Design of Helicopter Autopilot*

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Abstract: An automatic stabilization system was designed on a standard computer for a small electrical helicopter. A TV sensor consists of two Web cameras connected to the computer and four onboard diodes. Results of experiments are reported where the full autopilot system stabilizes a helicopter near a fixed 3-dimensional point that is not far from the ground. It is shown also that an onboard sensor with three gyroscopes provides reliable information about angular velocities.

Keywords: Helicopter control, image recognition, on-line control, nonlinear models, stability analysis.

1. INTRODUCTION

The problem of helicopter modeling and autopilot design appeared to be much more complicated than that of an aircraft. The system is essentially interconnected and aerodynamic forces are not derived directly from the standard blade theory. A small electrical helicopter as a dynamic plant has a time constant much less than 1 s, and therefore control system must be very sensitive to measurements. The plant is also nonlinear and unstable. An autopilot system presented in this paper was designed by complete nonlinear modeling of the main rotor dynamics, sensor study, experimental identification and LQ control design.

The electrical helicopter Walkera X450 was chosen for autopilot design. It has the Hiller hub, the main rotor diameter is 700 mm, the weight is 830 g, the useful load is 300 g. There are 4 controls which are sent by the radio signal from the control panel equipped with two joysticks. The first two controls determine angles of the swash plate, the other two controls determine collective pitches of the main and tail rotors. The main rotor frequency depends also on the third control and takes values between 27 and 29 Hz.

2. TV OBSERVATION

Two standard Web cameras were connected to computer through the USB ports. The cameras are located at arbitrary places in 2-4 meters from the helicopter. No calibration or position adjustment is needed. 4 small diodes were attached to the helicopter, and the distances between them were measured and known by the computer. This information is enough for calculation of the full helicopter state vector by one image in a camera coordinate system. The spots from 4 diodes on the screen are recognized, their centers and shapes are calculated. It is not a complicated geometrical task to find three dimensional coordinates of a tetrahedron tops by their two dimensional projections on the screen.

The navigation system consists of two parts: primary adjustment and tracking. Primary adjustment means calculation of



Fig. 1. Electrical helicopter Walkera X450.

the mutual positions of cameras including focus coordinates and Euler angles. The input is only one image of the diodes tetrahedron in each camera.

The helicopter state vector was converted to the earth coordinate system. The origin of this system is a point in the middle between the two camera focuses. The focuses lies in the OYZ plane while the axis OY is vertical. The vertical direction is recognized in the image by an additional vertical line with two diodes installed in the room. The axis OX is directed to the half space of the helicopter.

Tracking is made by the Extended Kalman-Bucy filter. Camera position parameters are estimated with errors that must be corrected during observation. These parameters are included in the full state vector of the system which contains 20 variables. Coefficients of the linearized plant equations are calculated near the balanced realization with zero linear and angular velocities.

3. TRACKING OF HELICOPTER MOTION

The TV images are received asynchronously with the averaged speed of 30 shots per second. The tracking system calculates the predicted diodes positions after receiving a new TV image. Then a strobe is selected around the predicted position for each

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diode. A spot from a diode is searched in the strobe in a similar way to the primary recognition.

When all spots are found their centers and approximating ellipsoids are determined. They describe the results of position estimates on the screen and their expected accuracy.

The spot positions on the screen are nonlinear functions of the state vector

$$y = h(S)$$

where the vector *y* consists of 2-dimensional coordinates of the diodes images on the camera screen. Some diodes can be lost or shadowed, and therefore the dimension of *y* may vary from image to image.

The state dynamics is described by

$$\hat{S} = f(S, u).$$

Assume the state vector *S* is estimated as S^0 to an appropriate accuracy such that the error $\tilde{S} = S - S^0$ is relatively small. Then linearization gives

$$\dot{\widehat{S}} \approx f(S^0, u) + f'_S(S^0, u)\widetilde{S},$$

$$y \approx h(S^0) + h'(S^0)\widetilde{S}.$$

The linearized Kalman filter is applied to obtain an estimate of the deviation \tilde{S} .

The functions f and h are linearized in the strobe of tracking to an appropriate accuracy. Experiments have shown that the measurement noises are small enough such that this filter provides good performance.

The observation system does not depend on the number of detected diode spots in the image. Some of spots can be shadowed or lost on the bright background. Even in the case of completely shadowed diodes for a couple of images the Kalman filter does not lose the helicopter unless it makes a new manoeuvre.

