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# ON STABILITY OF THE ELECTROMAGNETIC SUSPENSION ROTOR IN SPACE OF CONTROL PARAMETERS

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**Abstract**
The problem of stabilizing the rotor in an electromagnetic suspension is considered. It is shown that initially a nonlinear system can be reduced to a linear form via feedback linearization approach. For mathematical model of a rigid rotor, consideration of a delay in the control system results in restrictions on the maximum values of the control parameters of the linearized system. The numerical and experimental results of the model of a flexible rotor qualitatively coincide with analytical estimation of a domain of control parameters and reveal the advantage of a linear system over nonlinear one. Due to the absence of subharmonic vibration in the oscillation spectrum of the linearized system, the rotor oscillation amplitude is substantially reduced.

**Key words**
stability, electromagnetic suspension, rotor, control, bearing, external linearization.

## 1 Introduction

The non-contact electromagnetic suspension (EMS) of the rotor is an unstable object. Electromagnetic force only attracts the rotor and decreases with increasing distance between the rotor and the electromagnet. In order to stabilize the rotor, and also to give it the desired dynamic behavior a regulator is introduced into the control channel, namely, a dynamic link whose input receives a signal from the position sensor and the output signal current or voltage is fed to the electromagnet and controls the magnetic force.

In most of publications [Zhuravlev, 2003; Schweitzer and Maslen, 2009], to ensure the stability of the magnetic suspension of the rotor the simplest linear relationship between the displacement of the rotor and the current in the electromagnet based on a proportional-differential (PD) or proportional-integral-differential (PID) regulator is considered. The block diagram of the EMS control system with a PID controller is shown in Figure 1.

A sensor measures the displacement of the rotor from its reference position, a microprocessor as a controller derives a control signal from the measurement, a power amplifier transforms this control signal into a control current, and control current generates the magnetic forces within the actuating magnet in such a way that the rotor remains in its hovering position. The control law is responsible for the stability of the hovering state.

There are two control schemes which are called "voltage control" and "current control" [Schweitzer and Maslen, 2009]. Since current control requires a controller build into the amplifier, a current-controlled magnetic bearing should more precisely be described as voltage controller with a current control integrated in the amplifier.

In this paper the current control is considered, i.e. the control variable is the current in the windings of the electromagnet, while the control object is described by the system of differential equations each of which has a form similar to the following

$$m\ddot{x} = \frac{L_0 S_0}{2}\left[\left(\frac{I_2}{S_0 - x}\right)^2 - \left(\frac{I_1}{S_0 + x}\right)^2\right], \quad (1)$$

where $m$ is the mass of the rotor; $I_1$, $I_2$ are currents in windings; $L_0$ is a inductance at the central position of the rotor; $S_0$ is a nominal gap between the rotor and the stator of the EMS; $x$ is a displacement of the rotor from the central position.

For the linearization of the nonlinear function of (1), the bias currents are traditionally used. To do this, the same bias current is supplied to the opposite windings of the electromagnet. The control algorithms are implemented in such a way that the control current from the PD controller in one winding of the EMS is added

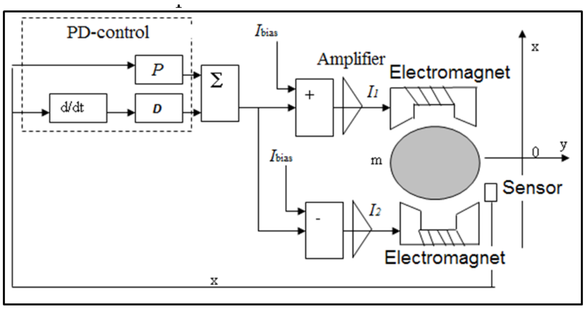


Figure 1. Magnetic bearing control loop with PD controller

to the constant bias current and in the other it is subtracted. This leads to the nonzero derivative $dF/dI$ near the point $= 0$ which theoretically contributes to the stability of the control system; the creation of unilateral traction forces acting simultaneously on the rotor on the side of each of the opposing magnets; the linearization of the dependence of the force on the current.

