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"BROWNIAN MOTION" BY MACROSCOPIC GROUP ROBOTS

Teturo Itami
Department of Mechanics and Robotics
Hiroshima International University
Japan
t-itami@it.hirokoku-u.ac.jp

Abstract
We apply microscopic Brownian motion to macroscopic transportation systems by group of robots with energy dissipation. We extend continuum mechanical picture of group robots so far developed by the author. We treat systems where frictional force proportional to velocity acts on each robot. Taking Hamiltonian with specified exponential function of time allows us to deal with motion of robots by canonical equation. We give methods how to explicitly calculate force between robots and the object that is to be transported. For this we set up an ordinary time-differential equation for temperature parameter. That parameter characterizes distribution function of robots in approximate equilibrium. We make use of the formula of the force in simulation studies. We obtain physically adequate results that the small/large frictional coefficient values lead to good/bad transportation characteristics. Comparison of our results based on continuum picture of robots with those by Newtonian mechanics for robots and objects leads to good agreement.

Key words
Group robotics, Liouville equation, energy disspation, friction proportional to velocity, temperature parameter.

1 Introduction

In cooperatively acting robots, we expect an intelligence that an individual robot will never achieve. We have various studies[Liu and Wu, 2001; Chen and Li, 2006; Li and Chen, 2006; Badano, 2008] on group robotics that have been thoroughly investigated[Ota, 2006] in a framework of multi-agent. Agents cooperation as a simple interaction based on sensors have enabled us to enhance the performance[Sugawara and Sano, 1997]. Sensing limitations have also been explicitly examined in robots similar to myopic ants[Gordon, 2010]. Some researchers[Schweitzer, 2003; Hänggi and Marcheson, 2009] have been inspired by a fact that a huge number of liquid atoms can move pollen floated on liquid, that have contributed to develop a new motor device in nano region. According to Brownian motion, it worked under temperature gradients. We can also notice that we do not need to equip the atoms with any sensing devices. To apply the idea of "Brownian motors" to macroscopic systems of robots, we have proposed[Itami, 2012; Itami, 2011] a transportation system of objects by group of robots. Robots corresponded to liquid atoms while objects that were transported by robots to floating pollen. By a word "macroscopic" we meant constituents of systems moved according to classical mechanics. We assumed $10^2 \sim 10^4$ constituent robots with extent $nm \sim m$. Under external potential field, our robots moved aimlessly and only collided with each other. They had a chance to collide with an object. Repetitive collision of robots with the object indirectly carried the object. As applications, we thought transporting garbage of various physical properties and removing obstacles at disaster spots. We also thought that when these robots in cylinders were controlled to efficiently push pistons, thermal cycles were designed to achieve high performance. It was obvious that calculating Newton equations of each robot becomes difficult when a number N of robots increases. However, in designing systems of group robots, we needed mathematical model that appropriately describes trends of the systems in time. It was better to calculate an average of dynamical state of robots than to directly follow locations and momenta of each robot and of the object in time. Based on the idea, a main dynamical variable in our formulation was a number density of the robots. A framework of Hamiltonian dynamics of robots enabled us to take such a number density. Liouville equation has been derived by canonical equations of motion for Hamiltonian systems. For macroscopic systems, we have to take a friction into account. When frictional force is present, energy of systems dissipates. In this article, we take systems with frictional force that is proportional to velocity. To the systems with energy dissipation, we apply a well-known extension[Kimura and Sugano, 1999] of canonical formalism. Distribution function represents continuum mechanical picture also for group robots under friction. The function turns out to take a specific form with temperature parameters in its equilibrium state. We set an ordinary differential equation for the temperature parameter by statistically averaging amount of potential energy of group robots.

First in Section **2** we explain results obtained in preceding articles[Itami, 2012; Itami, 2011]. For systems with energy dissipation, we introduce Hamiltonian in Section **3**. We assume friction proportional to velocity. We calculate an equilibrium distribution $f_v(v, x; t)$. The distribution gives number density of robots. The distribution function depends on “temperature” parameter $\tilde{\beta}(t)$. Section **4** gives an ordinary time differential equation that determines time trends of the parameter $\tilde{\beta}(t)$. We obtain expression of force acted on the object. For three combinations of values for proportionality constant of friction to velocity of both robots and the object, we show numerical simulation in Section **5**. Our results are physically acceptable as we have good/bad transportation characteristics under small/large friction force. Dependence of the features on strength of collision among robots and that on number of robots are also examined. Summary and discussion are given in Section **6**.This article shows contents those presented in conferences[Itami1, 2012; Itami2, 2012] with slight modifications.

