ADAPTIVE PAYLOAD AND DYNAMIC FRICTION COMPENSATION IN ROBOTIC SYSTEMS

Lőrinc Márton [*,**,1,2] **Béla Lantos** [*,1]

[*] *Dept. of Control Engineering and Information Technology*
Budapest University of Technology and Economics
H-1117 Budapest, Magyar Tudósok körútja 2. Hungary
martonl@seeger.iit.bme.hu lantos@iit.bme.hu

[**] *Dept. of Electrical Engineering*
Sapientia Hungarian University of Transylvania
540485 Tg. Mures, Op.9, Cp. 4, Romania
Tel:0040365-403030 Fax:0040265-206211
martonl@ms.sapientia.ro

Abstract: In this paper a tracking control algorithm is introduced for robotic systems with unknown friction in the joints and unknown payload mass and inertia. To perform the adaptive payload compensation, the effect of the payload on dynamic behaviour of the machine is separated from the robot arm dynamics. For adaptive compensation of dynamic LuGre friction, a piecewise linearly parameterized friction model is developed. The tracking performances of the introduced control algorithm is analysed using Lyapunov techniques. Simulation results are presented to show the applicability of the control algorithm.

Keywords: Adaptive control, Robot control, Friction

1. INTRODUCTION

To achieve increasing tracking performances and transient properties in robot control systems, the dynamic model of the robot should be taken into consideration in the control algorithm. It is necessary for effective compensation of the nonlinearities, which appears in the dynamics of the robot, on the controlled motion. There are analytical methods to determine the exact mathematical model of a robot, e.g. using the Lagrange method (Lewis *et al.*, 2004). The parameters of the dynamic model (masses, inertias, length of the links, position of the center of mass of the links) are often catalog data which are given by the manufacturer of the robotic system.

Nowadays there is a tendency in robotic industry to build robots with lightweight arms to avoid unnecessary energy consumption. For this reason in the robot model the mass and inertia of the payload cannot be neglected related to the masses and inertias of the robot arm links. In many applications the payload parameters are unknown, varies according to the specific task of the robot.

The friction phenomena, which should be considered in every mechanical system, can be described with models whose parameters are slowly time varying, depending on external factors such as the applied lubricant, the temperature and humidity of the environment in which the robot is used. The friction parameters in robotic systems can hardly be determined a-priori.

The trajectory tracking problem of robotic arms under model uncertainties was permanently in the focus of the researches in the last two decade. The proposed solutions try to obtain on-line more ac-

[1] Support for research is provided by the Hungarian National Research program under grant No. OTKA T 068686 respectively.
[2] The first author research was also supported by Institute of Research Programmes from Sapientia Hungarian University of Transylvania

curate system parameters to improve the quality of the used model in control. Early results can be found in the classical paper (Slotine and Li, 1988) in which it was explored that the robot model can be written in a linearly parameterized form.

In high precision trajectory tracking tasks the friction has a negative influence on tracking accuracy in the robotic systems. In a recent paper (Putra *et al.*, 2006) it was shown that the undercompensation of the friction force can lead to steady state error, the overcompensation can lead to limit cycles. Hence the exact compensation of the friction is necessary in motion control algorithms. On-line estimation of the unknown state and some of the parameters in the dynamic LuGre friction model in robotic systems were reported in (Tomei, 2000) and (Panteley *et al.*, 1998). Adaptive Coulomb+viscous friction combined with payload compensation in robotic systems has been proposed in (Márton, 2006). Review of friction compensation methods in robotic systems can be found in (Bona and Indri, 2005).

Soft computing methods has also been proposed for model free adaptive manipulator control. In (Leahy *et al.*, 1991) neural network based compensation of the payload for robot position control was introduced. Neural network based controllers for robot trajectory tracking were also presented in (Kim *et al.*, 2000) and (Xu and Ioannou, 2003). RBF network based compensation of the dynamic friction in robotic systems was proposed in (J.Wang and Lee, 2001).

2. ROBOT AND FRICTION MODELING

The mathematical model of an open chain, rigid, n Degree Of Freedom robot is given by:

$$H(\underline{q})\underline{\ddot{q}} + C(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D(\underline{q}) = \underline{\tau} - \underline{\tau}_F(\underline{\dot{q}}) \qquad (1)$$

where $\underline{q}$ is the joint position vector of the robot and the vector $\underline{\tau}$ contains the control torques for the joints. The following notations are used: $H(\underline{q})$ is the generalized inertia matrix, $C(\underline{q}, \underline{\dot{q}})$ describes the effect of the centripetal and Coriolis forces, $D(\underline{q})$ is the gravity effect, $\underline{\tau}_F(\underline{\dot{q}})$ represents the effect of the friction force which acts on the joints of the robot. Although (1) is generally a complex system of nonlinear differential equations, it has two fundamental proprieties, which can be exploited to develop control laws for the robot:

P.1 The inertia matrix $H(\underline{q})$ is symmetric and positive definite for every $\underline{q} \in R^n$.

