STELLAR-INERTIAL ATTITUDE NAVIGATION OF THE LARGE-SCALE INFORMATION SATELLITES

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Abstract

Agile large-scale information satellites (for communication, geodesy, radio- and opto-electronic observation of the Earth et al.) have some principle problems with respect to a precise attitude determination. In the paper these problems are considered and obtained results are presented. The original multiple discrete algorithms for filtering, integration and calibration of a strapped-down inertial navigation system (SINS) with astronomical correction for precise determining a spacecraft (SC) attitude are presented. This system consists of an inertial block (IB) based on the gyro sensors of the SC angular position quasi-coordinates and an astronomical system (AS) based on the star trackers with wide field-of-view that are fixed to the SC body. Some results of computer simulation are presented.

Key words

Spacecraft attitude navigation, filtering, identification

1 Introduction

The programmed guidance of the agile large-scale information SC is presented by a consequence of time intervals for a target application – in the general case, the routes (RTs) with the module of an angular rate vector $\boldsymbol{\omega}$ up to 0.5 deg/sec and the intervals of spatial rotational maneuvers (RMs) with variable direction of the vector $\boldsymbol{\omega}$, which module may be as large as 5 deg/sec, Fig. 1. Some authors' results on the in-flight SC equipment's alignment calibration, the SC attitude gyromoment guidance and control were presented on the PhysCon09, Catania (Somov *et al.*, 2009; Somov and Butyrin, 2009).

The problems of the SINS algorithmic software are connected with integration of kinematic equations in using the information on the quasi-coordinate increment vector $\mathbf{i}^{\omega}$ obtained by the IB over the main discrete period T_{o}, filtration of noises, identification and

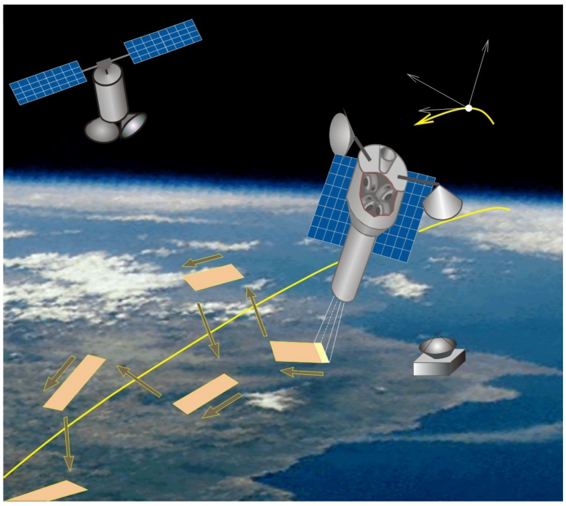

Figure 1. The scanning pattern of given targets

compensation of errors on a mutual angular position of the IB and the AS reference frames, variation of the measure scale coefficients, and the IB bias with respect to the vector $\boldsymbol{\omega}$. As kinematic parameters many authors applied the quaternion $\boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$, the vector of Euler's parameters $\mathbf{L} = \{l_0, \mathbf{l}\}$, the orientation matrix $\mathbf{C}$, the Euler vector $\boldsymbol{\phi} = \theta\,\mathbf{e}$, the vector of terminal rotation $\boldsymbol{\theta} = 2\,\mathrm{tg}(\theta/2)\,\mathbf{e}$ etc. Moreover, for the SC low angular motions with little variation of the angle θ and practically fixed direction of the Euler axis unit $\mathbf{e}$ during the discrete period T_{o}, the integration of the kinematic relations for the Euler vector $\boldsymbol{\phi}(t)$ with obtaining the values $\boldsymbol{\Lambda}_k \equiv \boldsymbol{\Lambda}(t_k)$ was usually carried out by the following scheme:

$$\boldsymbol{\phi}_{k+1} = \theta_{k+1}\mathbf{e}_{k+1} \equiv \mathbf{i}^{\omega}_{k+1} = \int_{t_k}^{t_{k+1}} \boldsymbol{\omega}(\tau)d\tau;$$

$$\mathbf{i}^{\omega}_{k+1} \equiv \mathbf{Int}(t_k, T_{\text{o}}, \boldsymbol{\omega}(t));\ t_{k+1} = t_k + T_{\text{o}};$$

$$\boldsymbol{\phi}_k \equiv \boldsymbol{\phi}(t_k) \Rightarrow \mathbf{C}_k \Rightarrow \boldsymbol{\Lambda}_k, k \in \mathbb{N}_0 \equiv [0, 1, 2, ...).$$

The solutions of other problems indicated, which are very topical ones for practical realization of the SINS,

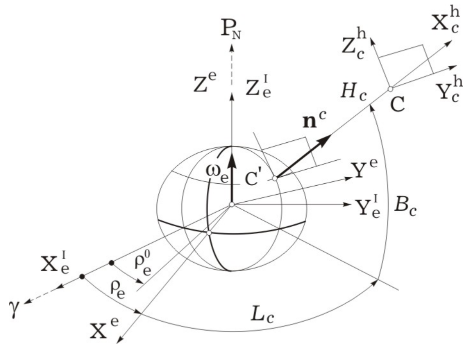

Figure 2. The bases $\mathbf{I}_{\oplus}$, $\mathbf{E}_{\rm e}$ and $\mathbf{E}_{\rm e}^{\rm h}$