2 Continuum Picture of Group Robots [Itami, 2012; Itami, 2011]

We have analyzed systems of group robots and objects in 2 dimensional plane region. Each robot with mass m was taken as point-like where its size was considered as its radius parameter a_R. For an object, an assumed form of a disc with its radius R_B and mass M was applied in a collision process. We assumed that a system of robots was described as a Hamiltonian dynamical system. Under the assumption, a continuum mechanical picture of a group of robots was developed. In Hamiltonian formulation, each robot with momentum p moving under potential energy $V(x; t)$ was assumed to have its free Hamiltonian

$$H_0^{'}(p, x; t) = \frac{p^2}{2m} + V(x; t) \tag{1}$$

Potential energy $V(x; t)$ was a sum of $V_0(x; t)$, control energy $V_{cnt}(x; t)$ of robots and reaction of surrounding walls, and $V_B(|x - X(t)|)$, collision between each robot at x and the object at $X(t)$.

$$V(x; t) = V_0(x; t) + V_B(|x - X(t)|) \tag{2}$$

An operation to control a movement of a group of robots was expressed as a potential $V_0(x; t)$ dependent on a location x of a robot. In our system, manipulating $V_0(x; t)$ **indirectly** transported the object by collision of robots on the object. Among any two robots, one centered at x and another at $x^{'}$, we assumed collision energy

$$V_{col}(|x - x^{'}|) \tag{3}$$

As a number density of robots with momentum p centered at x, we were able to define one-body distribution function $f_1(p, x; t)$. We introduce a correlation between robots, one with p, x and another one with $p^{'}$, $x^{'}$, in another number density, two-body distribution function $f_2(p, x, p^{'}, x^{'}; t)$. These distribution functions satisfied a following equation

$$\frac{\partial f_1}{\partial t} + [f_1, H_0^{'}] = -(N_0 - 1) \times \int d^2x^{'} d^2p^{'} [f_2, V_{col}] \tag{4}$$

We do not show in (4) arguments p, x and t of the functions on the left hand side and additionally $p^{'}$, $x^{'}$ of f_2 and V_{col} on the right hand side are not shown. We assumed that the distribution functions varied slowly in time. Under the assumption the following one-body distribution function

$$f_1^{eq}(p, x) = C \cdot e^{-\beta H_0^{'}(p,x;t)} \tag{5}$$

satisfied (4) in the absence of collision among robots, $V_{col} = 0$. Two constants C and β were calculated by the two conditions of total energy E_R and number N_0 of robots in the system.

Specific formulae of $V_0(x; t)$, $V_B(R)$ and $V_{col}(r)$ were given as shown below. External potential V_0 for robots was given by $V_{cnt}(x; t)$ added by repulsive force by walls. Under definitions $x_{\pm i} \equiv x_i \pm S_i$, the explicit form was

$$V_0(x; t) = V_{cnt}(x; t) + c_R \sum_{i=1}^{2} \left({x_{+i}}^{-n_{cR}} + {x_{-i}}^{-n_{cR}} \right) \tag{6}$$

For collision V_B between robots and the object and V_{col} among robots, following soft core potentials were adopted.

$$V_B(R) = \sigma_s \left(\frac{R_B}{R} \right)^{n_s} \tag{7}$$

$$V_{col}(r) = \sigma_v \left(\frac{a_R}{r} \right)^{n_v} \tag{8}$$

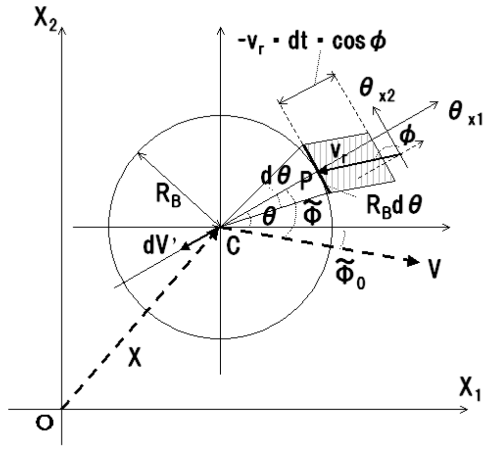

Figure 1. Collision of robots with a line element $R_B d\theta$ on the object B centered at $C(X)$. The angles $\tilde{\Phi}$ and $\tilde{\Phi}_0$ denote the direction of V clockwise measured from θ_{x_1} and X_1, respectively. Calculation of (12) needs these parameters $\tilde{\Phi}$ and $\tilde{\Phi}_0$.