P.2 The matrix $\dot{H}(\underline{q}) - 2C(\underline{q}, \underline{\dot{q}})$ is skew-symetric, namely:

$$\underline{x}^T(\dot{H} - 2C)\underline{x} = 0 \ \forall \underline{x} \in R^n \qquad (2)$$

In order to model the friction force in the i'th joint of the robot (τ_{Fi}), the dynamic LuGre friction model can be applied:

$$\frac{dz_i}{dt} = \dot{q}_i - \sigma_{0i}\frac{|\dot{q}_i|}{g(\dot{q}_i)}\dot{q}_i$$
$$\tau_{Fi}(\dot{q}_i) = \sigma_{0i}z_i + \sigma_{1i}\frac{dz_i}{dt} + F_{Vi}\dot{q}_i \qquad (3)$$

z_i is the unmeasurable internal state of the model, which value is always bounded, see (de Wit *et al.*, 1995). $\dot{q}_i$ is the velocity of the i'th joint, σ_{1i} is a damping coefficient, σ_{0i} is a constant parameter representing the stiffness, the function $g(q_i)$ is a positive continuous function which is meant to describe the Striebeck effect (decreasing friction force with increasing velocities in low velocity regime). It can be defined as an exponential function of velocity: $g(\dot{q}_i) = F_{Ci} + (F_{Si} - F_{Ci})e^{-|\dot{q}_i|/\dot{q}_{iS}}$, where F_{Ci} represents the Coulombic friction coefficient, F_{Vi} is the viscous friction coefficient, $F_{Si} > F_{Ci}$ the static friction term, $\dot{q}_{Si}$ is the Striebeck velocity in the i'th joint.

Reformulated robot model for adaptive payload compensation: Assume that the dimension of the payload can be neglected related to the dimension of the robot link. However, it does not mean that the mass (m_{PL}) and inertia of the payload do not influence the motion of the robot. Assume that the payload has a sphere-like shape, hence $I_{xxPL} = I_{yyPL} = I_{xxPL} = I_{PL}$. Rewrite the model in the following form:

$$\begin{aligned}&(H_R(\underline{q}) + m_{PL}H_m(\underline{q}) + I_{PL}H_I(\underline{q}))\underline{\ddot{q}} + \\ &+(C_R(\underline{q}, \underline{\dot{q}}) + m_{PL}C_m(\underline{q}, \underline{\dot{q}}))\underline{\dot{q}} + \\ &+D_R(\underline{q}) + m_{PL}D_m(\underline{q}) = \underline{\tau} - \underline{\tau}_F(\underline{\dot{q}})\end{aligned} \qquad (4)$$

The H_R, C_R, D_R terms represents the of the robotic arms without the payload. The H_m, C_m, D_m and H_I terms contain the parts from the robot model (1) which multiply the payload mass and inertia respectively. It was exploited that the product $m_{PL} \cdot I_{PL}$ can never appear in the robot model. With the parameters of the robotic arm known and the joint variables $\underline{q}$ and $\underline{\dot{q}}$ measured, the H_m, C_m, D_m, H_I terms are considered known.

Friction model for adaptive compensation: To apply the well known adaptive control schemes for friction compensation it is desirable that the friction force/torque in the i'th joint of the robot could be written in a linearly parameterized form, namely as a scalar product between a known regressor vector $\underline{\xi}_{Fi}(\dot{q}_i)$ and an unknown parameter vector $\underline{\theta}_{Fi}$ ($\tau_{Fi} = \underline{\theta}_{Fi}^T\underline{\xi}_{Fi}(\dot{q}_i)$). In the other hand the friction parameters could change even in the function of the sign of velocity. Hence it is recommended to use different friction parameters in positive and negative velocity regimes. Moreover the dynamics of the friction should also be taken into consideration.

