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DYNAMICS MODES OF A NUMBER IN DENSITY-DEPENDENT TWO-AGE-STRUCTURED MODEL

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Abstract

In this paper we research a mathematical model of dynamics for the population number. This population consists of two age classes. We investigate the dynamic modes of the model. It is shown that density-dependent factors of regulation for the population number can lead to generation of fluctuations and chaotic dynamics behavior of the population.

Key words

Population models equations, discrete-time systems, modeling, nonlinear systems, age distribution, density-dependent, stability, bifurcations, dynamic modes, chaos.

1 Introduction

This paper continues in the research of the two-componential discrete model.

This model describes the dynamics of the age structured population number. It has been considered in the articles [Frisman, etc.,1988; Shapiro,1983].

As well as in the previous research, we considered the population of the two age classes by the beginning of the next season: the younger, one including not reproductive individuals, and the senior class, consisting of the individuals participating in reproduction.

A change in the number is determined by the processes of reproduction and death rate.

In the previous articles [Frisman, etc.,1988; Frisman and Skaletskaya,1994; Shapiro,1983] it was proposed that it is only the number of the certain age class influences its survival rate.

The cases when survival rates represent the functions of the both age groups numbers are considered in this work.

2 A description of the models equations

In our mathematical model, x_n is a number of younger age classes during n season of reproduction; y_n is a number of the senior age class making a reproductive part of population during the n-th season of reproduction.

In this case the density-dependent two-age-structured models equations look as follows:

$$\begin{cases} x_{n+1} = a(y_n) \cdot y_n \\ y_{n+1} = s(x_n, y_n) \cdot x_n + v(x_n, y_n) \cdot y_n \end{cases}, \qquad (1)$$

where $s(x, y)$, $v(x, y)$ are the functions of survival for non reproductive and reproductive individuals, $a(y)$ is the function describing dependence of birth and survival rate of newborns on their number.

Taking into account the fact that the density-dependent factors restrict the development of population, all functions of the survival rate monotonously decrease and aspire to zero at infinite increase of argument.

3 The modes of dynamics

We have considered three cases of this system (1) when two parameters are fixed, and the third one is a function.

The situation when the density factors influence the survival rate of young individuals, and the survival and birth rate of adult individuals are constant is considered.

In this case the equations of dynamics have are as follows:

$$\begin{cases} x_{n+1} = a \cdot y_n \\ y_{n+1} = s(x_n, y_n) \cdot x_n + v \cdot y_n \end{cases}. \qquad (2)$$

The analytical research of the system on stability has shown that loss of stability occurs in two ways.

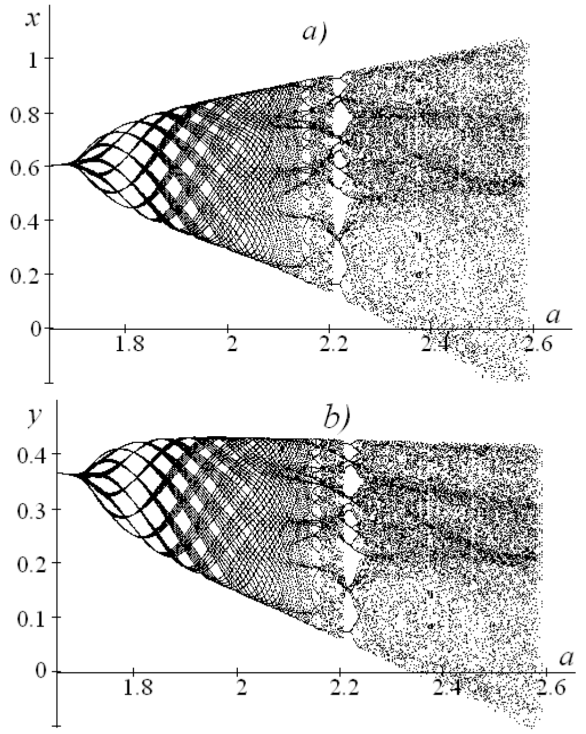

Figure 1. Bifurcation diagrams: a) Dynamic variable x; b) dynamic variable y systems (2) at increase of reproductive potential $a \in [1,65; 2,6]$, $x_0 = 0,3$, $y_0 = 0,1$, $v = 0,95$, $\alpha = 1$, $\beta = 1$.

