Efficient Divided Difference filters applied to low accuracy inertial navigation system

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Abstract: The application of the divided derivative filters is described, applied and compared to the most popular tool in estimation , the extended Kalman filter (EKF). In our work the first order divided difference filter and the second order divided difference filter were simulated and applied to integrated navigation system including inertial measurement unit and global positioning system. These filters were tested and applied for low cost inertial sensors with very low accuracy, finally, the results are discussed and compared in different conditions.

Keywords: inertial navigation, kalman filter, extended kalman filter, derivative filter, global positioning system, integrated system.

1. INTRODUCTION

In low cost integrated system, generally, the kalman filter is used to combine the outputs of IMU ; accelerations and angular rate for strapdown configuration, with kinematics' model of vehicle. Using too, GPS outputs as position and velocity to correct INS errors growing during time. This correction is possible using in the linear model case, the Kalman filter and in the non linear case , the extended Kalman filter witch is the most useful filter in integrated navigation system. The main important problem in inertial navigation system is the bias and drift of the accelerometers and gyrometers. To salve this problem different approaches can be used such as indirect and direct mode and using different filters . The first means estimate the errors of the state vector using linear Kalman filter (Kim, J.2004) and add these values at the output of the inertial system calculation. It is for what we said indirect mode. For the second ; direct mode, it means that we estimate directly the state vector through non linear estimator as EKF or other non linear filter as Sigma point Kalman filters(Bo,T; Cui,P; Chen ,Y,*2005*), (Cho,S,Y; Wan ,S,C 2006). So, a new were introduced by (Norgaard. M, N. Poulsen, and O. Ravn.2000)as a divided derivative filters using polynomial Stirling's interpolation (Simandl,M.and J.Duik 2006). To compare and test the efficiency of these new algorithms, we use very low accuracy inertial sensors with a very high bias and drift. They were compared and tested to estimated position, velocity and attitude of an aerial vehicle.

2. INERTIAL NAVIGATION SYSTEM

The inertial navigation system is based on the using of the inertial sensors as accelerometers and gyroscopes, for plateforme inertial navigation systems, but usually these are very expensive and in several applications , we used strapdown inertial navigation systems , using gyrometers than gyroscopes. To understand more the inertial navigation, we have to distinguish the different frames included in this kind of navigation as in the below in figure(1).

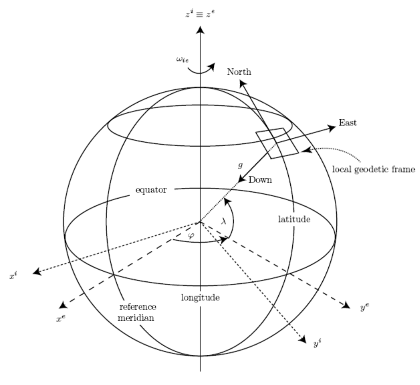

Fig.1. inertial (i) frame, earth frame (e) and navigation frame(n)

The mechanisation of strapdown inertial navigation is done as this (Savage 1998)(Crassidis, J.L 2006):

The attitude of the vehicle as, yaw , pitch and roll angle are obtained using the following integration :

$$\dot{\rho}=\begin{pmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{pmatrix}=C_{\dot{\phi}\dot{\theta}\dot{\psi}/pqr}\begin{pmatrix}P\\ q\\ r\end{pmatrix}=\frac{1}{\cos\theta}\begin{pmatrix}\cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta\\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta\\ 0 & \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}p\\ q\\ r\end{pmatrix} \quad (1)$$

According to the director cosines matrix and attitude integration's matrix:

$$R_{bn}=\begin{pmatrix}\cos\theta\cos\psi & -\cos\phi\sin\psi+\sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi\\ \cos\theta\sin\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi\\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta\end{pmatrix} \qquad (2)$$

And

$$C_{\dot\phi\dot\theta\dot\psi\,/\,pqr}=\begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta\end{bmatrix} \qquad (3)$$

With:

$$\begin{pmatrix}p\\ q\\ r\end{pmatrix}=\begin{pmatrix}p\\ q\\ r\end{pmatrix}_{mesuré}-R\begin{bmatrix}\Omega\cos\varphi+\dot\lambda\cos\varphi\\ -\dot\varphi\\ -\Omega\sin\varphi-\dot\lambda\sin\varphi\end{bmatrix}$$

φ : latitude of the vehicle.