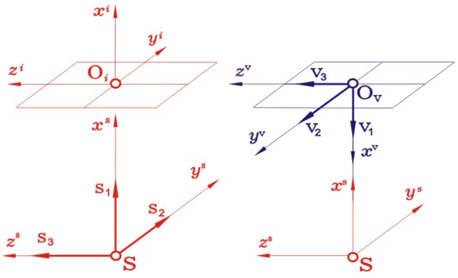

Figure 3. The bases $\boldsymbol{\mathcal{S}}$, $\boldsymbol{\mathcal{F}}$ and $\mathbf{V}$

were considered in Branetz and Shmyglevsky (1992); Gusinsky *et al.* (1997); Pittelkau (2001) , for example. Here the problems on the identification of the IB and the AS reference frames' "alignments" (errors on their mutual angular position) and variation of the measure scale coefficients by the vector $\boldsymbol{\omega}(t)$ into the IB reference frame at forming the vectors $\mathbf{i}_{k+1}^{\omega}$ are the most complicated ones. This is due to a multiplicative character of the interconnected parametric disturbances indicated.

Standard SINS scheme is applied to determine the attitude for spacecraft of this type: the IB is a principle measuring unit, and the AC signals are applied for its situational calibration and alignment. Here use is made of the measured information at the intermediate points with a period T_q multiple of the main sampling period $T_{\rm o}$, and integration of the kinematic equation for the modified Rodrigues parameters vector (hereafter, simply, the Rodrigues vector) is carried out with applying multiple discrete filtering and polynomial approximation. The quaternion $\boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$ is an one-one related to the Rodrigues vector $\boldsymbol{\sigma} = {\rm tg}(\theta/4)\,\mathbf{e}$ by the explicit analytic relations

$$\boldsymbol{\sigma} = \frac{\boldsymbol{\lambda}}{1+\lambda_0};\ \lambda_0 = \frac{1-\sigma^2}{1+\sigma^2};\ \boldsymbol{\lambda} = \frac{2\boldsymbol{\sigma}}{1+\sigma^2}.$$

To the well-known direct and backward quaternion kinematic equations there correspond the direct and backward equations for the Rodrigues vector

$$\dot{\boldsymbol{\sigma}} = \tfrac{1}{4}\{\mathbf{I}_3(1-\sigma^2) + 2[\boldsymbol{\sigma}\times] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^{\rm t}\}\boldsymbol{\omega};$$
$$\boldsymbol{\omega} = \tfrac{4}{(1+\sigma^2)^2}\{\mathbf{I}_3(1-\sigma^2) - 2[\boldsymbol{\sigma}\times] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^{\rm t}\}\dot{\boldsymbol{\sigma}}.$$