The last statement is valid for small deviations of the rotor from the point $x = 0$. The nonlinearity with respect to the coordinate is eliminated by including in the control signal a feedback signal along the coordinate.

It should be noted that when using bias currents in the control system, the electric energy consumed by the EMS grows and, accordingly, the heating of the EMS coils increases. Therefore, it is advisable to consider the possibility of using PID control without bias currents.

The special algorithms for compensating the nonlinear dependence of the electromagnetic force on current and gap are based on feedback linearization (or external linearization) method [Levine, Lottin and Ponsatr, 1996; Kato, Yoshida and Ohniwa, 2008; Balandin and Kogan, 2011; Malkin, 2011]. The advantage of this circuit in comparison with the introduction of bias current is obvious. The external linearization is introduced only in the winding into which the control current is applied. Therefore losses of consumed electric energy in EMS are reduced.

## 2 Theoretical analysis

We will consider a vertical rotor, hung out by eight radial and one axial electromagnets. Assuming that the displacement of the rotor in the axial direction is small in comparison with the nominal value of the gap we will consider separately the axial and radial motion of the rotor in electromagnetic bearings.

Dynamics of a rigid rotor is described by differential equations

$$\begin{aligned} &J\ddot{\alpha} = -l_1(F_2^{upp} - F_1^{upp}) + l_2(F_2^{low} - F_1^{low}) - J_z\omega\dot{\beta}, \\ &J\ddot{\beta} = l_1(F_3^{upp} - F_4^{upp}) - l_2(F_3^{low} - F_4^{low}) + J_z\omega\dot{\alpha}, \\ &m\ddot{x} = F_3^{upp} - F_4^{upp} + F_3^{low} - F_4^{low}, \\ &m\ddot{y} = F_2^{upp} - F_1^{upp} + F_2^{low} - F_1^{low}, \end{aligned} \tag{2}$$

where $x$ and $y$ are the coordinates of the center of mass of the rotor, $\alpha$ and $\beta$ are the rotational angles of the rotor relative to the $y$ and $x$ axes respectively, $l_1$, $l_2$ are the distances from the center of mass of the rotor to the upper and the lower electromagnetic bearings, respectively, superscripts *upp* and *low* indicate the electromagnetic forces acting on the rotor from the upper and lower electromagnetic bearings, $J$ is the main moment of inertia of the rotor, $\omega$ is a specified angular rotational speed of the rotor relative to the rotation axis.

The idea of feedback linearization for the object under consideration is presented in [Balandin and Kogan, 2011; Malkin, 2011] and involves the transition to new controls:

$$\begin{aligned} u_1 &= F_2^{upp} - F_1^{upp}, \quad u_2 = F_2^{low} - F_1^{low}, \\ u_3 &= F_3^{upp} - F_4^{upp}, \quad u_4 = F_3^{low} - F_4^{low}. \end{aligned} \tag{3}$$

For the controls introduced by this way the system (2) is transformed to the form

$$\begin{aligned} J\ddot{\alpha} &= -l_1u_1 + l_2u_2 - J_z\omega\dot{\beta}, \\ J\ddot{\beta} &= l_1u_3 - l_2u_4 + J_z\omega\dot{\alpha}, \\ m\ddot{x} &= u_3 + u_4, \\ m\ddot{y} &= u_1 + u_2. \end{aligned} \tag{4}$$

If the controls $u_i$ are given in the form

$$\begin{aligned} u_1 &= -(a_1y_{upp} + b_1\dot{y}_{upp}), \\ u_2 &= -(a_2y_{low} + b_2\dot{y}_{low}), \\ u_3 &= -(a_3x_{upp} + b_3\dot{x}_{upp}), \\ u_4 &= -(a_4x_{low} + b_4\dot{x}_{low}). \end{aligned} \tag{5}$$

where $x_{low}$, $y_{low}$ and $x_{upp}$, $y_{upp}$ are the coordinates of the rotor axis measured in the lower and upper bearings, respectively, and the parameters are defined as follows