4. AERODYNAMIC MODEL OF HELICOPTER

Helicopter dynamics is described by the equation

 $\dot{S} = f(S, u),$

where u is control containing cyclic and collective pitch control of the main rotor and directional control by the tail rotor. The function f is given by a system of nonlinear algebraic equations.

The state vector *S* contains 12 variables of the helicopter position in the earth coordinate system and 8 components of the camera positions. The cameras positions do not change. Therefore, the last 8 components of the vector $\dot{S} = f(S, u)$ are zeros.

The helicopter state vector consists: position of the center of gravity (x, y, z), its linear velocity $V = (V_x, V_y, V_z)$, the helicopter Euler angle (γ, Φ, θ) and the angular velocities $\Omega = (\Omega_x, \Omega_y, \Omega_z)$. The equations

$$\dot{x} = V_x, \qquad \dot{y} = V_y, \qquad \dot{z} = V_z$$

are trivial. Dynamics of the Euler angles is determined by the angular velocities according to the equations

$$\begin{pmatrix} \dot{\gamma} \\ \dot{\Phi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{\cos \Phi}{\cos \theta} & 0 & -\frac{\sin \Phi}{\cos \theta} \\ -\tan \theta \cos \Phi & 1 & \tan \theta \sin \Phi \\ \sin \Phi & 0 & \cos \Phi \end{pmatrix} \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix}.$$

The most important part of the helicopter dynamics are the equations for linear and angular velocities that are determined by forces and torques. In the helicopter system of coordinates the helicopter dynamics is described by the equations

$$\dot{V} = \frac{1}{m} (F_{main} + F_{gyro} + F_{tail} + F_{mg}),$$

$$\dot{\Omega} = I_{inert}^{-1} (M_{main} + M_{gyro} + M_{tail})$$

m is the mass of helicopter, $I_{inert} = diag(I_x, I_y, I_z)$ is the diagonal matrix with moments of inertia, $(F_{main}, F_{gyro}, F_{tail})$ and $(M_{main}, M_{gyro}, M_{tail})$ are the forces and torques vectors of main rotor blades, servo rotor blades and tail rotor blades, respectively, F_{mg} vector of gravity.

The forces and torques are the functions of the state vector. The explicit formulas for them and the corresponding numerical algorithm of their implementation were developed from the impulse theory of the main rotor Mil (1966); Esaulov (1977); Volodko (1984)

An effective numerical algorithms and software were also developed to calculate balanced realization with an arbitrary constant linear velocity vector V and an arbitrary constant yaw velocity Ω_{v} .

5. CONTROL DESIGN

The main goal of the controller is stabilization of the helicopter around a chosen balanced state vector. In particular, the stationary state was of the main interest. Stabilization was achieved by the linear feedback

$$u(t) = L_{\delta}\delta(t) + L_{\Omega}\Omega(t) + L_{V}V(t) + L_{X}X(t) + (a+bt)$$

where $\delta = \operatorname{col}(\gamma, \Phi, \theta)$ is the vector of Euler angles, $X = \operatorname{col}(x, y, z)$ is the helicopter position, $L_{\delta}, L_{\Omega}, L_{V}$ are the constant matrices to be designed, *a* and *b* correspond to the linear trend of the electrical part of the main rotor. The vectors Ω and *V* are in the helicopter coordinate system.

This controller was obtained from the standard LQ theory with appropriate choice of coefficients in the cost function. The helicopter equations were linearized

$$\widetilde{S} = A\widetilde{S} + B\widetilde{u}$$

where *A* and *B* contain partial derivatives in the balanced state vector.



Fig. 2. Estimates of the trends of "zero" levels in the 4 control channels

The cost function contains only deviations of the helicopter position (x, y, z) from its initial value and a regularizing term with deviation of control from the balanced values.

The linear trend of the electrical part of the main rotor is essential and quickly changes. It is necessary to track this change on-line. Results of the trend estimation in the flight tests are shown in Fig. 2.

6. SENSITIVITY

The Kalman-Bucy filter provides estimates of the full state helicopter vector. The standard LQ technique is used for regulator design. The closed loop system appears to be very sensitive to the parameters in the plant equations. Theoretical equations taken from the standard aerodynamic system description appeared to have inaccurate coefficients.

We observed two modes of instability in the experimental flights. The coordinate instability means motion back and forth with increasing amplitude. The frequency of a such swinging was always around 0.5 Hz. Roll and Pitch in this mode are shown in Fig. 3.