We calculated force of robots under equilibrium distribution. The force acted on an object according to a configuration shown in Fig.**1**. We parametrized relative velocity of collision v_r by its absolute value v_r and an angle ϕ. We measured the angle ϕ from the vector θ_{x_1} in Fig.1. This vector indicated a direction from the center of the object to a collision point. Let e be coefficient of restitution. An increment $dV^{'}$ of the object velocity by collision with robots was given by

$$dV^{'} = \frac{(1+e)v_r \cos\phi}{1+\frac{M}{m}} \tag{9}$$

The increment $dV^{'}$ was in the direction of θ_{x_1}. Since we saw $\frac{\pi}{2} < \phi < \frac{3\pi}{2}$ in Fig.1, $dV^{'} < 0$. In the unit area there were

$$d^2N = N_0 f_1(p, x; t) d^2 v_r \tag{10}$$

robots. Relative velocity of these robots was $v_r \sim v_r + dv_r$. During time interval dt, with line element $R_B d\theta$ of the object only robots in the area

$$dS = R_B d\theta(-v_r \cos\phi)dt \tag{11}$$

collided. Each robot in the number $d^2N \times dS$ gave impulse $MdV^{'}$ to the object. When we integrated the impulse, net force acting on the object by robots was given as

$$F(X;t) = \frac{1}{dt}\int_{\theta=0}^{2\pi}\int_{v_r=0}^{\infty}\int_{\phi=\frac{\pi}{2}}^{\frac{3\pi}{2}} MdV^{'} d^2N dS \tag{12}$$

Reaction of the walls to the objects was expressed by an external potential

$$V_{B0}(X) = c_B \sum_{i=1}^{2}\left(X_{+i}{}^{-n_{cB}} + X_{-i}{}^{-n_{cB}}\right) \tag{13}$$

where $X_{\pm i} \equiv X_i \pm (S_i - R_B)$. Even when we took no interaction between robots group robots approximately reached its equilibrium only by energy exchange between each robot and the object.

3 Liouville Equation Under Friction

When frictional force proportional to velocity, $-\gamma\dot{x}$, is present in equation of motion

$$m\ddot{x} = -\frac{\partial V(x;t)}{\partial x} - \gamma\dot{x} \tag{14}$$

a Lagrangian is set as

$$L(x, \dot{x}; t) = e^{\frac{\gamma}{m}t}\left(\frac{m}{2}\dot{x}^2 - V(x;t)\right) \tag{15}$$

Euler-Lagrange equation of motion $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x}$ gives (14). Usual canonical method allows us to calculate canonical momentum

$$\begin{aligned} p &= \frac{\partial L(x, \dot{x}; t)}{\partial \dot{x}} \\ &= e^{\frac{\gamma}{m}t} m\dot{x} \end{aligned} \tag{16}$$

and Hamiltonian $H(p, x; t) = p \cdot \dot{x} - L$. Existence of Hamiltonian makes us possible to take number density of robots as dynamical variable. We have in (16) special dependence $e^{\frac{\gamma t}{m}}$ on time and the potential energy $V(x;t)$ has explicit time dependence. However, infinitesimal phase volume $d\Gamma \equiv d^2p_1 \cdots d^2p_{N_0} d^2x_1 \cdots d^2x_{N_0}$ is easily shown to be invariant in time.

$$d\Gamma(t) = d\Gamma(0) \tag{17}$$

This fact is proved in straightforward calculations. According to Hamiltonian equations, a Jacobian $J(t)$ given by

$$d\Gamma(t) = J(t)d\Gamma(0) \tag{18}$$

is 1 even when the Hamiltonian explicitly depends on time. We understand the fact (17) also in an intuitive argument. Let $N_0 = 1$ only for simplicity. Under slowly varying potential $V(x;t)$, (14) shows that a robot or particle with coordinate x moves to a point of extremum of V. The particle gradually lessens its velocity $\dot{x}$. The velocity $v \equiv \dot{x}$ damps in a manner as $v \sim e^{-\frac{\gamma}{m}t}$, since $m\dot{v} \sim -\gamma v$ is seen by (14). An infinitesimal volume in a space by v and x damps exponentially to zero accordingly. On the other hand, (16) tells that in

$$d^2pd^2x = e^{2\frac{\gamma}{m}t}m^2d^2vd^2x \tag{19}$$

we have the exponential function $e^{2\frac{\gamma}{m}t}$. This $e^{2\frac{\gamma}{m}t}$ and the factor $e^{-2\frac{\gamma}{m}t}$ in $d^2v \sim e^{-2\frac{\gamma}{m}t}$ cancel each other out and the infinitesimal volume (19) conserves in time.