Denote the steady state value of the internal state in (3) z_i with z_{ssi}. It can be expressed as: $z_{ssi} = g(\dot{q}_i)sign(\dot{q}_i)/\sigma_{0i}$. From (3) yields:

$$\begin{aligned} \tau_{Fi} &= \sigma_{0i}z_{ssi} + \sigma_{0i}(z_i - z_{ssi}) + \sigma_{1i}\frac{dz_i}{dt} + F_{Vi}\dot{q}_i \\ &= g(\dot{q}_i)sign(\dot{q}_i) + F_{Vi}\dot{q}_i + \tau_{FDi},\ where \quad (5) \\ \tau_{FDi} &= \sigma_{0i}(z_i - z_{ssi}) + \left(\sigma_{1i}sign(\dot{q}_i) + \frac{\sigma_{0i}\sigma_{1i}z_i}{g(\dot{q}_i)}\right)|\dot{q}_i| \end{aligned}$$

In (5) the term $g(\dot{q}_i)sign(\dot{q}_i) + F_{Vi}\dot{q}_i$ represents the static part of the friction model and the rest of the expression represents the dynamic behaviour of the friction.

The dynamic part of the model is always bounded: since z_i, z_{ssi} and $g(\dot{q}_i) > 0$ are bounded, from the expression of τ_{FDi} defined in (5) yields that there exist two positive constants a_{Di}, b_{Di} satisfying:

$$\begin{aligned} &|\tau_{FDi}| < \underline{\theta}_{FDi}^T \underline{\xi}_{FDi}, \quad (6) \\ &where\ \underline{\theta}_{FDi} = (a_{Di} \quad b_{Di})^T;\ \underline{\xi}_{FDi} = (1 \quad |\dot{q}_i|)^T \end{aligned}$$

The static part of the model has the form:

$$\begin{aligned} &\tau_{FSi} = g(\dot{q}_i)sign(\dot{q}_i) + F_{Vi}\dot{q}_i = \quad (7) \\ &(F_{Ci} + (F_{Si} - F_{Ci})e^{-|\dot{q}_i|/\dot{q}_{Si}})sign(\dot{q}_i) + F_{Vi}\dot{q}_i \end{aligned}$$

For the simplicity only the positive velocity domain is considered, but similar study can be made for negative velocities. Consider that the i'th joint moves in $0 \ldots \dot{q}_{maxi}$ velocity domain. The model (7) is approximated using two lines: d_{1i_+} which crosses through the $(0, \tau_{Fi}(0))$ point and it is tangent to curve and d_{2i_+} which passes through the $(\dot{q}_{maxi}, \tau_{Fi}(\dot{q}_{maxi}))$ point and tangential to curve. These two lines meet each other at a $\dot{q}_{swi_+}$ velocity. (see Figure 1.) In the domain $0 \ldots \dot{q}_{swi_+}$ the d_{1i_+} can be used for the linearization of the curve and d_{2i_+} is used in the domain $\dot{q}_{swi_+} \ldots \dot{q}_{maxi}$. The equations for d_{1i_+} and d_{2i_+}, using Taylor expansion, are:

$$\begin{aligned} d_{1i_+} : \tau_{FSi_{1+}}(\dot{q}_i) &= F_{Si} + \frac{\partial \tau_{FSi}(0)}{\partial \dot{q}_i}\dot{q}_i = \quad (8) \\ &= F_{Si} + (F_{Vi} - (F_{Si} - F_{Ci})/\dot{q}_{Si})\dot{q}_i \\ d_{2i_+} : \tau_{SFi_{2+}}(\dot{q}_i) &= \tau_{FSi}(\dot{q}_{maxi}) + \quad (9) \\ &+ \frac{\partial \tau_{FSi}(\dot{q}_{maxi})}{\partial \dot{q}_i}(\dot{q}_i - \dot{q}_{maxi}) \end{aligned}$$

Thus the linearization of the static friction model in the $0 \ldots \dot{q}_{maxi}$ velocity domain can be realized using two lines:

$$\begin{aligned} \tau_{FSi_{1+}}(\dot{q}_i) &= a_{1i+} + b_{1i+}\dot{q}_i,\ if\ 0 \ \leq \dot{q}_i \ \leq \ \dot{q}_{swi+} \quad (10) \\ \tau_{FSi_{2+}}(\dot{q}_i) &= a_{2i+} + b_{2i+}\dot{q}_i,\ if\ \dot{q}_{swi+} \ < \dot{q}_i \ \leq \ \dot{q}_{maxi} \end{aligned}$$

