\psi}, \tau + \psi, \dot{\mathbf{U}}_0, \mathbf{U}_0\right) - \left\langle M\left(\omega + \dot{\psi}, \tau + \psi, \dot{\mathbf{U}}_0, \mathbf{U}_0\right)\right\rangle$.

Note that, in (7), the "frozen" angle α, which is insignificant for the equations of fast motions, was omitted. Assuming that the angle ψ is small, we expand the function $M_\nu\left(\omega + \dot{\psi}, \tau + \psi, \dot{U}_0, U_0\right)$ in a Taylor series. As a result, we obtain $M_\nu\left(\omega + \dot{\psi}, \tau + \psi, \dot{\mathbf{U}}_0, \mathbf{U}_0\right) \approx M_\nu\left(\omega + \dot{\psi}, \tau, \dot{\mathbf{U}}_0, \mathbf{U}_0\right) + \psi M_\nu^{/}\left(\omega + \dot{\psi}, \tau, \dot{\mathbf{U}}_0, \mathbf{U}_0\right)$ where the prime denotes the derivative with respect to τ. Then equation (7) can be written in the form

$$I\ddot{\psi} + k\dot{\psi} + c_\nu\psi = M_\nu\left(\omega + \dot{\psi}, \tau, \dot{\mathbf{U}}_0, \mathbf{U}_0\right), \tag{8}$$

where $c_\nu = -M_\nu^{/}\left(\omega + \dot{\psi}, \tau, \dot{\mathbf{U}}_0, \mathbf{U}_0\right)$ - "dynamic" stiffness coefficient.

Further decomposing the periodic function of time $M_\nu\left(\omega + \dot{\psi}, \tau, \dot{\mathbf{U}}_0, \mathbf{U}_0\right)$ in a Fourier series per the argument τ, we obtain $M_\nu\left(\omega + \dot{\psi}, \omega t, \dot{\mathbf{U}}_0, \mathbf{U}_0\right) = \sum_{n=1}^{\infty} A_n(n\omega)\sin(n\omega t + \gamma_n)$.

Note that when determining the Fourier coefficients in the expression for the moment $M_v\left(\omega+\dot{\psi},\tau,\dot{\mathbf{U}}_0,\mathbf{U}_0\right)$ we substitute the value of the vector $\mathbf{U}_0$, found for the steady oscillations of the driven object under the influence of the impact forces, that occur when the rotor rotates with an angular velocity $\dot{\varphi}_i=\omega$.

In the case of resonant effects in the driven object, the influence of all harmonics except for one - the resonant one, can be neglected. Then, in the considered approximation, we come to the equation of fast motions of the rotor of the machine assembly in the form

$$\ddot{\psi}+2b_\psi\dot{\psi}+p_\psi^2\psi=Q\cos n\omega t\,, \qquad (9)$$

where $b_\psi=\dfrac{k}{2I}$; $p_\psi=\sqrt{\dfrac{c_v}{I}}$; $Q=\dfrac{A_n(n\omega)}{I}$.

Thus, when a stationary mode occurs, which is a consequence of resonance oscillations in the driven object, the equation of fast motions of the machine assembly rotor takes form of an oscillatory-type equation. On the right side of (8), the value of "dynamic" stiffness coefficient c, representing the rate of change of the function $M_v\left(\omega+\dot{\psi},\tau,\dot{\mathbf{U}}_0,\mathbf{U}_0\right)$, in the case of resonance effects in the driven object will be significant.

The variable p_ψ is the frequency of small free oscillations of the assembly rotor or, according to [Blekhman, 2008; Blekhman 2018], the frequency of small free oscillations of the internal pendulum. In article [Tomchina and Gorlatov, 2021] this frequency is called the Blekhman frequency. Note that in the case under consideration, the Blekhman frequency can also be considered the critical frequency of the machine assembly rotor.

When resonance oscillations occur in the driven object, the amplitude of the driving force in the right side of (9) will be relatively large. Accordingly, in the considered mode of motion, the amplitudes of forced oscillations of the rotor speed of machine assembly will be increased.

Moreover, in the case of fulfilling the condition $n\omega>p_\psi$ (which is considered to be rather mild) when the stationary regime is established, there will have place a pronounced transient process with the main slow oscillation frequency p_ψ and relatively large initial amplitudes.