Let

$$f_p(p, x; t) \tag{20}$$

be a one-body distribution function of robots in phase space: we have a number of $f_p(p, x; t)d^2pd^2x$ robots with momentum around $p \sim p + d^2p$ cenetered at around $x \sim x + d^2x$. According to conservation of phase volume, this one-body distribution function satisfies the following equation similar to (4)

$$\frac{\partial f_p(p, x; t)}{\partial t} + [f_p(p, x; t), H(p, x; t)] = \int d^2p^{'} \int d^2x^{'} \mathcal{F}(p^{'}, x^{'}; p, x; t) \tag{21}$$

The right hand side of (21) is an integration of colliding energy of robots with layout $(p^{'}, x^{'})$ with robots with layout (p, x) in the left hand side over layout $(p^{'}, x^{'})$. The right hand side of (21) is often approximated[Prigogine, 1984] as difference $f_p(p, x; t)$ itself from equilibrium distribution. This approximation allows our distribution to approach to its equilibrium one. If we set

$$f_p(p, x; t) = C_p(t)e^{-\beta(t)H(p,x;t)} \tag{22}$$

the 2-nd term on the left hand side of (21) reduces to Poisson bracket among Hamiltonian themselves; hence, it becomes to zero. When we consider (22) as equilibrium distribution, the right hand side of (21) is zero. Moreover, if variations of $V(x; t)$ and $e^{\pm\frac{\gamma}{m}t}$ is slow enough compared to the motion of group of robots, we also can set the 1-st term on the left hand side of (21) to be zero. This means that (22) satisfies (21). An equilibrium distribution f_p given by (22) is applied to calculating collision force between robots and object. Now let us note that momentum p is an auxiliary variable. Actual physical quantity is velocity v. It is convenient to transform f_p into a function f_v of velocity. Let us define the following quantities.

$$\tilde{\beta}(t) \equiv \beta(t)e^{\frac{\gamma}{m}t} \tag{23}$$

$$\tilde{H}(v, x; t) \equiv \frac{m}{2}v^2 + V(x; t) \tag{24}$$

$$C_v(t) \equiv m^2e^{2\frac{\gamma}{m}t}C_p(t) \tag{25}$$

Under these (23), (24) and (25), distribution function is given by

$$f_v(v, x; t) = C_v(t)e^{-\tilde{\beta}(t)\tilde{H}(v,x;t)} \tag{26}$$

A condition that we have N_0 robots in the system gives

$$N_0 = \int d^2v \int d^2xf_v(v, x; t) \tag{27}$$

4 Time Differential Equation for $\tilde{\beta}(t)$

We present here a time differential equation for a quantity $\tilde{\beta}(t)$. Energy of robots $E_{Rob}(t)$, that of the object $E_{Obj}(t)$ and interaction energy $E_{int}(t)$ among them are given as

$$E_{Rob}(t) = \sum_{i=1}^{N_0}\Big(\frac{m}{2}v_i^2 + V_0(x_i; t)\Big) \tag{28}$$

$$E_{Obj}(t) = \frac{M}{2}V^2 + V_{B0}(X) \tag{29}$$

$$E_{int}(t) = \sum_{i=1}^{N_0} V_B(|x_i - X(t)|) \tag{30}$$

Total energy is a sum of these (28), (29) and (30). The following equations of motion of robots and the object

$$m\dot{v}_i = -\nabla_iV_0 - \nabla_iV_B - \gamma v_i \tag{31}$$

$$M\dot{V} = -\nabla V_{B0} - \nabla_X \sum_{i=1}^{N_0} V_B - \delta V \tag{32}$$

allow us to calculate time differentiation of (28), (29) and (30). The results are added together to lead to

$$\frac{dE_{tot}(t)}{dt} = -\gamma \sum_{i=1}^{N_0} v_i^2 - \delta V^2 + \frac{\partial}{\partial t}\sum_{i=1}^{N_0} V_0(x_i; t) \tag{33}$$

In the following we replace quantities that relate to robots with statistical averages calculated by the distribution function $f_v(v,x;t)$, (26). To calculate time differentials of statistical quantities, we use a time differential of $\tilde{\beta}(t)$. Equation (33) allows us to obtain a time differential equation for temperature paramter. Statistical average of kinetic energy is given by

$$\overline{\frac{m}{2}v^2} = \frac{\int d^2v \int d^2x \frac{m}{2}v^2 f_v(v,x;t)}{N_0} \equiv \overline{K} \quad (34)$$