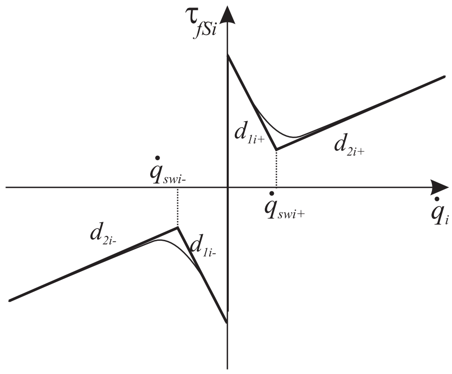

Fig. 1. Linearization of the static part of the friction

Using (8) and (9) can be shown that the switching velocity is equal with the Striebeck velocity ($\dot{q}_{Si}$). Its value can be calculated as:

$$\dot{q}_{swi+} = \dot{q}_{Si} = \frac{a_{1i+} - a_{2i+}}{b_{2i+} - b_{1i+}} \quad (11)$$

With same train of thoughts a similar model can be determined for the negative velocity domain. Combining the negative and positive velocity domains the obtained static friction model can be written as:

$$\begin{aligned} &\tau_{FSi}(\dot{q}_i) = \underline{\theta}_{FSi}^T \underline{\xi}_{FSi},\ where \quad (12) \\ &\underline{\theta}_{FSi} = (\ a_{1i+}\ b_{1i+}\ a_{2i+}\ b_{2i+}\ a_{1i-}\ b_{1i-}\ a_{2i-}\ b_{2i-})^T \\ &\underline{\xi}_{FSi} = (\ \mu_{1i+}\ \mu_{1i+}\dot{q}_i\ \mu_{2i+}\ \mu_{2i+}\dot{q}_i \\ &\qquad \mu_{1i-}\ \mu_{1i-}\dot{q}_i\ \mu_{2i-}\ \mu_{2i-}\dot{q}_i)^T \end{aligned}$$

The switching functions $\mu_i(\dot{q}_{swi}, \dot{q}_i)$ are introduced to separate the velocity regimes in which the different linearized friction models are applied. For example μ_{1i+} is defined as:

$$\mu_{1i+}(\dot{q}_{swi+}, \dot{q}_i) = \begin{cases} 1,\ if\ 0 \ \leq \dot{q}_i \ \leq \ \dot{q}_{swi+} \\ 0,\ otherwise \end{cases} \quad (13)$$

Hence, the friction in a joint of a robot can be modeled as a sum of a static friction model and a dynamic term ($\tau_{Fi} = \tau_{FSi}(\dot{q}_i) + \tau_{FDi}(\dot{q}_i, z_i)$). The static term can be written in piecewise linearly parameterized form with discontinuous regressor vector. The dynamic term is always bounded. Its bound can also be written in piecewise linearly parameterized form with discontinuous regressor vector.

The robot model, based on which the adaptive payload and friction compensation algorithm can be formulated, reads as:

$$\begin{aligned} &H_R(\underline{q})\underline{\ddot{q}} + C_R(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D_R(\underline{q}) + \quad (14) \\ &+ m_{PL}(H_m(\underline{q})\underline{\ddot{q}} + C_m(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D_m(\underline{q})) + \\ &+ I_{PL}H_I(\underline{q})\underline{\ddot{q}} + \underline{\tau}_{FS}(\underline{\dot{q}}) + \underline{\tau}_{FD} + \underline{d} = \tau \end{aligned}$$

The equation above is formulated based on (4) and using the presented friction modeling technique. The unknown frictional parameters, which differs in different velocity regimes are in parameter vectors $\underline{\theta}_{FSi}$ and $\underline{\theta}_{FDi}$, as it was presented in (12) and (6) respectively. The value of the switching velocity (11) which separates different velocity regimes in the friction model (12) and appears in the regressor vector is not known, its a-priori guess is necessary. It can introduce uncertainty at the beginning of the adaptation. Hence, an additive friction modeling error term ($\underline{d}$ with $|d_i| < D_{FM}$) is introduced in the model.

3. THE CONTROL LAW

Control task: The problem is to design a control input $\underline{\tau} = (\tau_1 \;\tau_2 \ldots \tau_n)^T$ such that the joint position $\underline{q} = (q_1 \;q_2 \ldots q_n)^T$ track the desired trajectory $\underline{q}_d = (q_{1d} \;q_{2d} \ldots q_{nd})^T$ with given precision. The desired joint trajectories q_{di} are known bounded functions of time with bounded, known first and second order derivatives.

Define the error metric S_i that describes the desired dynamics of the error system for the i'th joint: $S_i(t) = (\frac{d}{dt} + \lambda_i)e_i, \quad e_i = q_i - q_{di}$, where λ_i is strictly positive constant. For the entire robot the error metric can be defined as:

$$\underline{S} = (\underline{\dot{q}} - \underline{\dot{q}}_d) + \Lambda(\underline{q} - \underline{q}_d) \tag{15}$$

with $\Lambda > 0$ diagonal matrix with positive elements.

