CONTINUUM MECHANICAL DESCRIPTION OF GROUP ROBOTS

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Abstract

A continuum mechanical picture to describe dynamics of group robots is proposed. We introduce a density function $f_1(\vec{x}, \vec{p}; t)$ representing a number of robots located at around $\vec{x}$ with momentum around $\vec{p}$ at time t. This scheme is applied to new transportation systems by group robots. We control potential force that motivate the robots. The objects that do **not** respond to the potential can be transported by repetitive collisions with the robots moving aimlessly under the potential force, a negative gradient of the potential.Each robot of the group need not have specific sensors for mutual information. Increasing number of the robots makes it difficult to predict collective motion of the robots in transportation tasks. To give a well organized strategy of designing the tasks, we apply the continuum mechanical description of robots. Our analysis tells that the objects are motivated by a **positive** gradient of the potential. The design is validated by comparing paths of the object predicted by the continuum picture of robots with that calculated by Newton dynamics.

Key words

Liouville equation, continuum mechanics, aimless motion of robots, potential force acting only on robots, transportation system of objects

1 Introduction

In cooperatively acting robots, we expect an intelligence that an individual robot will never achieve. We have so far various studies on group robotics as has thoroughly been investigated [Ota, 2006] in a framework of multi-agent.In addition in [Shimizu, Kawakatsu and Ishiguro, 2005],whether each robot can obtain global information or not have been argued. A centralization of control strategies has been one of points focused in [Reynolds, 1987]. A cooperation between agents by a simple interaction based on sensors have been addressed in enhancing the performance [Sugawara and Sano, 1997]. Sensing limitations have also been explicitly examined regarding robots similar to myopic ants [Gordon, 2010]. Meanwhile, a huge number of liquid atoms can move pollen floated on liquid. Some researchers [Hänggi and Marcheson, 2009] have started to develop a new motor device in nano region that works according to Brownian motion under temperature gradients. It is also notable that we need not equip the atoms with sensing devices.

Let us imagine that we apply the idea of "Brownian motors" to macroscopic systems of robots. Robots correspond to liquid atoms while objects moved by the robots to floating pollen. By a word "macroscopic" we mean constituents of the systems obey laws of classical mechanics. We assume $10^2 \sim 10^4$ constituent robots with extent $nm \sim m$. Our robots move aimlessly and can only collide with each other. They can move and have chance to collide with the object. Repetitive collision of robots with the object indirectly carry the object. Garbage with various physical properties is transported only by collision with robots. The group of robots removes obstacles remained at disaster sites that are not accessible by vehicles. These robots in cylinders controlled to efficiently push pistons have abilities to perform thermal cycles as desired. Now, Brownian motion can be described by fluctuation terms in equations of motion. In contrast, we need dynamical model for macroscopic systems as physical properties of the systems as "liquids" are not made preparations in advance. In addition to this, temperature distribution cannot move robots as they are far and away heavier than liquid atoms. For full capabilities of the systems, however, it is essential that a mathematical model to predict trends of the systems be available. When a number of robots increases, difficulties in the predictions cannot be avoided. In this paper, we propose a continuum mechanical description of group of robots. A main dynamical variable is a number density of the robots. We handle our system with such a number density in a framework of Hamiltonian dynamics of robots. We also offer an idea of controlling robots by external potential fields. For the systems, we need an apparatus to receive potential signal and a wheel mechanism on each robot. On the contrary, specific sensors for mutual information are not necessary. A new scheme of transportation system can in this way be possible, where we manipulate potential field acting only on robots every moment. When we work with robots that respond to a potential field – electric field, for example, even objects without electric conductivity can be transported by the field. This strategy differs from the conventional potential method [Bennet and McInnes, 2008] in that the potential does not directly force the object. We also give comparisons of results based on continuum mechanical description of robots with those by direct dynamical simulation. Results obtained are valid both for nano and for macro systems as long as the systems behave according to classical mechanics.

We first show in **2** how we build a contiuum mechanical description of robots.Section **3** presents analysis of collision processes of robots and objects. We give a specific formula of calculating force on the objects in **4**. We apply the formula in **5** with appropriate parameter values to carrying out simulation studies. Results based on continuum mechanical description of robots are compared with those by directly applying Newton dynamics. Summary and discussion are given in **6**.