Ω : earth angular rate.

For position and velocity integration we have to use the following equations in North, East and Down direction of navigation frame (n):

$$v^n=\begin{pmatrix}v_N\\ v_E\\ v_B\end{pmatrix}=\begin{pmatrix}(r_M+h) & 0 & 0\\ 0 & (r_T+h)\cos\varphi & 0\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}\dot\varphi\\ \dot\lambda\\ \dot h\end{pmatrix} \qquad (4)$$

Where $r^{LLa}=[\varphi\ \lambda\ \mathrm{h}]^T$ is the vector of the three positions: latitude, longitude and altitude of the vehicle.

By this, we can integrate the last equation to obtain the position in the navigation frame using the following equation:

$$\dot r^{LLa}=\begin{pmatrix}\dot\varphi\\ \dot\lambda\\ \dot h\end{pmatrix}=\begin{pmatrix}(r_M+h) & 0 & 0\\ 0 & (r_T+h)\cos\varphi & 0\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}v_N\\ v_E\\ v_B\end{pmatrix}=Dv^n \qquad (5)$$

2.2 Errors in inertial sensors

The principals errors in the inertial system are: bias, scale factors and non linearity as in the following equation:

$$a_{output}=a_{input}+\delta a_{bias}+K_a.a_{input}+K_0+K_1.a_{input}+K_2.a^2_{input}+\ldots$$
$$\omega_{output}=\omega_{input}+\delta\omega_{bias}+K_\omega.\omega_{input}+L_0+L_1\omega_{input}+L_2\omega^2_{input}+\ldots$$

The inertial navigation system presents some advantages and disadvantages as follow :

-Advantages: complete output solution, good accuracy during short time, high data rate and small size.

-Disadvantages: accuracy decrease after a long time, gravity sensitivity and obligatory external aid for initialization.

3.EXTERNAL AID FOR INERTIAL NAVIGATION

To correct the inertial navigation system we have to use an external aid as radio navigation system providing solutions in position and velocity (Kim,J,2004), in the best case attitude of the vehicle as it is presented in the figure2(Van der Merwe.R, E. Wan, and S. J. Julier 2004).

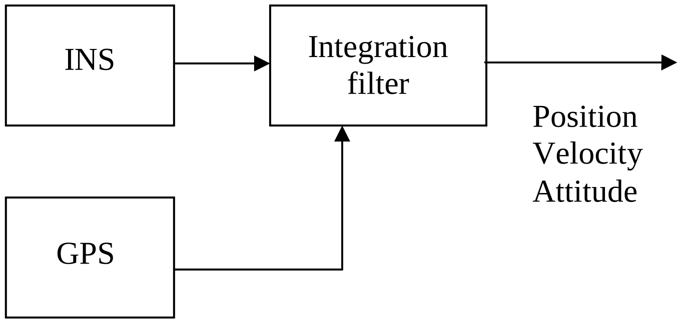

Figure.2. integrated navigation system using GPS as external aid of navigation.

In our work we assumed that the external aid is the global positioning system outputs as position ,velocity and using three receivers the attitude angles of the vehicle.