Based on the error metric, the control problem can be reformulated as: design a control input $\underline{\tau}$ such that $|S_i(t)| < \Phi$ if $t \to \infty$, $\forall\; i = 1..n$. $\Phi > 0$ is the given precision.

By using the original (1) and reformulated (14) model, the dynamics of the error metric reads:

$$\begin{aligned} &H\dot{S} = H(\underline{\ddot{q}} - \underline{\ddot{q}}_d + \Lambda(\underline{\dot{q}} - \underline{\dot{q}}_d)) = H(-\underline{\ddot{q}}_d + \Lambda(\underline{\dot{q}} - \underline{\dot{q}}_d)) \\ &-C(\underline{q},\underline{\dot{q}})\underline{\dot{q}} - D(\underline{q}) + \underline{\tau} - \underline{\tau}_F(\underline{\dot{q}}) = -CS + \\ &+H_R(-\underline{\ddot{q}}_d + \Lambda(\underline{\dot{q}} - \underline{\dot{q}}_d)) + C_R(-\underline{\dot{q}}_d + \Lambda(\underline{q} - \underline{q}_d)) - \\ &-D_R + m_{PL}\underline{\xi}_m + I_{PL}\underline{\xi}_I + \underline{\tau} - \underline{\tau}_{fS} - \underline{\tau}_{fD} - \underline{d} \quad (16) \\ &where: \;\underline{\xi}_I = H_I(-\underline{\ddot{q}}_d + \Lambda(\underline{\dot{q}} - \underline{\dot{q}}_d)) \\ &\underline{\xi}_m = H_m(-\underline{\ddot{q}}_d + \Lambda(\underline{\dot{q}} - \underline{\dot{q}}_d)) + \\ &+C_m(-\underline{\dot{q}}_d + \Lambda(\underline{q} - \underline{q}_d)) - D_m \end{aligned}$$

Control law: The payload and friction parameters are unknown, only its estimated values can be used in the control algorithm. Denote with $\widehat{m}_{PL}$, $\widehat{I}_{PL}$ the estimated payload parameters and with $\widehat{\underline{\theta}}_{FSi}$ and $\widehat{\underline{\theta}}_{FDi}$ the vectors of estimated static and dynamic friction parameters in the i'th joint.

In function of known robot arm parameters and estimated friction and payload parameters the control signal $\underline{\tau}$ can be formulated as follows:

$$\begin{aligned} &\underline{\tau} = -H_R(-\underline{\ddot{q}}_d + \Lambda(\underline{\dot{q}} - \underline{\dot{q}}_d)) - \\ &-C_R(-\underline{\dot{q}}_d + \Lambda(\underline{q} - \underline{q}_d)) + D_R - \\ &-K_S\underline{S} - \widehat{m}_{PL}\underline{\xi}_m - \widehat{I}_{PL}\underline{\xi}_I + \widehat{\underline{\tau}}_{FS} + \\ &-\widehat{\underline{\tau}}_{FD}\,sat(\underline{S}/\Phi) - D_{FM}\,sat(\underline{S}/\Phi) \\ &where \;\; \widehat{\tau}_{FSi} = \widehat{\underline{\theta}}^T_{FSi}\underline{\xi}_{FSi} \;\; \widehat{\tau}_{FDi} = \widehat{\underline{\theta}}^T_{FDi}\underline{\xi}_{FDi} \end{aligned} \tag{17}$$

$sat(\cdot)$ denote the saturation function.

The last two terms in the control law are introduced to compensate the effect of dynamic friction behaviour and uncertainties of friction modeling respectively. For the estimation of unknown parameters the following adaption laws are applied:

$$\begin{aligned} &\dot{\widehat{m}}_{PL} = \gamma_m\underline{S}^T\underline{\xi}_m \quad \dot{\widehat{I}}_{PL} = \gamma_I\underline{S}^T\underline{\xi}_I \\ &\dot{\widehat{\underline{\theta}}}_{fSi} = \Gamma_{fSi}\underline{\xi}_{FSi}S_i \quad \dot{\widehat{\underline{\theta}}}_{fDi} = \Gamma_{fDi}\underline{\xi}_{FDi}|S_i| \end{aligned} \tag{18}$$

with $\gamma_m, \gamma_I > 0$. $\Gamma_{fSi}, \Gamma_{fDi}$ are diagonal matrices with only positive elements on the diagonal.