2 Continuum mechanical description of robots

In n dimensional space, both a robot and an object to be transported by a group of robots are dealt with as a classical mechanical mass point with freedom n. In a potential energy that models collision between robots, size of a robot and that of an object are considered as their radius parameters a_R and R_B, respectively. An assumed form of a disk(we restrict ourselves to $n = 2$ in the paper) with its radius R_B for the object is applied when we analyze a collision process. Under an assumption that a system of robots is described as a Hamiltonian dynamical system, a continuum mechanical picture of a group of robots is developed. Meanwhile,the object is still handled in mechanics of mass point. We can easily extend the Hamiltonian dynamics to systems with friction force proportional to velocity. This point will be discussed in forthcoming papers.

Let $\omega \equiv \{\vec{x}, \vec{p}\}$ be a set of a location $\vec{x}$ of a robot and a momentum $\vec{p}$ with which the robot moves. When we set aside a potential energy $V_{col}(r_{ij})$, (3), that expresses a collision process among robots, under existence of an object located at $X(t)$, a motion of each robot is determined by a Hamiltonian

$$H_0^{'}(\omega; t) = H_0(\omega; t) + V_B(|\vec{x} - \vec{X}(t)|) \qquad (1)$$

that consists of a free Hamiltonian

$$H_0(\omega; t) = \frac{\vec{p}^2}{2m} + V_0(\vec{x}; t) \qquad (2)$$

and a potential energy $V_B(|\vec{x} - \vec{X}(t)|)$ that represents external action of the object located at $\vec{X}(t)$ at time t on the robot located at $\vec{x}$. In the free Hamiltonian $H_0(\omega, t)$, (2), an operation to control a movement of a group of robots is expressed as a potential $V_0(\vec{x}; t)$ dependent on a location $\vec{x}$ of a robot. In our system, manipulating $V_0(\vec{x}; t)$ **indirectly** transports the object by collision of robots on the object. In that sense, the external field V_0 depends on time t. The potential V_0 also expresses action of boundary walls that encounter a finite area of a system. With defining a distance $r_{ij} \equiv |\vec{x}_i - \vec{x}_j|$ between centers of robots i and j, potential energy

$$V_{col}(r_{ij}) \qquad (3)$$

represents collision between these robots. A total Hamiltonian of a group of robots in the transportation system is given as

$$H(\omega_1, \omega_2, \cdots, \omega_N; t) = \sum_{i=1}^{N} H_0^{'}(\omega_i; t) + \sum_{i<j} V_{col}(r_{ij}) \qquad (4)$$

Canonical equations are given as follows

$$\frac{d}{dt}\vec{x}_i = \frac{\partial H}{\partial \vec{p}_i} \qquad (5)$$

$$\frac{d}{dt}\vec{p}_i = -\frac{\partial H}{\partial \vec{x}_i} \qquad (6)$$

It is obvious that calculation based on these equations (5) and (6) to be utilized for feedback action in control systems becomes difficult, when a number N of robots increases. It is better to calculate an average of dynamical state of robots than to directly follow locations and momenta of each robot and of the object in time. Such average calculation will be more efficiently done than direct dynamical simulation. Based on an idea above stated, we introduce a function $f(\omega_1, \omega_2, \cdots \omega_N; t)$ to describe average behavior of the system. A value

$$f(\omega_1, \omega_2, \cdots \omega_N; t) d^{2n}\omega_1 d^{2n}\omega_2 \cdots d^{2n}\omega_N \qquad (7)$$

is a probability that the location and the momentum of a robot No.1 lie in an interval $\vec{x}_1 + d^n\vec{x}_1$ and $\vec{p}_1 + d^n\vec{p}_1$, $\cdots$ those of No.N lie in an interval $\vec{x}_N + d^n\vec{x}_N$ and $\vec{p}_N + d^n\vec{p}_N$.