A definition

$$\begin{aligned} Iv[\tilde{\beta}(t)] &\equiv \int d^2v e^{-\tilde{\beta}(t)\frac{m}{2}v^2} \\ &= \frac{2\pi}{m\tilde{\beta}(t)} \end{aligned} \quad (35)$$

allows us to express the quantity (34) in a simple form

$$\begin{aligned} \overline{K} &= -\frac{\partial Iv[\tilde{\beta}(t)]}{\partial\tilde{\beta}(t)} \cdot Iv[\tilde{\beta}(t)]^{-1} \\ &= \frac{1}{\tilde{\beta}(t)} \end{aligned} \quad (36)$$

Equation (36) gives a meaning that average of kinetic energy of robots reduces to an inverse of the quantity $\tilde{\beta}(t)$. Kinetic theory of gas has an interpretation that average value of kinetic energy $= k_BT(k_B$:Boltzmann constant). According to the explanation, we call $\tilde{\beta}(t)$ an inverse of temperature or simply temperature parameter. Use of (36) allows us to represent a sum of (33), (28) and (30) as statistical average as

$$\begin{aligned} \frac{1}{N_0}\frac{dE_{tot}(t)}{dt} &= -\frac{2\gamma}{m}\overline{K} - \frac{\delta V^2}{N_0} + \overline{\frac{\partial V_0(x;t)}{\partial t}} \\ &= -\frac{2\gamma}{m}\frac{1}{\tilde{\beta}(t)} - \frac{\delta V^2}{N_0} + \overline{\frac{\partial V_0(x;t)}{\partial t}} \end{aligned} \quad (37)$$

In the same manner we have

$$N_0\overline{V(x;t)} = \int d^2v \int d^2x V(x;t) f_v(v,x;t) \quad (38)$$

Under a definition

$$Ix[\tilde{\beta}(t);t] \equiv \int d^2x e^{-\tilde{\beta}(t)V(x;t)} \quad (39)$$

(27) gives

$$\overline{V(x;t)} = -\frac{\partial Ix[\tilde{\beta}(t);t]}{\partial\tilde{\beta}(t)} \cdot Ix[\tilde{\beta}(t);t]^{-1} \quad (40)$$

We represent statistical average of a sum of (28) and (30) as

$$\begin{aligned} \frac{E_{Rob}+E_{int}}{N_0} &= \overline{K} + \overline{V(x;t)} \\ &= \tfrac{1}{\tilde{\beta}(t)} + \left(-\frac{\partial Ix[\tilde{\beta}(t);t]}{\partial\tilde{\beta}(t)}\right) \cdot Ix[\tilde{\beta}(t);t]^{-1} \\ &\equiv f(\tilde{\beta}(t);t) \end{aligned} \quad (41)$$

Time differentiation of (41) added by $E_{Obj} \cdot {N_0}^{-1}$ leads to a time differential of total energy per robot. (37) allows us to obtain

$$\begin{aligned} -\frac{2\gamma}{m}\frac{1}{\tilde{\beta}(t)} - \frac{\delta V^2}{N_0} + \overline{\frac{\partial V_0(x;t)}{\partial t}} \\ = \frac{d}{dt}f(\tilde{\beta}(t);t) + \frac{1}{N_0}\frac{dE_{Obj}(t)}{dt} \end{aligned} \quad (42)$$

From (42) we obtain an ordinary differential equation for $\tilde{\beta}(t)$ in time

$$\begin{aligned} \dot{\tilde{\beta}}(t) = \big(-\frac{2\gamma}{m}\frac{1}{\tilde{\beta}} - \frac{\delta V^2}{N_0} + \overline{\frac{\partial V_0(x;t)}{\partial t}} \\ -\frac{1}{N_0}\frac{dE_{Obj}}{dt} - \frac{\partial f}{\partial t}\big)\big(\frac{\partial f}{\partial\tilde{\beta}}\big)^{-1} \end{aligned} \quad (43)$$

The 3-rd and the 4-th terms of the numerator on the right hand side is explicitly calculated by time difference. As $E_{Rob}+E_{int}$ equals to $E_{tot}-E_{Obj}$, setting $t=0$ in (41) gives initial value $\tilde{\beta}(0)$ as

$$\frac{E_{tot}(0) - E_{Obj}(0)}{N_0} = f(\tilde{\beta}(0);0) \quad (44)$$

We calculate total energy $E_{tot}(0)$ on the left hand side and energy of the object $E_{Obj}(0)$ when we know initial layout and initial velocity of robots and object. Let us note that left hand side of (44) depends on number of robots N_0. At each stage of calculation, $C_v(t)$ is given by the condition (27) using $\tilde{\beta}(t)$ at that stage of time.