4. STATE EQUATIONS

In this case, we have choose to integrate the positions using the distances north, east and down without use latitude ,longitude and altitude as presented in previous paragraph, the velocities are integrated in north ,east and down directions, and the angles integration provide yaw, pitch and roll angles of the vehicle the state equations in discrete time could be written as the following forms (Sukkarieh.S 1999):

$$\begin{bmatrix}\mathrm{p}_n(k)\\ \mathrm{v}_n(k)\\ \psi_n(k)\end{bmatrix}=\begin{bmatrix}\mathrm{p}_n(k-1)+\mathrm{v}_n(k-1)\Delta t\\ \mathrm{v}_n(k-1)+\{\mathrm{C}_n^b(k-1)[\mathrm{f}_b(k)+\delta\mathrm{f}_b(k)+g^n]\}\Delta t\\ \psi_n(k-1)+\mathrm{E}_b^n(k-1)[\omega^b(k)+\delta\omega^b(k)]\Delta t\end{bmatrix}+\begin{bmatrix}\mathrm{w}_{p^n}(k)\\ \mathrm{w}_{v^n}(k)\\ \mathrm{w}_{\psi^n}(k)\end{bmatrix} \qquad (7)$$

Where $\mathrm{x}(k)=\mathrm{f}(\mathrm{x}(k-1),\mathrm{u}(k),\mathrm{w}(k))$ is the state vector to estimate and contains three positions, three velocities and three angles of vehicles attitude (Sukkarieh.S 1999).

$$\mathrm{E}[\mathrm{w}_v(k)]=0$$
$$\mathrm{E}[\mathrm{w}_v(k)\mathrm{w}_v(k)^T]=Q(k)=\begin{bmatrix}\sigma^2_{f^b} & 0\\ 0 & \sigma^2_{\omega^b}\end{bmatrix} \qquad (8)$$

Where

$$\nabla\mathrm{f}_x(k)=\begin{bmatrix}\dfrac{\partial\mathrm{p}^n(k)}{\partial\mathrm{p}^n(k-1)} & \dfrac{\partial\mathrm{p}^n(k)}{\partial\mathrm{v}^n(k-1)} & \dfrac{\partial\mathrm{p}^n(k)}{\partial\psi^n(k-1)}\\ \dfrac{\partial\mathrm{v}^n(k)}{\partial\mathrm{p}^n(k-1)} & \dfrac{\partial\mathrm{v}^n(k)}{\partial\mathrm{v}^n(k-1)} & \dfrac{\partial\mathrm{v}^n(k)}{\partial\psi^n(k-1)}\\ \dfrac{\partial\psi^n(k)}{\partial\mathrm{p}^n(k-1)} & \dfrac{\partial\psi^n(k)}{\partial\mathrm{v}^n(k-1)} & \dfrac{\partial\psi^n(k)}{\partial\psi^n(k-1)}\end{bmatrix} \qquad (9)$$

The observation equation from GPS is linear as:

$$Z_{k+1} = H(X_k) + V_k \qquad (10)$$

Where observation matrix is as follow:

$$H_k = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad (11)$$

Where :

$$E[V_k(k)] = 0$$
$$E[V_k(k)V_k(k)^T] = Q(k) \qquad (12)$$

$Q(k)$ is the noise covariance matrix of GPS measurement. The noise is assumed white Gaussian additive noise.

5. FILTERING

In this section the three algorithms are presented, the extended Kalman filter, the two divided difference filters at the first order and the second order interpolation and are as in the following subsections.

5.1Extended Kalman filter (EKF)

It is the most used technique in non linear filtering . for each time of calculation of the algorithm, the non linear dynamic and the measurement functions are approximated to the first order of Taylor development around the current estimates. The algorithm of EKF is done as this (Haykin, S. 2000) (Kim.J2004):

- Initialisation : $\hat{X}_0$ et P_0 .
- Prediction :

$$\hat{X}_{k+1/k} = f_k(\hat{X}_k)$$
$$P_{k/k-1} = F_k(\hat{X}_k)P_{k-1}F_k(\hat{X}_k)^T + Q_k$$

- Filtering :

$$K_k = P_{k/k-1}H_k^T(\hat{X}_{k/k-1})\left[H_k(\hat{X}_{k/k-1})P_{k/k-1}H_k^T(\hat{X}_{k/k-1}) + R_k\right]^{-1}$$
$$\hat{X}_k = \hat{X}_{k/k-1} + K_k\left[Z_k - h_k(\hat{X}_{k/k-1})\right]$$
$$P_k = P_{k/k-1} - K_k H_k(\hat{X}_{k/k-1})P_{k/k-1}$$

- $k = k + 1$

The extended Kalman filter is the useful filter in all engineering domains and especially in aerospace.