Lyapunov analysis of the control: In order to analyse the convergence of the tracking error metric, define the following Lyapunov function:

$$\begin{aligned} &V(t) = \frac{1}{2}\underline{S}^T H(\underline{q})\underline{S} + \frac{\gamma_m^{-1}}{2}\widetilde{m}^2_{PL} + \frac{\gamma_I^{-1}}{2}\widetilde{I}^2_{PL} + \\ &+\frac{1}{2}\sum_{i=1}^{n}\widetilde{\underline{\theta}}^T_{FSi}\Gamma^{-1}_{FSi}\widetilde{\underline{\theta}}_{FSi} + \frac{1}{2}\sum_{i=1}^{n}\widetilde{\underline{\theta}}^T_{FDi}\Gamma^{-1}_{FDi}\widetilde{\underline{\theta}}_{FDi} \end{aligned} \tag{19}$$

in which the estimation errors are defined as $\widetilde{m}_{PL} = m_{PL} - \widehat{m}_{PL}$, $\widetilde{I}_{PL} = I_{PL} - \widehat{I}_{PL}$, $\widetilde{\underline{\theta}}_{FSi} = \underline{\theta}_{FSi} - \widehat{\underline{\theta}}_{FSi}$, $\widetilde{\underline{\theta}}_{FDi} = \underline{\theta}_{FDi} - \widehat{\underline{\theta}}_{FDi}$.

The time derivative of (19) can be calculated as:

$$\begin{aligned} &\dot{V}(t) = \underline{S}^T H(\underline{q})\underline{\dot{S}} + \frac{1}{2}\underline{S}^T \dot{H}(\underline{q})\underline{S} \\ &-\widetilde{m}_{PL}\gamma_m^{-1}\dot{\widehat{m}}_{PL} - \widetilde{I}_{PL}\gamma_I^{-1}\dot{\widehat{I}}_{PL} - \\ &-\sum_{i=1}^{n}\widetilde{\underline{\theta}}^T_{FSi}\Gamma^{-1}_{FSi}\dot{\widehat{\underline{\theta}}}_{FSi} - \sum_{i=1}^{n}\widetilde{\underline{\theta}}^T_{FDi}\Gamma^{-1}_{FDi}\dot{\widehat{\underline{\theta}}}_{FDi} \end{aligned} \tag{20}$$

By substituting the expression of the control law (17) in the equation of error dynamics (16), yields:

$$\begin{aligned} &H\underline{\dot{S}} = -C\underline{S} + \widetilde{m}_{PL}\underline{\xi}_m + \widetilde{I}_{PL}\underline{\xi}_I - \widetilde{\underline{\tau}}_{FS} + \\ &+(\underline{\tau}_{FD} - \widehat{\underline{\tau}}_{FD}sat(\underline{S}/\Phi)) - K_S\underline{S} - \underline{d} - D_{FM}sat(\underline{S}/\Phi) \\ &where: \;\widetilde{\tau}_{FSi} = \widetilde{\underline{\theta}}^T_{FSi}\underline{\xi}_{FSi} \end{aligned} \tag{21}$$

Outside the boundary layer $(|S_i| > \Phi \;\forall\; i = 1..n)$ we have: $sat(\underline{S}/\Phi) = sign(\underline{S})$

Introduce the error dynamics (21) and the adaptation laws (18) into (20) and applying the property (2) of the robot model, the time derivative of the Lypaunov function reads as:

$$\dot{V}(t) = -\underline{S}^T K_S \underline{S} - \sum_{i=1}^{n} \left(\tau_{FDi} S_i + \underline{\widehat{\theta}}_{FDi}^T \underline{\xi}_{FDi} |S_i| \right) - \\ - \sum_{i=1}^{n} \left(\underline{\widetilde{\theta}}_{FDi}^T \underline{\xi}_{FDi} |S_i| \right) - \underline{S}^T \underline{d} - \underline{|S|}^T \underline{D}_{FM} \quad (22)$$

Applying that $-d_i S_i < D_{FM}|S_i|$, if $|d_i| < D_{FM}$, yields:

$$\dot{V}(t) < -\underline{S}^T K_S \underline{S} - \sum_{i=1}^{n} \left(\tau_{FDi} S_i - \underline{\theta}_{FDi}^T \underline{\xi}_{FDi} |S_i| \right) \quad (23)$$