5 Simulation

In our formula (12) for net force, d^2N contains distribution function $f_1(p,x;t)$ as seen in (10). We replace this function f_1 by $f_v(v,x;t)$ calculated by (26) when friction force is present. The temperature parameter $\tilde{\beta}(t)$ in f_v is to be calculated along with motion of the object. In this section, numerical simulation is done by the formula (12). We expand the formula in a Taylor series in $|V|$. The 0-th approximation is quite easily calculated only by substitution, $C \to C_v$ and $\beta \to \tilde{\beta}$, in (32)[Itami, 2011] [1] as

$$F(X;t)|_{V=0} = -M\frac{1+e}{1+\frac{M}{m}}R_B N_0 C_v(\frac{\overline{v^2}}{2})^2$$
$$\cdot R_B \cdot \pi^2 \cdot c_{VB} \cdot \frac{\partial e^{-\tilde{\beta}V_0(X;t)}}{\partial X}$$
$$= +\frac{(1+e)M}{1+\frac{M}{m}}R_B^2\pi^2 N_0 C_v \frac{1}{\tilde{\beta}m^2}c_{VB}$$
$$\cdot e^{-\tilde{\beta}V_0(X;t)}\frac{\partial V_0(X;t)}{\partial X} \quad (45)$$

where $c_{VB} \equiv e^{-\tilde{\beta}V_B(|x-X|)}$ and we do not show dependence of F, C_v, $\tilde{\beta}$ and c_{VB} on time. The **negative** gradient of V_{cnt} forces the robots to move towards the area where we have small potential values. The robots collide more frequently than those in the area with large potential values. This resulted in the **positive** sign in (45) for the force on the object. In actual calculation, we also applied formulae with higher degrees $|V|^1$, $|V|^2 \cdots$. To clarify the point discussed above, we set a linear function as a specific form for a potential

$$V_{cnt}(x) = \alpha_1 x_1 + \alpha_2 x_2 \quad (46)$$

Parameters in MKS units are:

1. walls are modeled as $[-S_1, S_1] \times [-S_2, S_2] = [-1, 1] \times [-1, 1]$,
2. a number of robots is $N_0 = 200$,
3. mass and radius of robots are set as $m = 0.01$ and $a_R = 0.01$, respectively,
4. for the object we set its radius $R_B = 0.1$ and mass $M = 0.5$, respectively,
5. as coefficient of restitution in (9) and (45), we set $e = 1$
6. for interaction potentials given by (6), (7), (8) and (13), we set $c_R = 3 \times 10^{-5}$, $n_{cR} = 4$, $c_B = 3 \times 10^{-7}$, $n_{cB} = 4$, $\sigma_v = 0$(no collision between robots), $\sigma_s = 10$ and $n_s = 4$,
7. in (46) we set as $\alpha_1 = \alpha_2 = 0.1$ to make robots move from **upper right** $\rightarrow$**lower left** .

Calculations with various values of $\sigma_v = 1$, 10, and 100, and $N_0 = 50$ and 100 are also tried as shown in Fig.**4** and Fig.**5**, respectively. In the parameters, radii are set as $R_B = \frac{S_1}{20}, i = 1, 2$ and $a_R = \frac{R_B}{10}$. For mass, we set $m = \frac{M}{50}$. We take computational burden in direct simulation into account when setting parameters for interaction potentials and α_1, α_2. In an initial state at $t = 0.0$, the object is set at the origin, while robots are randomly laid out. Both are put at rest. Energy conditions are calculated as $E_{tot}(0) = 1443$ and $E_{obj}(0) = 0$ in (44) to give the initial value of $\tilde{\beta}(0)$.

[1] Coefficient of restitution e, that is tacitly taken $e = 1$ in [Itami, 2011], is explicitly shown in (45).

In the calculation we take up to $n = 3$ degree in the Taylor expansion of (12).

To finite difference of space and time, we apply

space: a numerical integration of (39) with 20 division,
time: a forward difference with $dt = 5 \times 10^{-6}$.