$$\begin{aligned} a_1 = a_3 &= \frac{l_2}{l_1 + l_2}a, \quad a_2 = a_4 = \frac{l_1}{l_1 + l_2}a, \\ b_1 = b_3 &= \frac{l_2}{l_1 + l_2}b, \quad b_2 = b_4 = \frac{l_1}{l_1 + l_2}b, \end{aligned} \tag{6}$$

the system (2) takes the simplest form of a linear system with constant parameters

$$\begin{aligned} J\ddot{\alpha} &= -l_1 l_2(a\alpha + b\dot{\alpha}) - J_z\omega\dot{\beta}, \\ J\ddot{\beta} &= -l_1 l_2(a\beta + b\dot{\beta}) + J_z\omega\dot{\alpha}, \\ m\ddot{x} &= -(ax + b\dot{x}), \\ m\ddot{y} &= -(ay + b\dot{y}). \end{aligned} \quad (7)$$

The currents can be presented, for example, in the upper electromagnet as follows

$$I_1^{upp} = \begin{cases} \sqrt{\dfrac{2|u_1|}{L_0 S_0}}(S_0 + y_{upp}), & \text{if } u_1 < 0, \\ 0, & \text{if } u_1 \geq 0, \end{cases} \quad (8)$$

$$I_2^{upp} = \begin{cases} 0, & \text{if } u_1 < 0, \\ \sqrt{\dfrac{2|u_1|}{L_0 S_0}}(S_0 - y_{upp}), & \text{if } u_1 \geq 0. \end{cases} \quad (9)$$

A simple analysis shows that for positive values of the parameters $a$, $b$ the system (7) is asymptotically stable. To improve the quality of transients, we need to increase the values of these parameters. However, in practice the values of these parameters cannot be taken too large. The main reason for this phenomenon is a time lag in the circuit of formation of the control signals. For the simplest explanation of the delay effect, we will neglect the gyroscopic forces in the first two equations of system (7).Then all equations of this system become the same. Let us consider for definiteness the third equation in (7) written in the form

$$m\ddot{x}(t) = -\big(ax(t-\tau) + b\dot{x}(t-\tau)\big), \quad (10)$$

where the parameter $\tau$ determines the delay in the controller. The expansion of the right-hand side of this equation in series with respect to a small parameter $\tau$ with holding of terms of the first order allows us to obtain the stability conditions in the form

$$a > 0, \quad b > 0, \quad b - \tau a > 0, \quad b\tau < m. \quad (11)$$

## 3 Test rig

The study of control algorithms was carried out at the test rig of the scale model of the rotor of a turbomachine on electromagnetic bearings of SC “Afrikantov OKBM” which was created as part of an international project of a high-temperature reactor with GT-MGR gas-turbine cycle of energy conversion [http://www.okbm.nnov.ru].

The vertical rotor of the test rig consists of two flexible shafts (generator rotor model and rotor model of turbocharger) connected by a diaphragm clutch. The main technical data of the test rig are given in the following table.

| Data | Value |
|---|---|
| Total mass, kg | 1200 |
| Total length, m | 11 |
| First flexible mode, Hz | 17 |
| Second flexible mode, Hz | 45 |
| Third flexible mode, Hz | 85 |
| Maximum speed of rotation, rpm | 6000 |
| Nominal inductivity of radial magnet bearings $L_0$, H | 0.3 |
| Nominal air gap $S_0$, mm | 1.5 |
| Air gap to retainer bearing, mm | 0.4 |

The test rig ensures the identity of the amplitude-frequency characteristics with the rotor of the full-scale turbomachine.

## 4 Numerical analysis

The numerical analysis was carried out using the software “DIROM” developed by the experts from Institute of Mechanics of Lobachevsky State University of Nizhni Novgorod. Figures 2 and 3 show the test run of a scale model of the turbomachine rotor with a given uniform unbalance of 10 $\mu$m and the absence of traction with an acceleration of 1 Hz/s. It can be seen that in this case the external linearization gives a considerable reduction of the oscillation amplitude of the rotor.