The basis for this conclusion is also the fact that the considered stationary modes occur when there is an abrupt increase in the vibration moment, the effect of which for a low-frequency oscillatory system can be considered shock.

Next, we will again give a consideration to the dynamic model of an elastic machine assembly (Fig. 1).

Taking into account that driven moment of inertia of rotor of the driven object is much more than the moment of inertia of the motor $I_2>>I_1$, we come (as in [Blekhman, 2017) to the equation of fast motions of driven object's rotor in the form (9).

Consequently, the conclusions made above when analyzing Eq. (9) are also valid in the case under consideration.

5 Example. Vibration machine with inertial drive

The design model consists (Fig. 2) of a rigid platform with an unbalanced vibration exciter attached to it; at the same time, in contrast to [Blekhman, 2008], its rotor is connected to the rotor of the electric motor (for definiteness - of the asynchronous type) by an elastic-damping element (transmission). Note that the elastic-damping connection is considered to be inertialess; let it be an elastic coupling for brevity.

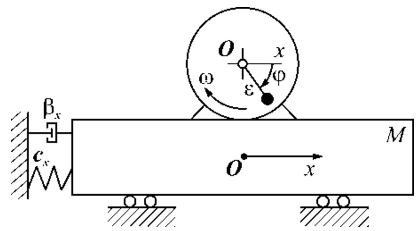

Figure 2. Dynamic model of vibration machine

The equations of motion of the system can be represented in the form of equations (1) and the equation of platform oscillations

$$M\ddot{x}+\beta_x\dot{x}+c_x x=m\varepsilon\left(\ddot{\varphi}_2\sin\varphi_2+\dot{\varphi}_2^2\cos\varphi_2\right), \qquad (10)$$

where M – platform weight; x - horizontal displacement of the platform; β_x, c_x – coefficients of viscous friction and stiffness of platform fasteners; m,ε – vibration exciter mass and eccentricity; φ_1, φ_2 – angles of rotation of the motor and exciter rotors; the function $M(\dot{\varphi}_2,\varphi_2,\dot{\mathbf{U}},\mathbf{U})$ has the form $M(\dot{\varphi}_2,\varphi_2,\dot{\mathbf{U}},\mathbf{U})=m\varepsilon\ddot{x}\sin\varphi_2$.

We are looking for solutions of the equations of motion in the form (3). Following the method used, we obtain the equations of slow and fast motions of the motor and exciter rotors in the form (4), (5), where $\Psi(\dot{\omega},\omega,\dot{\psi}_2,\psi_2)=m\varepsilon\big(\ddot{x}\sin(\omega t+\alpha_2+\psi_2)-$ $-\langle\ddot{x}\sin(\omega t+\alpha_2+\psi_2)\rangle\big)$.

Summing up the equations of slow motions, we arrive at the equation of dynamics of machine assembly (6); where $V(\omega)=-\dfrac{1}{2}A_{st}^2\beta_x p_x\,k_d^2$, $p_x=\sqrt{\dfrac{c_x}{M}}$; where A_{st} – amplitude of the platform over-resonance oscillations; k_d – dynamic coefficient.

Thus, the resonance oscillations of vibration machine lead to the occurrence of a braking vibration moment. It seems important that the value of this moment is proportional to the square of the dynamic coefficient.

Considering the "peak" type of the vibration moment change, we can assume that it has a noticeable effect on fast motions.

We represent the equations of fast motions (7) in the form [Blekhman, 2008]:

$$I_2\ddot{\psi}_2 + k_2\dot{\psi}_2 = m\varepsilon\left(\ddot{x}\sin\omega t + \ddot{x}\psi_2\cos\omega t - \langle\ddot{x}\sin\omega t\rangle\right). \quad (11)$$

Taking into account the solution of equation (10), corresponding to the steady-state forced oscillations of the bearing body when the vibration exciter rotates with a constant angular velocity ω, it is easy to represent equations (11) in the form (8), or (9), where $c_v = V_{\max}\cos\gamma_x$; $M_v\left(\omega+\dot{\psi},\tau,\dot{U}_0,U_0\right) = -V_{\max}\sin 2\omega t$; $p_\psi = \sqrt{\dfrac{c_v}{I_2}}$.