5.2 The 1st order Divided difference filter DD1

As a starting point of the derivation of the divided difference 1st order the basic structure of the Kalman filter can be supposed (Norgaard,M.,N.K.Poulsen and O.Ravn ,2000).

$$S_k = chol\ (P_k^{'});$$
$$\hat{z}_k^{'} = h_k\left(\hat{x}_k^{'}\right);$$
$$P_{z,k}^{'} = H\left(\hat{x}_k^{'}, S_k, h\right)H^T\left(\hat{x}_k^{'}, S_k, h\right) + R_k$$
$$P_{x,z}^{'} = S_k H^T\left(\hat{x}_k^{'}, S_k, h\right);$$
$$K_k = P_{x,z}^{'}\left(P_{z,k}^{'}\right)^{-1};$$
$$\hat{x}_k = \hat{x}_k^{'} + K_k\left(z_k - \hat{z}_k^{'}\right);$$
$$P_k = P_k^{'} - K_k P_{z,k}^{'} K_k^T;$$
$$S_k = chol\ (P_k);$$
$$\hat{x}_{k+1}^{'} = f_k\left(\hat{x}_k\right);$$
$$P_{k+1}^{'} = F\left(\hat{x}_k^{'}, S_k, h\right)F^T\left(\hat{x}_k^{'}, S_k, h\right) + Q_k$$

Where :

$$H\left(\hat{x}_k^{'}, S_k, h\right) = \left\{H_{j,i}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{h_{j,k}\left(\hat{x}_k^{'} + h*s_{x,i}\right) - h_{j,k}\left(\hat{x}_k^{'} - h*s_{x,i}\right)\right\}/2*h \qquad (13)$$

$$F\left(\hat{x}_k^{'}, S_k, h\right) = \left\{F_{j,i}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{f_{j,k}\left(\hat{x}_k^{'} + h*s_{x,i}\right) - f_{j,k}\left(\hat{x}_k^{'} - h*s_{x,i}\right)\right\}/2*h \qquad (14)$$

S_k is the Cholesky decomposition of the covariance matrix P_k .The functions f_k and h_k are approximated using a polynomial interpolation of Stirling's first order.

5.3 The 2nd order Divided difference filter DD2

As presented , the algorithm use the second order polynomial interpolation of Stirling's and we have to define two other matrix comparing with the 1st order divided difference filter DD1. these matrix are presented in the following expressions:

$$H^{(2)}\left(\hat{x}_k^{'}, S_k, h\right) = \left\{H_{j,i}^{(2)}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{\frac{\sqrt{h^2-1}\left(h_{j,k}\left(\hat{x}_k^{'} + hs_{x,i}\right) + h_{j,k}\left(\hat{x}_k^{'} - hs_{x,i}\right) - 2h_{j,k}\left(\hat{x}_k^{'}\right)\right)}{2h^2}\right\} \qquad (15)$$

$$F^{(2)}\left(\hat{x}_k^{'}, S_k, h\right) = \left\{F_{j,i}^{(2)}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{\frac{\sqrt{h^2-1}\left(f_{j,k}\left(\hat{x}_k^{'} + hs_{x,i}\right) + f_{j,k}\left(\hat{x}_k^{'} - hs_{x,i}\right) - 2f_{j,k}\left(\hat{x}_k^{'}\right)\right)}{2h^2}\right\} \qquad (16)$$

$$H\left(\hat{x}_k^{'}, S_k, h\right) = \left\{H_{j,i}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{h_{j,k}\left(\hat{x}_k^{'} + h*s_{x,i}\right) - h_{j,k}\left(\hat{x}_k^{'} - h*s_{x,i}\right)\right\}/2*h$$

$$F\left(\hat{x}_k^{'}, S_k, h\right) = \left\{F_{j,i}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{f_{j,k}\left(\hat{x}_k^{'} + h*s_{x,i}\right) - f_{j,k}\left(\hat{x}_k^{'} - h*s_{x,i}\right)\right\}/2*h$$