The angular instability is oscillations of the helicopter with the frequency always around 2 Hz. Controls almost always achieve their limit stop values. The results are shown in Fig. 4.

The stable closed loop system was obtained after detailed study of the experimental data with identification of the plant equation parameters.

7. ONBOARD SENSOR

An onboard sensor was attached to the helicopter fuselage. It contains three gyroscopes and three acceleration gauges.

The gyroscopes measure angular velocities of the helicopter. The range between -3 and +3 rad/s is covered by a grid with 1024 points. The sample rate is around 273.5 samples per second. It corresponds to 10 samples per revolution of the main rotor.

The data from gyroscopes show big and fast oscillations of fuselage forced by the rotation of the main rotor. Angles variations are small but the angular velocities of oscillations appear to be 3–4 times greater than their smoothed values. The Pitch and Roll velocities of the helicopter are shown in Fig. 5. The sensor data are blue, Kalman filter estimates by camera observations are red.



Fig. 3. Position instability. Frequency = 0.3 Hz



Fig. 4. Pitch-Roll unstable oscillations. Frequency = 2 Hz

A typical spectra of the onboard sensor data from the Roll and Pitch gyroscopes are shown in Fig. 6. The spectra contain small values for the frequency less than 2 Hz, that express angular motion of the helicopter. The highest peak corresponds to the frequency of the main rotor rotation. The second, third and fourth harmonics are clearly seen. The last harmonic near the Nyquist frequency of 137 Hz corresponds to rotation of the tail rotor.

The difference of the curves for each angle reflects the fuselage oscillation and therefore, its period is around 10 samples of the sensor data or 1 revolution of the main rotor. It was noticed that the difference signal is nearly periodic. Its period was estimated to a high accuracy. It is a precise estimate of the main rotor revolution time up to 1%. Estimates of frequency of the main rotor rotation on the time interval of 37 s is shown on Fig. 7. The gyroscope measurements in the roll and pitch channels were processed independently. It follows from Fig. 7 that the results coincide and accuracy is high. Even fluctuations of the main rotor frequency fluctuations include control signals and an onboard battery charge trend.

A detailed study of the oscillation signal have shown that it is a sum of pure harmonics with frequencies multiple to the main rotor rotation frequency F. Variations of the frequency F are



Fig. 5. Angular velocities measured by the onboard sensor and by TV cameras

shown in Fig. 7. All the harmonics were extracted from the signal by corresponding filtering. The biggest amplitude of the harmonics is achieved on the frequency F. The phases of the main harmonic were estimated on the sequential intervals of 256 samples. The roll and pitch signals were processed separately. The phase difference of the main harmonic estimated by roll measurement and by pitch measurement is shown in the second subplot of Fig. 7. It is seen that the phase difference is nearly the same for all frequencies of the main rotor. A deviation from the central value of 155° is less than 5° . Phases are very sensitive to the frequency errors. Therefore, small deviations of phases prove a high accuracy of the main rotor frequency estimate. The constant delay of 155° can be derived only from mechanics and aerodynamics of the main rotor.

The disturbances in the angular velocity measurements forced by the main rotor rotation can be attenuated by means of the corresponding filtering. A simple time invariant FIR filter of the 10-th order was designed with zeros in the areas of the 5 first multiple harmonics frequencies. Results of filtering of the



Fig. 6. Frequency response of the body oscillation



Fig. 7. Estimates of frequency and phase of the main rotor by two gyroscopes



Fig. 8. Filtered data from the onboard sensor compared with TV data

sensor measurements are close to the estimates of the angular velocities obtained by Kalman-Bucy filtering of the camera measurements. Estimates of the roll and pitch angular velocities obtained by filtering of the sensor measurements and of the camera measurements are shown in Fig. 8.

It can be concluded that the helicopter angular velocities can be successfully estimated from the onboard sensor measurements.

The onboard sensor cannot provide sufficiently accurate estimates of the helicopter Euler angles and of position of the center of gravity. It is necessary to use additional information about position, for instance, from GPS.

8. CONCLUSION

A full autopilot system was designed for an electrical helicopter. It shows correct behavior in the hover mode. The system was implemented on a standard computer and receives TV images from two Web cameras. Mathematical background contains image processing, aerodynamic modeling, parameter identification, LQ control design. It is shown that angular stabilization can be made by onboard sensor containing three gyroscopes.

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