After applying canonical equations (5) and (6),we can calculate a time derivative of f using the Poisson bracket as

$$\frac{d}{dt} f(\omega_1, \cdots, \omega_N) = -[f(\omega_1, \cdots, \omega_N), H] + \frac{\partial f(\omega_1, \cdots, \omega_N)}{\partial t} \qquad (8)$$

that becomes equivalent to Liouville equation when we note $\frac{df}{dt} = 0$ that generally holds in Hamiltonian dynamics. If we are interested only in behavior of ω_N, integration over $\omega_1, \cdots, \omega_{N-1}$ gives the following one body density function [1].

$$f_1(\omega_N) \equiv \int d^{2n}\omega_1 \cdots \int d^{2n}\omega_{N-1} f(\omega_1, \cdots, \omega_N) \tag{9}$$

Definitions of two body density function

$$\begin{aligned} &f_2(\omega_{N-1}, \omega_N) \equiv \\ &\int d^{2n}\omega_1 \cdots \int d^{2n}\omega_{N-2} f(\omega_1, \cdots, \omega_N) \end{aligned} \tag{10}$$

allows us to write down our governing equation as

$$\begin{aligned} &\frac{\partial f_1(\omega_N)}{\partial t} + [f_1(\omega_N), H_0^{'}(\omega_N)] = \\ &-(N-1) \times \\ &\int d^{2n}\omega_{N-1} [f_2(\omega_N, \omega_{N-1}), V_{col}(r_{N-1,N})] \end{aligned} \tag{11}$$

With the object, a Hamiltonian is given as

$$\begin{aligned} H_B(\vec{X}, \vec{P}; t) = \frac{\vec{P}^2}{2M} + V_{B0}(\vec{X}; t) \\ + \sum_{i=1}^{N} V_B(R_i) \end{aligned} \tag{12}$$

where the potential V_{B0} simply expresses action of the boundary walls, force inevitably acted on the object, like gravitation. It is emphasized that we **do not manipulate** V_{B0}. Robots that we move by the external potential $V_0(\vec{x}; t)$ carry the object via an interaction potential

$$V_B(R_i) \tag{13}$$

with defining $R_i \equiv |\vec{x}_i - \vec{X}|$ as a distance between the center of robot i and an object.

In the following, we give specific formulae of $V_0(\vec{x}; t)$ in (2), $V_{B0}(\vec{X}; t)$ in (12), $V_{col}(r)$ in (3) and $V_B(R)$ in (13). External potential V_0 for robots is given by $V_{cnt}(\vec{x}; t)$ in (34) added by repulsive force by walls. Under supposition that closed region in n-dimensional space is given as $[-S_1, +S_1] \times \cdots \times [-S_n, +S_n]$, we assume for repulsive potential as inverse power of $x_{\pm i} \equiv x_i \pm S_i$,

$$\begin{aligned} V_0(\vec{x}) &= V_{cnt}(\vec{x}; t) \\ &+ c_R \sum_{i=1}^{n} \left\{ \left(\frac{1}{x_{+i}}\right)^{n_{cR}} + \left(\frac{1}{x_{-i}}\right)^{n_{cR}} \right\} \end{aligned} \tag{14}$$

External potential V_{B0} for the objects are also given by inverse power of $X_{\pm i} \equiv X_i \pm (S_i - R_B)$,

$$V_{B0}(\vec{X}) = c_B \sum_{i=1}^{n} \left\{ \left(\frac{1}{X_{+i}}\right)^{n_{cB}} + \left(\frac{1}{X_{-i}}\right)^{n_{cB}} \right\} \tag{15}$$

For collision between robots expressed by the potential V_{col} and the potential V_B between robots and the object, we adopt soft core potential given by

$$V_{col}(r) = \sigma_v \left(\frac{a_R}{r}\right)^{n_v} \tag{16}$$

$$V_B(R) = \sigma_s \left(\frac{R_B}{R}\right)^{n_s} \tag{17}$$

3 Collision process

Let us consider a collision process of a robot with an object in a frame where the object is at rest.Take a configuration as shown in Fig.**1**. A disc (robot) with radius

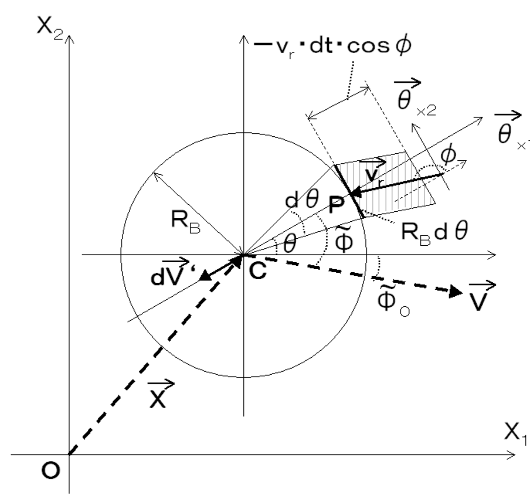

Figure 1. Collision of robots with a line element $R_B d\theta$ on the object B centered at $C(\vec{X})$.