It was shown in [Blekhman, 2008] that for the equations of fast motions to be valid, the frequency p_ψ must be significantly lower than ω; usually enough $\omega/p_\psi > 3$. In this case, when a stationary mode is established, an intense transient process will take place, which is damped biharmonic oscillations of the rotor with a slow (fundamental) frequency p_ψ.

Rotor oscillations are best judged by rotor speed oscillations. Taking into account the shock application of the vibration moment ($\psi_{20} = 0$, $\dot{\psi}_{20} = V_{\max}/I_2\omega$), from equation (9) we obtain (without taking into account the resistance forces): $\dot{\psi}_2 \approx 3A_\psi\cos p_\psi t + A_\psi\cos 2\omega t$, where $A_\psi = \dfrac{V_{\max}}{2\omega I_2}$. Consequently, during this period of motion, both the initial and steady-state oscillation amplitudes of the rotor speed will be relatively large, since they contain the resonant value of the vibration moment, $V_{\max}$.

The obtained results are confirmed by the results of computer simulation (Fig. 3).

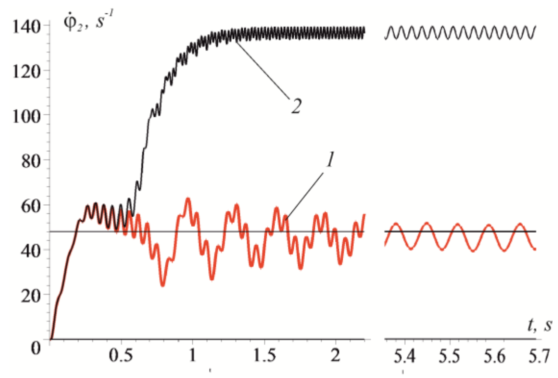

Figure 3. Changing of the vibration exciter speed in case of: 1 - «stucking» of the speed ($m\varepsilon = 1,2\,kg\,m$); 2 - passage of vibration exciter through the resonance zone ($m\varepsilon = 1{,}197\,kg\,m$)

Numerical integration of the system of equations (1), (2) and equations of the dynamic model of an asynchronous electric motor [Yaroshevich, 2020] was carried out with the following parameters: $M = 300\,kg$; $I_1 = 0{,}0068\,kg\cdot m^2$; $I_2 = 0{,}068\,kg\cdot m^2$; $\beta_x = 1500\,kg/s$; $c_x = 6{,}9\cdot 10^5\,N/m$; $\beta_c = 0{,}162\,kg\cdot m^2/s$; $c_c = 48N\cdot m$; electric motor 4A serie with power $P_{mot} = 2{,}2\,kW$ and synchronous frequency $n_s = 1500\,rtm$. Fig. 3 demonstrates the occurrence of slow oscillations of the vibration exciter rotor speed in the resonance zone of the vibration machine ($p_\psi = 18\,s^{-1}$, $\omega = 47\,s^{-1}$).

Note also that the presented results show good agreement with the results obtained in [Blekhman, 2008].

6 Conclusion

Using the method of direct separation of motions, drive dynamics of the machine assembly with additional degrees of freedom is considered; in addition, the elasticity of drive is taken into account.

It is shown that the equation of vibration mechanics of a complex real machine assembly proposed by I.I. Blekhman, retains its form in the case of an assembly with an elastic transmission mechanism, thereby indicating the generality of the results obtained by him.

The efficiency of using the approach of vibration mechanics and the method of direct separation of motions for solving applied problems of the dynamics of machine assemblies is demonstrated.

It is shown that along with the basic equation of vibration mechanics (the equation of slow motions), the equations of fast motions also allow one to obtain important practical results.

It has been found out that the occurrence of resonance oscillations in the driven object of the assembly can lead to the excitation of slow oscillations of the rotor; the initial amplitudes of such oscillations will be large; in addition, during this period of motion, there will encounter increased deformation of the drive and increased oscillations in the rotor speed.

The obtained results can be useful in the development of control algorithms for the passage of the resonance zone by an unbalanced rotor.

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