The algorithm DD2

The length of the difference interval is h=1 (Norgaard,M2000).

$$S_k = chol(\bar{P}_k);$$
$$\hat{z}_k = \frac{h^2 - n_x}{h^2} h_k(\bar{x}_k) + \frac{1}{2h^2}\sum_{i=1}^{n_x}\left(h_k(\bar{x}_k + h s_{x,i}) + h_k(\bar{x}_k - h s_{x,i})\right)$$
$$\bar{P}_{z,k} = H(\bar{x}_k, S_k, h)H^T(\bar{x}_k, S_k, h) + H^{(2)}(\bar{x}_k, S_k, h)H^{(2)T}(\bar{x}_k, S_k, h) + R_k$$
$$\bar{P}_{xzk} = S_k H^T(\bar{x}_k, S_k, h)$$
$$K_k = \bar{P}_{xzk}(\bar{P}_{z,k})^{-1}$$
$$\hat{x}_k = \bar{x}_k + K_k(z_k - \hat{z}_k)$$
$$P_k = \bar{P}_k - K_k \bar{P}_{z,k} K_k^T$$
$$S_k = chol(P_k)$$
$$\bar{x}_{k+1} = \frac{h^2 - n_x}{h^2} h_k(\hat{x}_k) + \frac{1}{2h^2}\sum_{i=1}^{n_x}\left(f_k(\hat{x}_k + h s_{x,i}) + f_k(\hat{x}_k - h s_{x,i})\right)$$
$$\bar{P}_{k+1} = F(\hat{x}_k, S_k, h)F^T(\hat{x} S_k, h) + F^{(2)}(\hat{x}_k, S_k, h)F^{(2)T}(F^{(2)}(\hat{x}_k, S_k, h)) + Q_k$$

Note: *in our case the state model is non linear only according to the system equation , the measurement equation is linear as for GPS observations ,position and velocity.*

6. SIMULATION

In our work the duration of simulation was 25 s at first time and 100s at second time, EKF, DD1 and DD2 were implemented using MATLAB software, with and without selective Availability of GPS outputs conditions, an we assumed that all noises are white Gaussian noises ,the simulations data are as in following:

Sample time Δt=0.005s, receiver noise=10m, accelerometer bias=1m/s ^{2} , gyrometer's bias=2°/s ,velocity=150-220m/s, initial Uncertainty in North distance : 1000m, initial Uncertainty in East distance : 1000m, initial Uncertainty initial in Down distance : 100m , initial Uncertainty in VN : 5m/s, initial Uncertainty in VE : 5m/s, initial Uncertainty in VD : 10m/s, initial Uncertainty in φ(yaw): 1°, Uncertainty in θ (pitch) : 1° , initial Uncertainty initial in ψ (roll) : 1°, and GPS data in SA conditions are augmented from 10m to 1000 m during 40 seconds for positions ,from 5 m/s to 50m/s for velocities and from1° to 10° for attitudes angles . concerning the initialization step of the three filters , it was the same and the following value: 80% from the true values of the state vector. GPS data were used at the frequency of 10Hz and the inertial integration process was made at the frequency of 200 Hz. We can observe in the following figures the simulation results and comment them easily.

6.1. velocity estimation

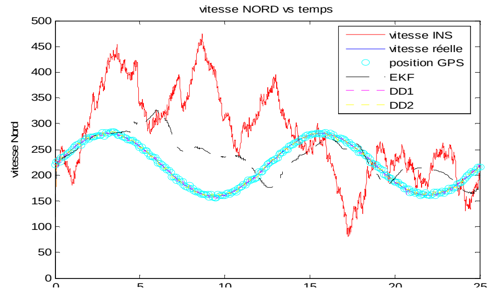

Fig.3. North velocity estimation (m/s) during time (s)