As friction acts on robots proportional to velocity, we take two values of proportionality constant $\gamma = 0.1$ and 0.2. Also for frictional force of object, we apply $\delta = 0.2$ and 0.5. We give results of simulation in Fig.**2**. As we set $\alpha_1 = \alpha_2$ in (46), $X_1(t) = X_2(t)$ and $V_1(t) = V_2(t)$ by symmetry between directions in x_1 and x_2. Trends in the figure show that small/large

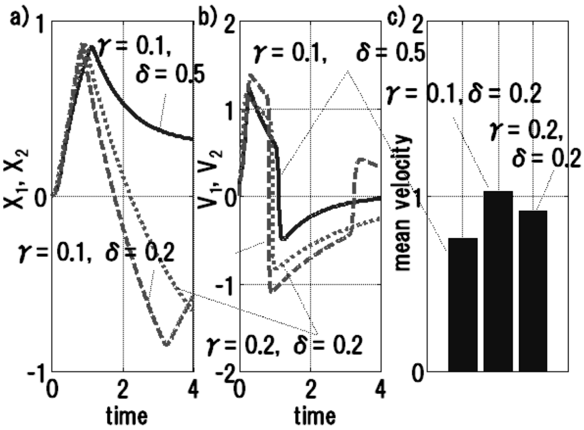

Figure 2. For three combinations of proportionality constants γ and δ, we show in a) trends of the coordinate $X_1(t)$ and in b) the velocity $V_1(t)$ of the object. Among these three trends, we compare transportation capabilities in c) as values of mean velocity $\overline{V_1(t)}$.

friction force corresponds good/bad characteristics of transporting object. We calculate each value of mean velocity in Fig.2c) simply as

$$\overline{V_1(t)} \equiv \frac{1}{T}\int_0^T dt\, V_1(t) \quad (47)$$

In (47) T is a time when $X_1(t)$ takes a maximum value: $T = 1.12$ for a combination ($\gamma = 0.1$, $\delta = 0.5$), 0.84 for (0.1, 0.2) and 0.94 for (0.2, 0.2). For a fixed $\gamma = 0.1$, we have better(faster) $\overline{V_1(t)} = 1.03$ for $\delta = 0.2$ than 0.76 for $\delta = 0.5$. Meanwhile, the result $\overline{V_1(t)} = 1.03$ for $\gamma = 0.1$ is better than 0.92 for $\gamma = 0.2$, when we set $\delta = 0.2$.

We compare the results with those by Newtonian mechanics for robots and objects. In direct calculation, integration method developed by Verlet[Gould and Tobochnik, 1996]

$$v(t+\frac{dt}{2}) = v(t) + \frac{dt}{2m}F(t) \quad (48)$$

$$x(t+dt) = x(t) + dt \cdot v(t + \frac{dt}{2}) \tag{49}$$

$$v(t+dt) = v(t + \frac{dt}{2}) + \frac{dt}{2m}F(t+dt) \tag{50}$$

is applied. We set time difference $\sim 0.01 \times R_B\sqrt{\frac{m}{\sigma_s}}$. This is required for conservation of total energy when our system would have no friction. Results, as shown in Fig.**3**a), present that transporting velocity is approximately calculated by continuum mechanics. Prediction of dead times must appropriately be done in our future studies. In Fig.3b), we show both a path of the object and paths of robots. Three robots that we arbitrarily choose are forced by a potential (46) gradually from upper right(I_R) to lower left(F_R). On the trips, one(dashed line) of them is recoiled by the object near the origin O. After that he obeys the potential force to go to his final position F_R, to where other two robots also move. As we assume no collision among robots, all robots can arrive at the same point. The object that starts at O is transported by collision with robots to W near the wall. In the time duration $t \in [0, 4]$ the object finally arrives at A. We compare trends with various values of σ_v, strength of collision between robots (8), in Fig.**4** with continuum mechanical calculations.

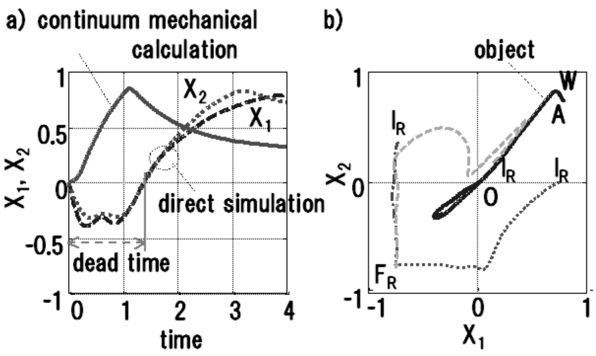

Figure 3. In a), for $\gamma = 0.1$ and $\delta = 0.5$, we compare the time trend $X_1(t)(= X_2(t))$(solid line) calculated by continuum mechanical method with the trends $X_1(t)$(dashed line) and $X_2(t)$(dotted line) by direct Newtonian mechanics. The solid line in this figure a) represents the same time trend of the solid line in Fig.2a). Calculation by Newtonian mechanics is represented in a $X_1 - X_2$ form by a solid line in b). Typical paths of robots are also shown in b) by a dashed, dotted and dash-dotted line.