In the first way the loss of stability occurs during the moment of the pair of complex-interfaced roots of the linearized system characteristic equation (2) passing through the unit circle.

As a result there appears a quasi-periodic motion, acquiring a chaotic character in the system parameters evolution.

In the second way the loss of stability occurs, when the root of the linearized system characteristic equation passes through -1.

Transition to chaos occurs through the cascade doubling of the period.

We had developed the system of programs, allowing to receive dynamic modes maps of a system, phase portraits, bifurcation trees, and diagrams on the Liapunov exponents dependence on the system parameter.

It is shown, that different types of density regulation of the population number growth correspond to the completely different limited structures.

We have made some numerical experiments at the admissible (biologically substantial) parameter values of the system (2) parameters when the survival rate function of a younger age class looks as follows: $s(x, y) = 1 - \alpha \cdot x - \beta \cdot y$

Bifurcation diagrams show possible dynamics modes at different values of the parameter a. Figure 1 represents the occurrence of different length cycles.

An evolution of model (2) phase portraits on the plane (x, y) is represented in figure 2.

Figure 3 illustrates a map of the dynamic modes ob-

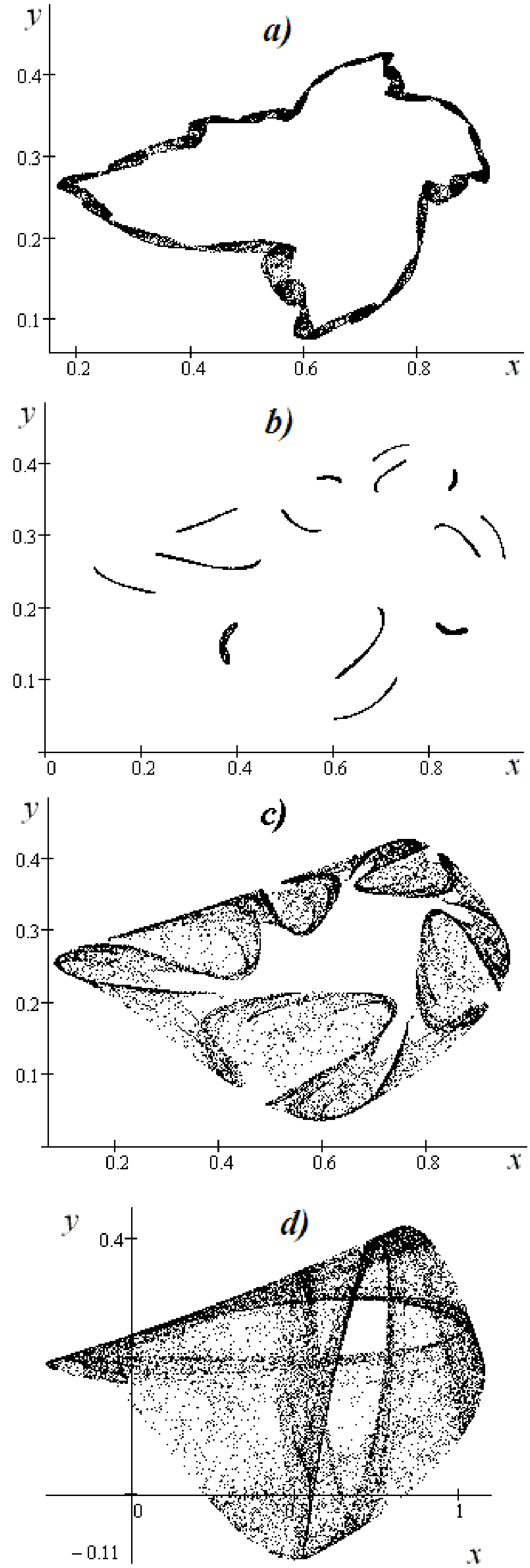

Figure 2. A phase portrait at values a) $a = 2,164$, $v = 0,95$; b) $a = 2,234$, $v = 0,95$; c) $a = 2,254$, $v = 0,95$; d) $a = 2,56$, $v = 0,95$.