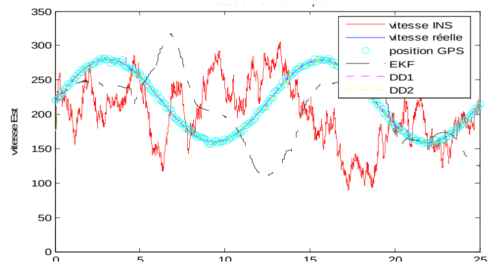

Fig.4. East velocity estimation (m/s) during time (s)

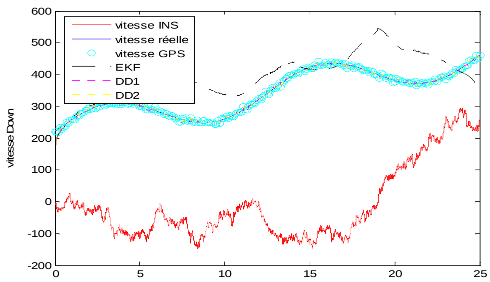

Fig.5 .Down velocity estimation(m/s) during time in second (s)

6.2. Attitude estimation (yaw,pitch and roll)

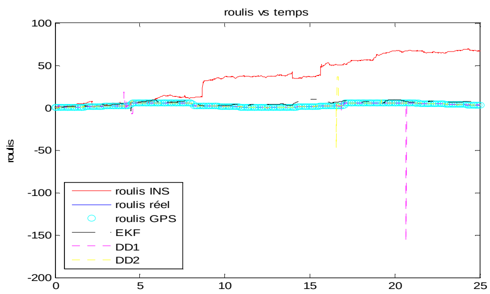

Fig.6. Roll angle estimation in degrees (°) during time in second (s)

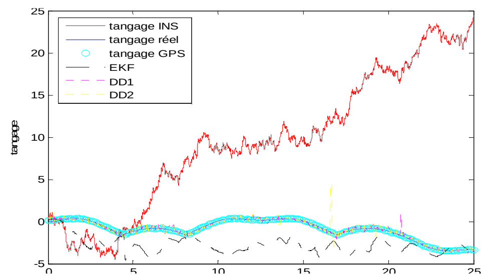

Fig.7 .pitch angle estimation in degrees (°) during timle in second (s)

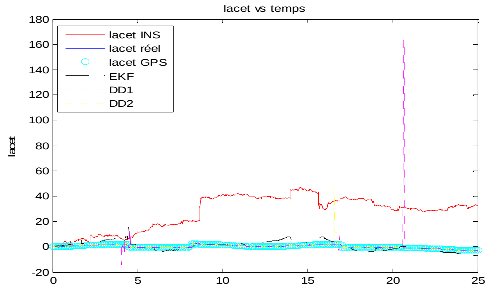

Fig.8. Yaw angle estimation in degrees (°) during time in second (s)

The simulations were repeated in selective availability conditions and have provided the following results:

6.3. velocity estimation with selective availability

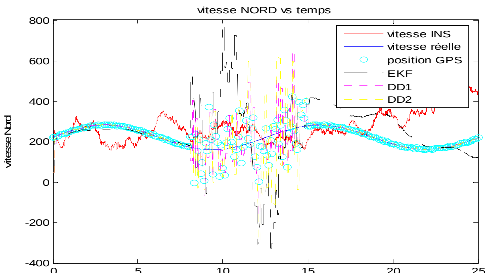

Fig.9. North velocity estimation (m/s) during time (s)

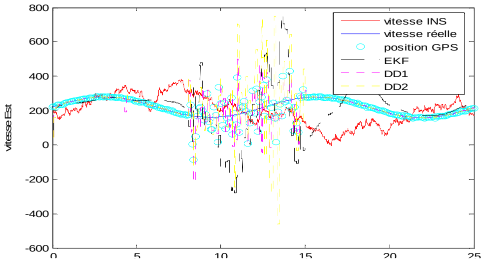

Fig.10. East velocity estimation (m/s) during time (s)