According to (6) the second term in (23) is always negative. It yields:

$$\dot{V}(t) < -\underline{S}^T K_S \underline{S} \quad (24)$$

Outside the boundary layer $\underline{S}^T K_S \underline{S} > 0$, hence $\dot{V}(t) < 0$, V is a positive, strictly decreasing function. It guarantees the convergence of the elements of the vector $\underline{S}$ inside the boundary layer with bound Φ. Form $|S_i| \leq \Phi$ yield that the position tracking error and velocity tracking error of the i'th joint are bounded as follows: $|q_i(t) - q_{di}(t)| \leq \Phi/\Lambda_i$, $|\dot{q}_i(t) - \dot{q}_{di}(t)| \leq 2\Phi$ (see eg. (Márton, 2006))

4. SIMULATION RESULTS

The performance of the proposed control algorithms is demonstrated for a SCARA type robotic arm in Matlab/Simulink using the SimMechanics toolbox. The first and second rotational joints of the robot performs the positioning of the payload in the vertical plane. There is a strong nonlinear coupling between these joint. The second prismatic joint carries out the vertical motion of the payload, the fourth rotational joint assures the rotation of the payload around vertical axis. The payload mass and inertia was taken around 20% of the mass and payload of the robotic arm links. The friction parameters have been chosen in such way that around $10\% - 15\%$ of the control effort was used to compensate the friction. The chosen reference trajectory guarantees acceleration, deceleration and constant velocities both in positive and negative velocity regimes for all joints.

Simulations has been performed with a well tuned PID controller, with a robot arm model based controller, without friction and payload compensation (by omitting the last five terms in the control law (17)), and with adaptive friction and payload compensation. In the last experiment the control law (17) together with the adaptation laws (18) were implemented. All initial values of the estimated parameters were taken 0. In the Figure 2 can be seen that the proposed adaptive compensation method guarantees not only better transients but precise tracking of the desired time varying position. The adaptation laws guarantees fast parameter convergence of the unknown parameters. Note that because of the chosen switching regressor vector in the model (12), the different friction parameters are tuned only when the machine moves in the corresponding velocity regime.

For the numerical evaluation of tracking performances, the average of the absolute values of the position error ($E_A = \frac{1}{nT} \int_0^T \left(\sum_{i=1}^n |q_i - q_{di}| \right)$) was calculated during the first $T = 2.2$ seconds of the simulation with the PID controller, model based and the adaptive friction and payload compensator controller. The following values has been obtained: $E_A(PID) = 13E - 4$ $[mm]$, $E_A(MODEL\ BASED) = 9.65E - 4$ $[mm]$, $E_A(ADAPTIVE) = 5.34E - 4$ $[mm]$.

5. CONCLUSIONS

To deal with payload and friction induced uncertainties in robotic systems, an adaptive control algorithm was introduced for high precision position tracking tasks. In order to develop the control algorithm, in the robot model the effect of the payload was separated from the robot arm dynamics. To describe the friction force in the robot joints a piecewise linearly parameterized friction model was introduced. Using Lyapunov analysis it was shown that the control algorithm guarantees the convergence of the tracking error inside a predefined boundary layer. Simulation results show that the introduced adaptive control algorithm assures high precision tracking of time varying trajectories.

REFERENCES

Bona, Basilio and Marina Indri (2005). Friction compensation in robotics: an overview. In: *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*. Seville, Spain.

de Wit, C. Caundas, H. Ollson, K. J. Åstrom and P. Lischinsky (1995). A new model for control of systems with friction. *IEEE Trans. on Automatic Control* **40**(3), 419–425.

J.Wang, S. S. Ge and T. H. Lee (2001). Adaptive NN control of robot manipulator with unknown dynamic friction. In: *Proc. of Asian Conference on Robotics and Applications*. Singapore.

Kim, Young, Frank Lewis and Darren Dawson (2000). Intelligent optimal control of robotic