a_R collides with relative velocity $\vec{v}_r$ towards a surface point P on a disc (object) with radius R_B. Assume

[1] In the following, we do not indicate time dependence.

that robot takes velocity $\vec{v}_r^{'}$ and the object takes $d\vec{V}^{'}$ after collision. The process of collision is governed by momentum conservation law

$$m\vec{v}_r = Md\vec{V}^{'} + m\vec{v}_r^{'} \tag{18}$$

When a_R is sufficiently smaller than R_B, the velocity $d\vec{V}^{'}$ is parallel to a vector $\vec{\theta}_{x1}$ that gives a direction of $\overrightarrow{CP}$. Coefficient of restitution is calculated as

$$e = \frac{\vec{v}_r^{'}|_{\theta_{x1}} - dV^{'}}{0 - \vec{v}_r|_{\theta_{x1}}} \tag{19}$$

Unknown variables $dV^{'}$ and $\vec{v}_r^{'}$ are determined by equations (18) and (19). If robot takes an angle ϕ anticlockwise from $\vec{\theta}_{x1}$, component of the object velocity $d\vec{V}^{'}$ takes

$$dV^{'} = \frac{(1+e)v_r \cos\phi}{1 + \frac{M}{m}} \tag{20}$$

Let $\vec{\theta}_{x2}$ be a direction vector rotated by a right angle anticlockwise from $\vec{\theta}_{x1}$. The vector $d\vec{V}^{'}$ is in the direction as shown in Fig.1, $dV' < 0$, as only robots with relative velocity

$$\vec{v}_r = v_r \cos\phi \vec{\theta}_{x1} + v_r \sin\phi \vec{\theta}_{x2} \tag{21}$$

with angle ϕ satisfying $\frac{\pi}{2} < \phi < \frac{3\pi}{2}$ could collide to the point P on the object. We further assume that the collision is completely elastic to take $e = 1$. In the Fig.1, $\tilde{\Phi}$ and $\tilde{\Phi}_0$ represent directions of $\vec{V}$ clockwise from $\vec{\theta}_{x1}$ and X_1, respectively. We apply anticlockwise directions $\Phi \equiv 2\pi - \tilde{\Phi}$ and $\Phi_0 \equiv 2\pi - \tilde{\Phi}_0$ in calculating force on an object in 4.

Let N_0 be a total number of robots in the system. A number of robots we find in each unit area with relative velocity between $\vec{v}_r$ and $\vec{v}_r + d^2\vec{v}_r$ is calculated to be

$$d^2N = N_0 \cdot f_1(\omega) d^2\vec{v}_r \tag{22}$$

The integral element is $d^2\vec{v}_r = v_r dv_r d\phi$. In a time interval dt, each robot in the shadowed region

$$dS = R_B d\theta \cdot (-v_r \cos\phi) dt \tag{23}$$

can collide the line element $R_B d\theta$ on the point P to give the impulse $Md\vec{V}^{'}$ to the object. We count total number of such robots as a product $d^2N \cdot dS$, (22) multiplied by (23). Force on the object is an integration of the impulse $Md\vec{V}^{'} \cdot d^2N \cdot dS$ acted on the line element $R_B d\theta$ from $\theta = 0$ to 2π divided by the time interval dt.

$$F(\vec{X}) = \frac{1}{dt} \int_{\theta=0}^{2\pi} \int_{v_r=0}^{\infty} \int_{\phi=\frac{\pi}{2}}^{\frac{3\pi}{2}} Md\vec{V}^{'} \cdot d^2N \cdot dS \tag{24}$$

Substituting specific formulae of potential energies to (24) allows us to numerically integrate Newton equation of the object in time. Note that the walls give repulsive force on the object in a form V_{B0} in (12) whether the force as shown by (24) acts or not.