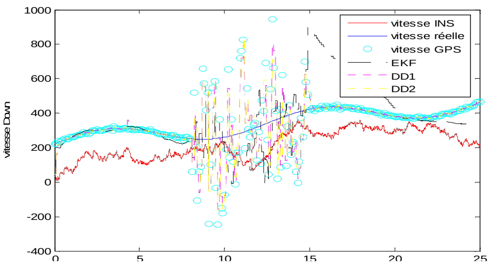

Fig.11. North velocity estimation (m/s) during time (s)

6.5 Observation

These simulations were made in order to proof the efficiency of the efficiency of the difference divided filters comparing with the extended Kalman filter as it is clear on the pictures witch show that EKF can't track the true trajectories or the true values of the state vector elements. So, when the selective availability is introduced , we observe that DD1 and DD2 return immediately on the tracked trajectory at the opposite of the EKF witch take more time to track and return on the desired values.

6.4. Attitude estimation

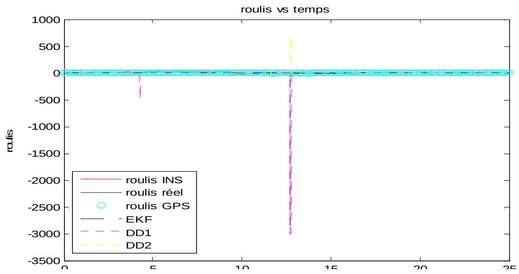

Fig.12. Roll angle estimation in degrees (°) during time in second (s)

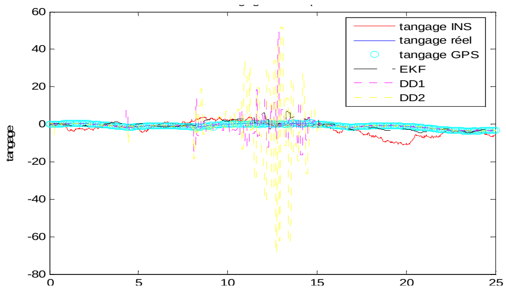

Fig.13. Pitch angle estimation in degrees (°) during time in second

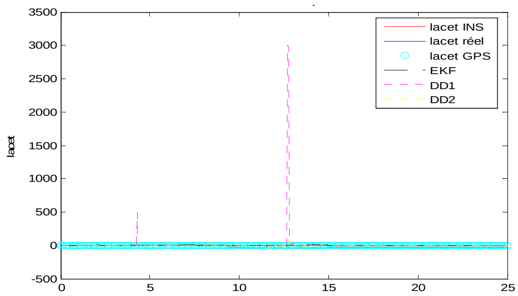

Fig.15. Yaw angle estimation in degrees (°) during time in second (s)

For the attitude estimation, the same observations ,in addition to the fact that DD1 and DD2 present some instabilities to estimate the angles of attitude during selective availability period.

7. CONCLUSION

After testing the various algorithms to estimate a nonlinear kinematics' model, we can conclude that DD1 and DD2 provide better results than EKF in all conditions of simulations, due to the low accuracy of the inertial sensors , the EKF can't estimate accurately the true positions, velocities and attitudes, at the opposite of the interpolation filters , they are centered on the true trajectories and provide high efficiency too, in selective availability conditions, when DD1 and DD2 return immediately after the end of Selective availability period, the EKF take more time to track the true values . To obtain the equivalent results with EKF, it will be necessary to estimate and augment the state vector with bias and drift estimation however with DD1 and DD2, with the simple state vector , we can estimate with very high accuracy the positions , velocities and the attitude of the vehicle. We can also said that in our case, DD1 and D2 provide same estimation's accuracy because the non linear function in our model is only the dynamic equation witch is used in the predictive step only , however the measurement equation is linear. In other way, some instabilities are observed using the interpolation filters in angle’s integration due to the non linear model of the three angles ; yaw, pitch and roll, so, in the future work we will try to modify the angle integration model to obtain a stable estimators an confirm these results by real experience on digital signal processing and will apply these algorithms using a non linear state model both on dynamic equation and measurement equation.

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