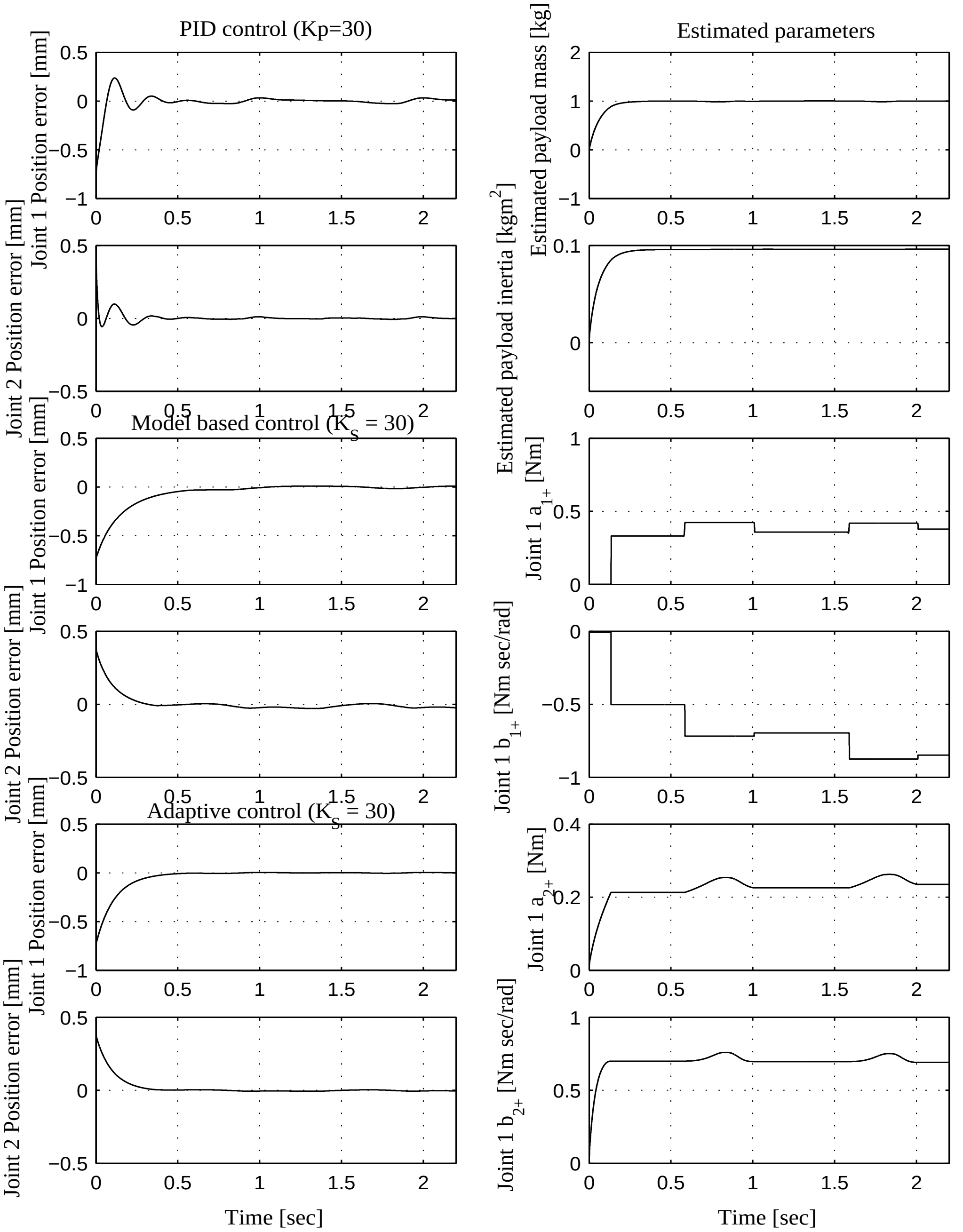

Fig. 2. Simulation results

mainpulators using neural fnetworks. *Automatica* **36**, 1355–1364.

Leahy, M., M. Johnson and S. Rogers (1991). Neural network payload estimation for adaptive robot control. *IEEE Trans. On Neural Networks* **2**(1), 93–100.

Lewis, Frank L., Darren M. Dawson and Chaouki T. Abdallah (2004). *Robot Manipulator Control.* Marcel Dekker, Inc.. New York.

Márton, Lőrinc (2006). Robust-Adaptive Control of Nonlinear Singlevariable Mechatronic Systems and Robots. PhD thesis. Budapest University of Technology and Economics. www.ms.sapientia.ro/˜ martonl.

Panteley, E., R. Ortega and M. Gafvert (1998). An adaptive friction compensator for global tracking in robotic manipulator. *Systems and Control Letters* **33**, 307–313.

Putra, D., H. Nijmeijer, N. van deWouw, O.C.L. Haas and K.J. Burnham (2006). Analysis of friction compensation effects in controlled mechanical systems. In: *Proceedings of the International Conference on Systems Engineering.* Coventry, England.

Slotine, Jean-Jaques and Weiping Li (1988). Adaptive manipulator control: A case study. *IEEE Trans. on Automatic Control* **33**(11), 995–1003.

Tomei, Patrizio (2000). Robust adaptive friction compensation for tracking control of robot manipulators. *IEEE Trans. on Automatic Control* **45**(6), 2164–2169.

Xu, Haojian and Petros A. Ioannou (2003). Robust adaptive control for a class of MIMO nonlinear systems with guaranteed error bounds. *IEEE Trans. on Automatic Control* **48**(5), 728–743.