4 Approximate formula

When it varies slowly in time, the density function

$$\begin{aligned} f_1^{eq}(\omega) &= C \cdot e^{-\beta H_0^{'}(\omega;t)} \\ &= C \cdot e^{-\beta \frac{\vec{p}^2}{2m}} e^{-\beta V_0} e^{-\beta V_B} \end{aligned} \tag{25}$$

approximately satisfies (11) with $V_{col} = 0$, i.e. $\sigma_v = 0$ in (16). The condition $V_{col} = 0$ means that we ignore collision among robots. Although this condition is unrealistic, it is only relaxation time [Prigogine, 1984] to bring $f_1(\omega)$ to $f_1^{eq}(\omega)$ that mainly depends on strength of collision V_{col}. The weaker the collision tends, the longer the relaxation time becomes. In calculating force acted on the object according to (24), we assume that our one body density function $f_1(\omega)$ **immediately** approaches to its equilibrium $f_1^{eq}(\omega)$. Two constants C and β in (25) are determined by the two conditions that in the system we have total number of robots N_0 and total energy E_0. Substitution of an expression $\vec{p} = m(\vec{v}_r + \vec{V})$ in the argument of exponential function in the right hand side of (25) into (24) leads to

$$\begin{aligned} \vec{F}(\vec{X}) = \frac{1}{dt} \int_{\theta=0}^{2\pi} \int_{v_r=0}^{\infty} \int_{\phi=\frac{\pi}{2}}^{\frac{3\pi}{2}} M \frac{2v_r \cos\phi}{1+\frac{M}{m}} \vec{\theta}_{x1} \\ \cdot N_0 \cdot C \cdot e^{-\beta \frac{m(\vec{v}_r + \vec{V})^2}{2}} \cdot e^{-\beta V_0(\vec{x};t)} \\ \cdot e^{-\beta V_B(|\vec{x} - \vec{X}|)} \cdot v_r dv_r d\phi \\ \cdot R_B d\theta \cdot (-v_r \cos\phi) dt \end{aligned} \tag{26}$$

Squared mean value of velocity is calculated by distribution function (25) as

$$\begin{aligned} \overline{\vec{v}^2} &= \frac{\int d^2\vec{x} d^2\vec{v} \vec{v}^2 f_1^{eq}(\omega)}{\int d^2\vec{x} d^2\vec{v} f_1^{eq}(\omega)} \\ &= \frac{2}{m\beta} \end{aligned} \tag{27}$$

Relative velocity $\vec{v}_r$ and the object velocity $\vec{V}$ are normalized by $\sqrt{\overline{\vec{v}^2}/2} = \sqrt{1/(m\beta)}$ to give nondimensional vectors $\vec{\psi}$ and $\vec{\Theta}$, respectively. Integration of (26) by variable $\vec{v}_r$ is calculated as the following nondimensional integral

$$\mathcal{I}(\vec{\Theta}) \equiv \int_{\phi=\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2\phi d\phi \int_{\psi=0}^{\infty} \psi^2 e^{-\frac{(\vec{\psi}+\vec{\Theta})^2}{2}} \psi d\psi \tag{28}$$

multiplied with $(\sqrt{\overline{\vec{v}^2}/2})^4$. Apart from a constant, (26) is evaluated as

$$\int_0^{2\pi} d\theta e^{-\beta V_0(\vec{x})} \mathcal{I}(\vec{\Theta}) \vec{\theta}_{x1} \equiv \sum_{n=0}^{\infty} \frac{\vec{\mathcal{F}}_n(\vec{X})}{n!} \tag{29}$$

Degree n on the right hand side of (29) corresponds to a Taylor expansion of the integrand function in the ψ integral (28) around $\Theta = 0$,

$$e^{-\frac{(\vec{\psi}+\vec{\Theta})^2}{2}} = e^{-\frac{\vec{\psi}^2}{2}} \cdot \left\{ \frac{1}{0!} + \frac{-\vec{\Theta}\cdot\vec{\psi}}{1!} + \frac{-\Theta^2 + (\vec{\Theta}\cdot\vec{\psi})^2}{2!} + \cdots \right\} \tag{30}$$