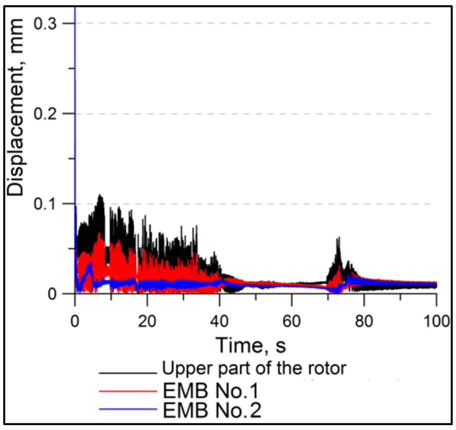


Figure 2. Time history of the displacements of the rotor with the PD regulator

Figures 4 and 5 show numerical results for the oscillation spectra of the test rotor under control by the PD and External Linearization regulators, respectively.

It can be seen that in the spectrum with the External Linearization algorithm only the current rotation frequency appears and there are no extraneous frequencies.

Figures 6 and 7 show the regions of the control parameters for scale model of the turbomachine rotor of generator using the PD and External Linearization algorithms.

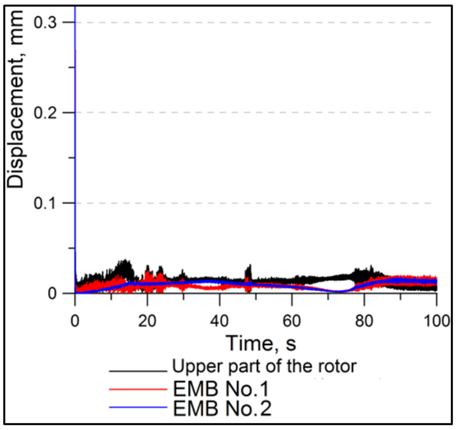


Figure 3. Time history of the displacements of the rotor with the External Linearization regulator

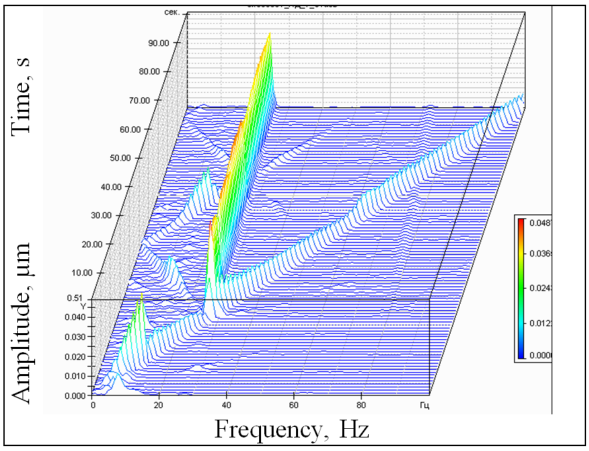


Figure 4. The oscillations spectra of the rotor with the PD regulator

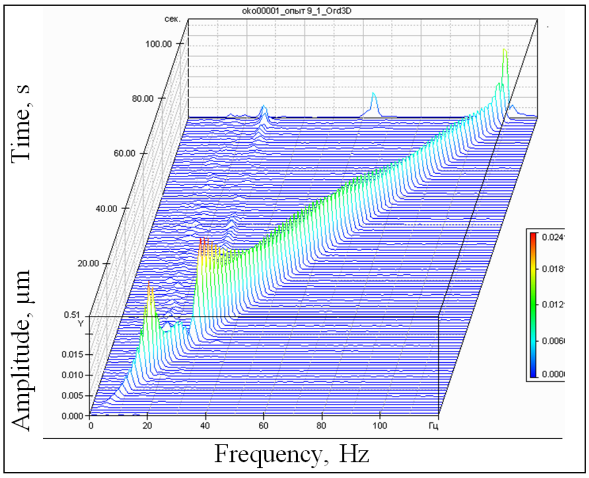


Figure 5. The oscillations spectra of the rotor with the External Linearization regulator

