ENOC-2008, Saint Petersburg, Russia, June, 30–July, 4 2008

TARGETED ENERGY TRANSFER IN A SYSTEM WITH SOFT NONLINEARITY

Oleg V. Gendelman

Faculty of Mechanical Engineering
Technion – Israel Institute of Technology
Technion City, Haifa, 32000, Israel
E-mail: ovgend@tx.technion.ac.il

Abstract

Targeted energy transfer (TET) in a 2DOF system consisting of primary linear oscillator and nonlinear energy sink (NES) with non – polynomial potential is investigated. Use of non – polynomial and even non – analytic potential functions is motivated by needs of practical design. It is demonstrated that the "complexification – averaging" technique of analysis developed before can be successfully extended for these cases with proper modifications.

Key words:
Targeted energy transfer, nonlinear energy sink, soft nonlinearity

1 Description of the model and numeric demonstration

Targeted energy transfer, i.e. almost irreversible passive transfer of mechanical energy from linear substructure to essentially nonlinear attachment (nonlinear energy sink, NES) has attracted a lot of attention of researches in few last years [Gendelman, 2001, Gendelman, 2004, Lee et al, 2006, Panagopulos et al, 2007, Gourdon and Lamarque, 2006]. In majority of these models, stiff nonlinear attachments (commonly, purely cubic spring) were used. In many applications – especially those where geometric nonlinearity is involved – the nonlinear springs are soft. Demonstration and investigation of the effect of the targeted energy transfer in a system with soft essentially nonlinear attachment is the goal of this work.

For the sake of modelling, the potential of the attachment is chosen in the form $V(z) = \varepsilon k \ln(1+z^2)$, providing linear limit with stiffness coefficient 2εk for small deformations z and softening while z grows. Primary system is chosen to consist of linear oscillator with unit frequency and mass, without damping. The nonlinear attachment is adopted to have small mass ε<<1 and linear damping with coefficient ελ. Equations describing the system dynamics are

$$\begin{aligned} &\ddot{y}_1 + y_1 + \varepsilon\lambda(\dot{y}_1 - \dot{y}_2) + \frac{2\varepsilon k(y_1 - y_2)}{1+(y_1-y_2)^2} = 0 \\ &\varepsilon\ddot{y}_2 + \varepsilon\lambda(\dot{y}_2 - \dot{y}_1) + \frac{2\varepsilon k(y_2 - y_1)}{1+(y_1-y_2)^2} = 0 \end{aligned} \qquad (1)$$

where y_1 and y_2 are the displacements of the primary oscillator and the attachment respectively. The phenomenon of the targeted energy transfer may be demonstrated numerically if System (1) is modelled with initial conditions:

$y_1(0) = 0, \dot{y}_1(0) = A, y_2(0) = 0, \dot{y}_2(0) = 0$

In order to demonstrate the targeted transfer, we plot the relative instantaneous energy $R = E_{att} / (E_{primary} + E_{att})$ stored in the attachment with respect to total energy of the system, for two different values of initial velocity A and parameters of the system ε=0.05, k=2, λ=0.2 (Figures 1 and 2):

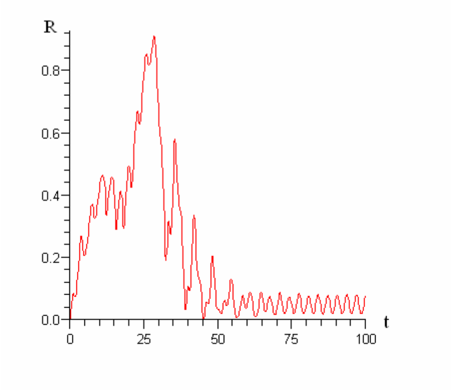

Figure 1. Relative instantaneous energy in the attachment, A=1.3

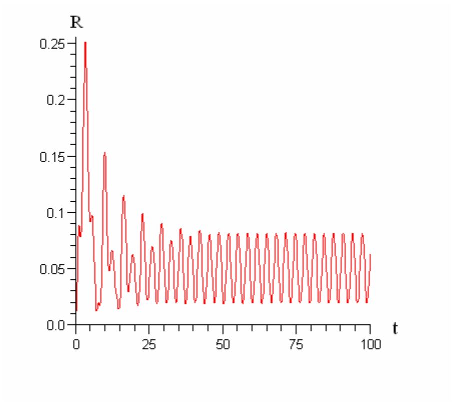

Figure 2. Relative instantaneous energy in the attachment, A=0.8

It is clear that for sufficient initial excitation the system exhibits vigorous targeted transfer of energy to the light attachment (for t~30 about 90% of the total energy is concentrated at the attachment, whereas the linear characteristic time of the system is 1/ελ=100). The mechanism of the energy transfer is resonance capture, which occurs due to soft nonlinearity. At small deformations, the linear frequency of the attachment is $\omega_0 = \sqrt{2k\varepsilon/\varepsilon} = 2$. Due to softening, when the deformations are large enough, the attachment can achieve 1:1 resonance with primary oscillator in the vicinity of unit frequency. Then, due to damping, the system is taken out from the resonance and energy remains at the attachment.