Inner-product formula $\vec{\Theta}\cdot\vec{\psi} = \Theta\psi\cos(\phi - \Phi)$ makes us possible to do straightforward integration of (28). The 0-th order value corresponding to the object at rest is calculated as

$$\begin{aligned}\vec{\mathcal{F}}_0(\vec{X}) = &\int_0^{2\pi} d\theta \Big(e^{-\beta V_0(\vec{X})} \\ &+ R_B \big(\cos\theta \frac{\partial e^{-\beta V_0(\vec{X})}}{\partial X_1} + \sin\theta \frac{\partial e^{-\beta V_0(\vec{X})}}{\partial X_2}\big) \\ &+ O(R_B^2)\Big) \pi \vec{\theta}_{x1} \\ = &\, R_B \pi^2 \frac{\partial e^{-\beta V_0(\vec{X})}}{\partial \vec{X}} + O(R_B^3)\end{aligned} \tag{31}$$

We finally obtain the 0-th approximation formula of the force (26) acted on the object as

$$\begin{aligned}\vec{F}(\vec{X})|_{V=0} = &-M \frac{2}{1+\frac{M}{m}} R_B N_0 C (\frac{\overline{\vec{v}^2}}{2})^2 \\ &\cdot R_B \cdot \pi^2 \cdot c_{VB} \cdot \frac{\partial e^{-\beta V_0(\vec{X})}}{\partial \vec{X}} \\ = &+ \frac{2M}{1+\frac{M}{m}} R_B^2 \pi^2 N_0 C \frac{1}{\beta m^2} c_{VB} \\ &\cdot e^{-\beta V_0(\vec{X})} \frac{\partial V_0(\vec{X})}{\partial \vec{X}}\end{aligned} \tag{32}$$

where c_{VB} is evaluated as $e^{-\beta V_B(|\vec{x}-\vec{X}|)}$. Note that, in the approximation that the object is at rest, the force (32) acted on the object is proportional to the **positive** gradient $\frac{\partial V_0}{\partial \vec{X}}$. That the force is in the positive direction is originated from the robot density pressure. Robots moves towards the area with small potential values according to the **negative** gradient of the external potential. This results in high robot density that can move the object towards a region with lower density of robots. In actual calculation, not only (32) but also formulae with higher degree n in (29) or (30) are applied. The formula for $n = 1$ is given as

$$\vec{\mathcal{F}}_1(\vec{X}) = 2\pi\sqrt{2\pi}\Theta e^{-\beta V_0(\vec{X})} \begin{pmatrix} \cos\Phi_0 \\ \sin\Phi_0 \end{pmatrix} \tag{33}$$

that is parallel to the object velocity $\vec{V}$.

5 Simulation

The negative gradient of V_{cnt} forces the robots to move, whereas, as shown in (32), its positive gradient acts on the object in an approximation that the object is at rest. To clarify the point discussed above, we set a linear function as a specific form for a potential V_{cnt}.

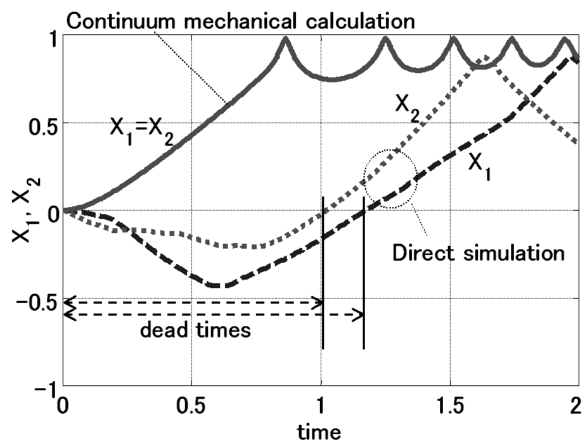

Figure 2. Trends X_1 and X_2 in time calculated based on continuum picture of robots are compared with those by dynamical simulation of Newton equations.

$$V_{cnt}(\vec{x}) = \alpha_1 x_1 + \alpha_2 x_2 \tag{34}$$

Parameters in MKS units are:

1. walls are modeled as $[-S_1, S_1] \times [-S_2, S_2] = [-1, 1] \times [-1, 1]$,
2. a number of robots is $N_0 = 50$,
3. mass and radius of robots are set as $a_R = 0.01$ and $m = 0.01$, respectively,
4. for the object we set its radius $R_B = 0.1$ and mass $M = 0.5$, respectively,
5. for interaction potentials given by (14) to (17), we set $c_R = 3 \times 10^{-5}$, $n_{cR} = 4$, $c_B = 3 \times 10^{-7}$, $n_{cB} = 4$, $\sigma_v = 0$, $\sigma_s = 10$ and $n_s = 4$,
6. in (34) we set as $\alpha_1 = \alpha_2 = 0.1$ to make robots move from **upper right** →**lower left** .