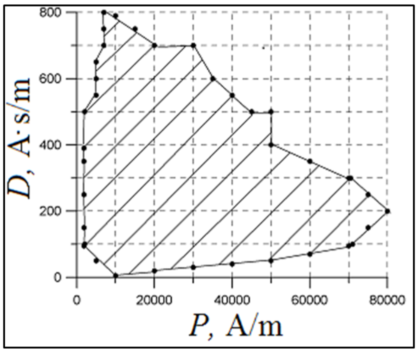


Figure 6. The region of the control parameters for the PD regulator

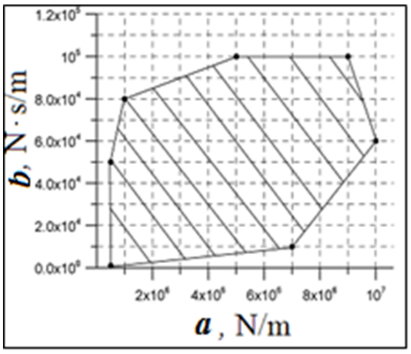


Figure 7. The region of the control parameters for the External Linearization regulator

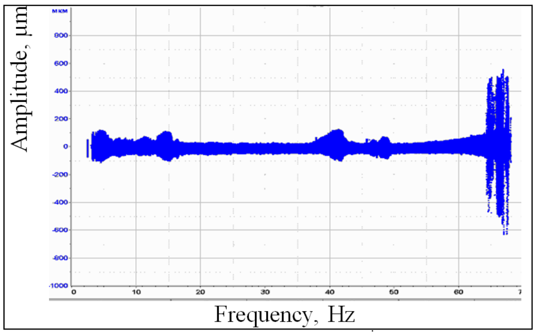


Figure 8. The amplitude of the oscillations of the rotor with the PD regulator

## 5 Experiment and concluding remarks

Figures 8 and 9 show plots of the dependence of the amplitude of the oscillations of the rotor on the rotational speed in the lower radial electromagnetic bearings (EMB). Control parameters for the PD controller are the following: upper EMB — $P = 10000$ A/m, $D = 50$ A · s/m; lower EMB — $P = 6000$ A/m, $D = 30$ A · s/m. Control parameters for the External Linearization regulator are the following: $a = 1.8 \cdot 10^6$ N/m, $b = 1.8 \cdot 10^6$ N · s/m.

On the Figure 9, one can see that the amplitude of the oscillations of the rotor does not exceed 160 $\mu$m at

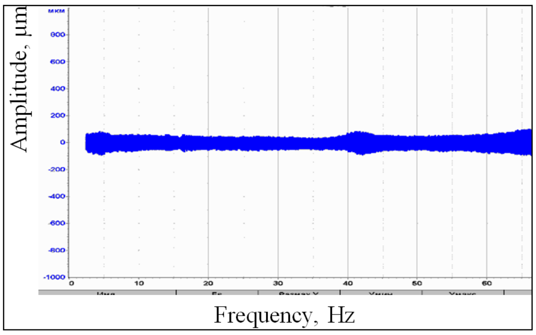


Figure 9. The amplitude of the oscillations of the rotor with the External Linearization regulator

frequencies up to 74 $s^{-1}$ that meets the requirements for this EMB systems.

These experiments allow us to conclude that the control scheme based on feedback linearization is rather promising and has obvious advantages over traditional scheme based on using bias currents.

## 6 Acknowledgement

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## References

Zhuravlev, Yu.N. (2003). *Active magnetic bearings. Theory, calculation, application*. SPb.: Politechnica.

Schweitzer, G. and Maslen, E. (2009) *Magnetic bearings. Theory, Design, and Application to Rotating Machinery*. Berlin: Springer.

Levine, J., Lottin, J. and Ponsatr, J.-C. (1996). A nonlinear approach to the control of magnetic bearings. *IEEE Trans. on Control Systems Technology*, **4**(5), pp. 524–544.

Kato, Y., Yoshida, T. and Ohniwa, K. (2008). Self-Sensing Active Magnetic Bearings with Zero-bias-current control. *Electrical Engineering in Japan*, **165**(2), pp. 69–76.

Balandin, D.V. and Kogan, M.M. (2011). Motion control for a vertical rigid rotor rotating in electromagnetic bearings. *Journal of Computer and Systems Sciences International*, **50**(5), pp. 683–697.

Malkin, S.A. (2011). Control of rotor motion in an electromagnetic suspension. *Bulletin of the UNN*, **4**(2), pp. 219–221.