2 Averaging procedure – peculiarities and verification

Analytic description of the process of targeted transfer has been performed by applying combined method of complexification and averaging with multiple – scale analysis [Gendelman, 2004]. This method works directly only for polynomial – type nonlinearities, which is not the case in the problem under consideration. For equations (1), one may be tempted to use similar method, by presenting the nonlinear potential of the attachment as Taylor series with respect to y_1-y_2 and keeping few first terms. Such approach is not valid if one is interested in the regime where displacements can be comparatively large (of course, it is the case for the problem of the targeted transfer) since the Taylor series will converge only for $|y_1 - y_2| < 1$. In order to circumvent this obstacle, Fourier series expansion with respect to fast time scale is used. Namely, the following change of variables is performed:

$$\begin{gathered} v = y_1 + \varepsilon y_2, w = y_1 - y_2, \\ \varphi_1(\tau)\exp(it) = \dot{v} + iv, \\ \varphi_2(\tau)\exp(it) = \dot{w} + iw \end{gathered} \tag{2}$$

where τ relates to the slow time scale of the problem. Anzats (2) is justified due to conditions of 1:1 resonance. The nonlinear term is presented as:

$$\begin{gathered} \frac{-ik(\varphi_2 \exp(it) + \varphi_2^* \exp(-it))}{1-(\varphi_2 \exp(it) + \varphi_2^* \exp(-it))^2/4} = \\ = \sum_{j=-\infty}^{\infty} a_j(\varphi_2, \varphi_2^*)\exp(ijt) \end{gathered} \tag{3}$$

The asterisk denotes a complex conjugation.

Standard averaging procedure yields the following slow – flow equations:

$$\dot{\varphi}_1 + \frac{i\varepsilon}{2(1+\varepsilon)}(\varphi_1 - \varphi_2) = 0$$

$$\begin{aligned} &\dot{\varphi}_2 + \frac{i}{2(1+\varepsilon)}(\varphi_2 - \varphi_1) + \\ &+ \frac{\lambda(1+\varepsilon)}{2}\varphi_2 - \frac{i(1+\varepsilon)}{2}G(|\varphi_2|^2)\varphi_2 = 0 \end{aligned} \quad (4)$$

$$G(z) = \frac{4k}{z}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

Validity of the averaging procedure for description of the targeted transfer is illustrated at Fig. 3 by direct comparison between simulated flows of systems (1) and (4) with appropriate corresponding initial conditions, computed with the help of anzats (2). We compare the values $Z = \sqrt{(y_1(t) - y_2(t))^2 + (\dot{y}_1(t) - \dot{y}_2(t))^2}$ (solid line) and $Z_{average} = |\varphi_2|$ (dotted line).

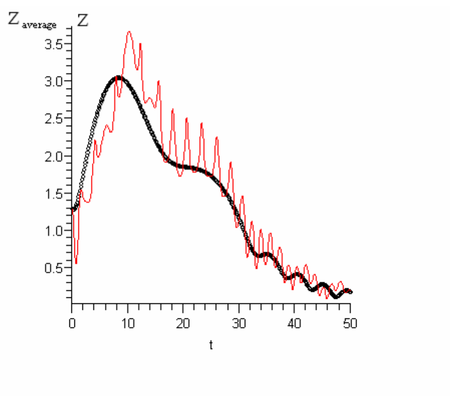

Figure 3. Comparison between initial flow (solid line) and averaged flow (dotted line)

The averaged flow, as expected, does not reflect the fast oscillations of the transient response but clearly predicts the characteristic shape of the response curve, at least at important initial stages of the process (time scale of order O(1/ε)). Therefore, despite non-polynomial nonlinear function, the approach based on Fourier series expansion (which may be treated as enhanced harmonic balance with respect to the fast time scale) yields rather reliable results. System (4) is much easier for analysis than system (1) - it is possible to prove that the averaging here reduces a dimension of effective state space of the system by one – but still is not solvable analytically. Asymptotic analysis is possible based on small parameter ε.