In the parameters, radii are set as $R_B = \frac{S_1}{20}, i = 1, 2$ and $a_R = \frac{R_B}{10}$. Regarding mass, we set $m = \frac{M}{50}$.

We take computational burden in direct simulation into account when setting parameters for interaction potentials and α_1, α_2. In an initial state $t = 0.0$, the object is set at the origin, while robots are randomly laid out. Both are put at rest. In describing the process by the continuum mechanical picture of robots, the results are those as shown in solid lines[2] in Fig.**2**. In the calculation we take up to $n = 3$ degree in the Taylor expansion (29) or (30). Dashed and dotted lines correspond to those calculated by direct simulation. Detail of the result of direct simulation along with a few snap shots is shown in Fig.**3**. We see that, in direct dynamical simulation, at the beginning robots under the potential V_{cnt} push the object in the negative gradient direction $-\frac{\partial V_{cnt}}{\partial x}$. Such process explains dead time characteristic seen in Fig.2. As one body density function $f_1(\omega)$ was assumed to immediately approach to its equilibrium $f_1^{eq}(\omega)$, the dead time is not represented by the continuum mechanical calculation of robots in its present form. Meanwhile, after dead time, we see in Fig.2 that dashed and dotted lines of direct simulation are well fitted by solid lines based on the continuum picture. This means that slopes of dashed and dotted lines have almost the same values as those of the solid lines. To examine dead times, we need information on how fast $f_1(\omega)$ approaches to its equilibrium one $f_1^{eq}(\omega)$ [Prigogine,1984].

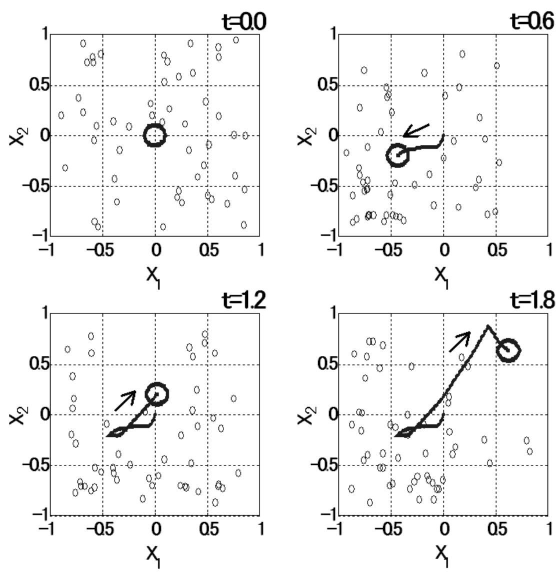

Figure 3. The object is shown as a circle with solid line and the robots are shown as $N = 50$ small circles. Arrows in three figures at $t = 0.6, 1.2$ and 1.8 indicate directions where the object moves towards. Solid lines in these figures show paths of the object.

6 Summary and discussion

We proposed a continuum mechanical method to describe collective dynamics of group robots. Under an assumption that Hamilton dynamics applies to the group, we described behavior of the robots by one body probability density function f_1. To state a concept, we examined how to transport objects by an ensemble of robots. An explicit formula of force acted on the object was obtained by analyzing collision process based on the function f_1. We found that the objects are moved along a positive gradient of the potential that only applies to robots. The formula for force acted on the object was applied to simulate transportation system. Results were compared with that by direct dynamical calculation. Velocity of transportation was well simulated by our continuum mechanical description of robots. For the reason that we assumed that the function f_1 immediately approches to the equilibrium one, dead time found in dynamical collision process was not seen in the continuum mechanical calculation. A scheme to calculate dead time is to be developed. Although robots were prepared without sensing devices for mutual information in this paper, the proposed method of describing group robots in the continuum mechanical way also applies to robots that can communicate each other. Mounting a wheel mechanism under a potential signal on many robots requires examination by experiments.

[2] In Fig.2, we have two solid lines, which overlap one another.

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