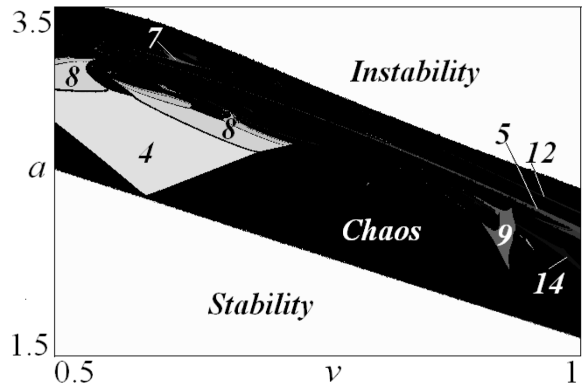

Figure 3. The dynamic modes map on the parameter plane (a, v).

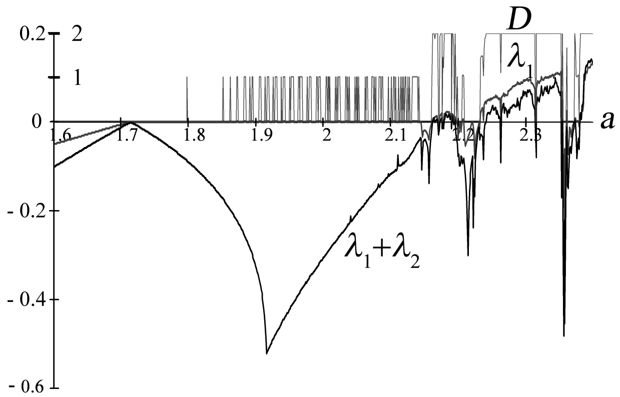

Figure 4. A change of the attractor dimension D (diagram 1), the maximal Liapunov exponent (diagram 2) and the sums Liapunov exponents (diagram 3). The parameter value a varies from $1,6$ up to $2,4$, with the step of $0,001$. The parameter value v is $0,95$.

tained in the numerical experiment on the parameter plane (a, v). Bifurcation lines of the period doubling, accumulating to the chaos border can be seen in it.

Numerals mark the periods of fluctuations.

The lower part of the map illustrates an establishment of a stationary condition.

The white area in the upper part of the figure illustrates a divergence of mapping iterations.

We have made the diagram of Liapunov's exponents for visualization of the area of the regular, quasi-periodic and chaotic behavior.

Figure 4 illustrates the Liapunov's exponents diagrams and the dimension attractor diagram.

We have calculated the Liapunov exponents with the Benettin algorithm and found the attractor dimension according to the Kaplan-York formula [Nejmark and Landa,1987] .

Parameter value $a = 1,7$ is a critical point. It separates the areas of regular and complex dynamics (figure 4). The maximal Liapunov exponent is negative in the pre rejection area.

In the after rejection area the maximal Liapunov exponent is positive. The chaos manifests itself in this area.

The diagram of the maximal Liapunov exponent contains falls into the negative values area, corresponding to the so-called.

As the attractor chaotic character of the parameter a increases, accordingly the attractor dimension increases as well.

For the majority of the birth rate parameter values ($a > 2,2$) the sum of the Liapunov exponents is positive. Hence, the mapping is tensile, and dimension of a chaotic attractor equals 2. The attractor movement can be described by two variables.

We have obtained similar results for exponential functions of the survival rate: $s(x, y) = e^{-\alpha \cdot x - \beta \cdot y}$.

4 Conclusion

We have shown that the density-dependent factors of the population number regulation can lead to the occurrence of number fluctuations and to chaotic dynamic behavior of a population.

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