3 Asymptotic analysis

Asymptotic analysis of system (4) has two main goals. Analysis of initial stage of the transfer process (time scale O(1)) is necessary to establish the critical amplitude of initial impact when the nonlinear attachment will be excited. Analysis of later stages of the process (time scale O(1/ε)) establishes the conditions of efficient dissipation of energy in the system.

3.1 O(1) time scale

For time scale O(1) system (4) is reduced to the form

$$\begin{aligned} &\varphi_1 = A, \\ &\dot{\varphi}_2 + \frac{i}{2}\varphi_2 + \frac{\lambda}{2}\varphi_2 - \frac{i}{2}G(|\varphi_2|^2)\varphi_2 = \frac{i}{2}A \end{aligned} \quad (5)$$

where A is the initial velocity of the primary mass (in other terms, we simulate the solutions of system (1) with initial conditions $y_1(0) = 0, \dot{y}_1(0) = A, y_2(0) = 0, \dot{y}_2(0) = 0$. It is easy to demonstrate that equation (5) can have one or three fixed points, depending on values of k, A and λ. If the fixed point is single it is stable node; pair of saddle and additional node may appear depending on parameters of the system. Efficient energy transfer to the attachment is provided if the only fixed point is "upper" node. Simple (but only numeric, due to complicated shape of function G) estimation yields the minimal value for initial velocity $A_{crit} = 1.1$ for values of the parameters λ=0.2 and k=2. It is rather close to the results of numeric observations.

3.2 O(1/ε) time scale

At time scale O(1/ε) system (3) is reduced to

$$\varphi_1 = (1 - i\lambda - G(N^2))N\exp(i\delta),$$
$$\varphi_2 = N\exp(i\delta), \tau = \varepsilon t$$
$$\frac{\partial N}{\partial \tau} = \frac{-\lambda N}{D}$$
$$\frac{\partial \delta}{\partial \tau} = (-\lambda^2 + G(N^2) - G^2(N^2) - \qquad (6)$$
$$-2N^2 G(N^2)G'(N^2))/D$$
$$D = 2(1 + \lambda^2 + G^2(N^2) - 2G(N^2) -$$
$$-2N^2 G'(N^2) + 2N^2 G(N^2)G'(N^2))$$

Despite apparent complexity, system (6) is completely integrable for any function G. It is possible to prove that for any value of k>0.5 (which naturally corresponds to possibility of 1:1 resonance capture in the system under consideration) there exists interval of values for the damping coefficient $\lambda \in (0, \lambda_{\max})$ for which the slow flow described by (6) will exhibit breakdown, leading to efficient dissipation of energy. So, against an intuition, in order to get efficient dissipation one should keep the dissipation coefficient small enough.

4 Conclusions

- Targeted energy transfer may be achieved in the systems with soft nonlinearity.
- Transient responses of these systems with non-polynomial nonlinearities may be successfully treated with the help of Fourier expansion with respect to fast time scale and subsequent averaging
- Asymptotic analysis based on small parameter related to the mass ratio yields reliable analytic description of the transfer process and allows optimization of the parameter values.

Acknowledgments

The author is grateful to Israel Science Foundation (grant 486/05) for financial support.

References

Gendelman O.V. (2001) Transition of Energy to Nonlinear Localized Mode in Highly Asymmetric System of Nonlinear Oscillators, *Nonlinear Dynamics*, **25**, 237-253

Gendelman O.V. (2004) Bifurcations of Nonlinear Normal Modes of Linear Oscillator with Strongly Nonlinear Damped Attachment, *Nonlinear Dynamics,* **37**, 115-128.

Gourdon, E. and Lamarque, C.H., (2006) Nonlinear energy sinks with uncertain parameters, *Journal of Computational and Nonlinear Dynamics* **1,** 187-195

Lee, Y.S., Kerschen, G., Vakakis, A.F., Panagopoulos, P.N., Bergman, L.A., McFarland, D.M. (2006) Complicated dynamics of a linear oscillator with an essentially nonlinear local attachment, *Physica D* **204**, 41-69.

Panagopoulos P.N., Gendelman O., Vakakis A.F. (2007) Robustness of Nonlinear Targeted Energy Transfer in Coupled Oscillators to Changes of Initial Conditions, *Nonlinear Dynamics* , **47**, 377-387