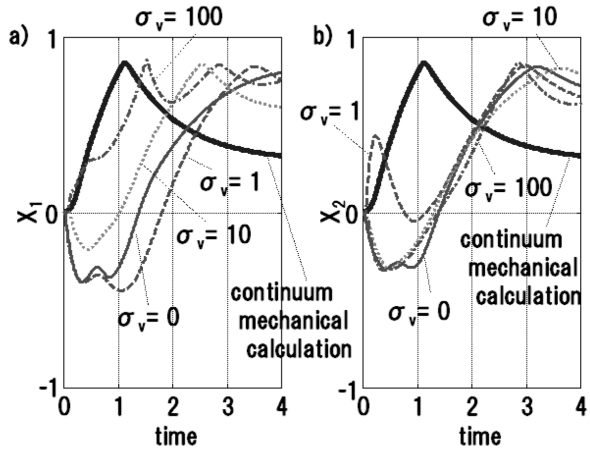

Figure 4. Newton mechanical calculations of coordinate $X_1(t)$ and $X_2(t)$ are compared with continuum mechanical simulation results (thick solid lines) in a) and b), respectively. In both figures, σ_v in (8) takes 0(no collision) for solid lines, 1 for dashed lines, 10 for dotted lines and 100 for dash-dotted lines. Thick solid lines in a) and b) are the same as the solid line in Fig.3a).

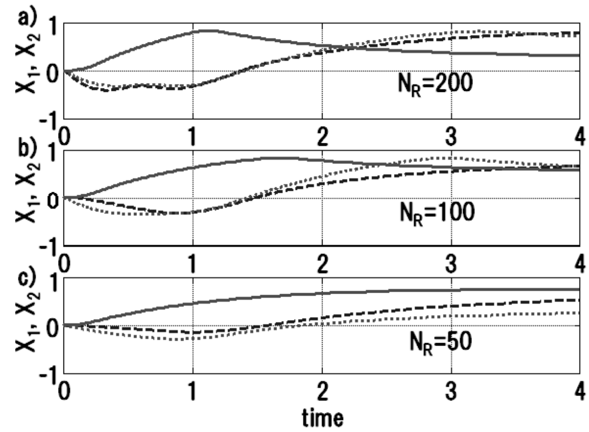

Figure 5. Solid lines in a)($N_0 = 200$), b)($N_0 = 100$) and c)($N_0 = 50$) represent time trends of the object calculated by continuum mechanics. Simulation results by Newton mechanics are shown in each graph as dashed lines for $X_1(t)$ and dotted lines for $X_2(t)$. Solid line in a) is the same as the solid line in Fig.3a).

We see no monotonic relation that larger/smaller σ_v corresponds to shorter/longer dead time at least in this figure. Regarding the number of robots N_0, simply the more robots we prepare, the faster transportation speed as shown in Fig.**5** is obtained. Friction coefficients take $\gamma = 0.1$ and $\delta = 0.5$ in Figs.4 and 5. In trends of the object simulated by Newton mechanics, we have those that take large deviation from the continuum mechanical prediction. Number of robots is not sufficient to completely describe the systems only in statistical methods. We must examine dependence of trends of the object on initial layout of robots. Such additional information can help our method. We have methods of molecular dynamics[Fincham, 1980]. When calculational space is uniform, we can set periodic boundary conditions. The periodicity reduces calculation volume to a great extent. Accordingly, simulation with $\sim 10^2$ molecules can give behavior of actual matter that contains $\sim 10^{23}$ order of molecules. In our transportation systems, however, robots are subject to external field (,including reaction force by objects) that breaks the uniformity. If molecular dynamical methods could be developed for systems under external field, our direct simulation by Newtonian mechanics would be enormously facilitated.

6 Summary and Discussion

In analyzing group of robots that transports object in continuum mechanical way, we studied more realistic condition. In this article, energies of robots and objects dissipate by friction. As we restricted ourselves to friction proportional to velocity, we treated energy disspation by Hamiltonian. This Hamiltonian allowed us to build equilibrium distribution of robots number as a function defined over velocity-coordinate space in the absence of collision among robots. The equilibrium distribution function was expressed by temperature paramter. This parameter gives average value of kinetic energy of robots. We gave a time differential equation that determines the temperature parameter. We calculated net force on the object under the equilibrium distribution. We analyzed motion of the object also under frictional force proportional to its velocity. Results allow physically acceptable explanation. We have the good/bad performance of transportation under the small/large frictional force. Good agreement with calculation by Newtonian mechanics was also obtained under appropriate initial layout of robots. By the method developed in this paper, we can design transportation system by group robots in a more realistic manner. We must examine our scheme of transportation by group of robots, especially without sensors for mutual information, in experiments.

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