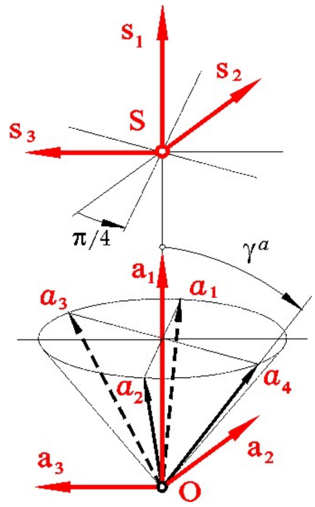

Figure 4. Scheme of the AS base

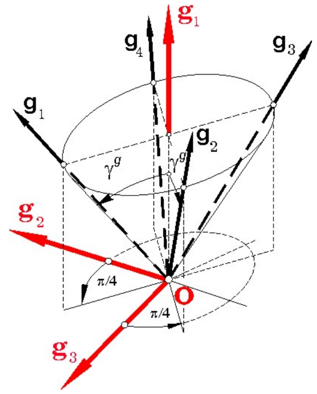

Figure 5. Scheme of the IB base

2 The Problem Statement

By analogy with Somov and Butyrin (2009) the bases formed from units and reference frames are introduced: the inertial reference frame (IRF) $\mathrm{O}_{\oplus}\mathrm{X}_{\rm e}^{\rm I}\mathrm{Y}_{\rm e}^{\rm I}\mathrm{Z}_{\rm e}^{\rm I}$ and the base $\mathbf{I}_{\oplus} = \{\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3\}$ of the present time; geodesic Greenwich reference frame (GRF) $\mathbf{E}_{\rm e}$ ($\mathrm{O}_{\oplus}\mathrm{X}^{\rm e}\mathrm{Y}^{\rm e}\mathrm{Z}^{\rm e}$) which is rotated with respect to the IRF by angular rate vector $\boldsymbol{\omega}_{\oplus} \equiv \boldsymbol{\omega}_{\rm e} = \omega_{\oplus}\mathbf{i}_3$, Fig. 2; the geodesic horizon reference frame (HRF) $\mathbf{E}_{\rm e}^{\rm h}$ ($C\,\mathrm{X}_c^{\rm h}\mathrm{Y}_c^{\rm h}\mathrm{Z}_c^{\rm h}$) with origin in a point C and ellipsoidal geodesic coordinates – longitude L_c, latitude B_c and altitude H_c : axis $C\,\mathrm{X}_c^{\rm h}$ is the local vertical, axes $C\,\mathrm{Y}_c^{\rm h}$ and $C\,\mathrm{Z}_c^{\rm h}$ lie in the local horizon plane and directed to local East (E) and local North (N), respectively, Fig. 2; the body reference frame (BRF) $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ ($\mathrm{O}xyz$) with origin in the SC mass center O; the optical telescope (antenna, sensor) reference frame (SRF) $\boldsymbol{\mathcal{S}} = \{\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3\}$ ($\mathrm{S}\ x^{\rm s}y^{\rm s}z^{\rm s}$) with origin in point S – the center of optical projection, Fig. 3; the image field reference frame (FRF) $\boldsymbol{\mathcal{F}}$ ($\mathrm{O}_i x^i y^i z^i$) with origin in center O_i of the telescope focal plane $y^i\mathrm{O}_i z^i$; the visual (sighting) reference frame (VRF) $\mathbf{V}$ ($\mathrm{O}_{\rm v}x^{\rm v}y^{\rm v}z^{\rm v}$) with origin in center $\mathrm{O}_{\rm v}$ of the OEC array, see Fig. 3; the AS virtual base $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ that is calculated by processing an accessible measuring information from the star trackers (Fig. 4), and also the IB virtual base $\mathbf{G} = \{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3\}$

which is computed by processing the measuring information from the integrating gyro sensors (Fig. 5).

For simplicity we will propose that the bases $\mathbf{B}$ and $\mathbf{S}$ are coincident. It is assumed that during a mode of astronomical checking axes concordance (Somov and Butyrin, 2009), we solved the problem on defining a fixed mutual angular position of the bases $\mathbf{S}$ and $\mathbf{A}$, when the measuring information obtained both from a telescope (antenna) and star trackers is applied. Let the measured values of a vector $\mathbf{i}^{g\omega}_{m\,s+1}, s \in \mathbb{N}_0$, be obtained from the IB with a period $T_q << T_{\mathrm{o}}$, and from the AS – the measured values of quaternion $\mathbf{\Lambda}^a_{m\,k}$ with a period T_{o}:

$$\begin{gathered} \boldsymbol{\omega}^g_m(t) = (1-m)\mathbf{S}^{\Delta}(\boldsymbol{\omega}(t) - \mathbf{b}^g); \\ \mathbf{i}^{g\omega}_{m\,k+1} = \mathbf{Int}(t_s, T_q, \boldsymbol{\omega}^g_m(t)) + \boldsymbol{\delta}^n_{s+1}; \\ \mathbf{\Lambda}^a_{m\,k} = \mathbf{\Lambda}_k \circ \mathbf{\Lambda}^n_k. \end{gathered} \tag{1}$$

Here the vector $\boldsymbol{\omega}^g_m(t)$ presents the measured angular rate vector $\boldsymbol{\omega}^g(t)$ at the base $\mathbf{G}$ taking into account the unknown small and slow variations of the IB bias vector $\mathbf{b}^g$ on angular rate; to the orthogonal matrix $\mathbf{S}^{\Delta}$ of an error on mutual angular position of the IB and the AC reference frames there corresponds the quaternion $\mathbf{\Lambda}^{\Delta}(C(\varphi^{\Delta}/2), \mathbf{e}^{\Delta}S(\varphi^{\Delta}/2))$ with the unknown Euler unit $\mathbf{e}^{\Delta}$ and angle φ^{Δ}, and the scalar m presents an unknown error of the IB scale coefficient. The vector $\mathbf{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ components of the IB and AC reference frames "alignments" are computed by the formula $\Delta_i = 2\varphi^{\Delta}\mathbf{e}^{\Delta}, i = x, y, z$. The discrete noises $\boldsymbol{\delta}^n_{s+1}$ and $\mathbf{\Lambda}^n_k$ of the IB and AS output signals were taken into account in relations (1).

The problem is to develop the algorithms for estimating the coordinated values $\hat{\mathbf{\Lambda}}_l$ and $\hat{\boldsymbol{\omega}}^g_l, l \in \mathbb{N}_0$, with an arbitrary period $T_p = t_{l+1} - t_l$, multiple of T_q, moreover $T_q < T_p \leq T_{\mathrm{o}}$, and also the algorithms for the SINS calibration and alignment at the SC *agile* rotational maneuvers.

3 An Approach to the Problem Solution

To integrate the kinematic equations with a small computing drift, an idea is being developed to use the measured information in the intermediate points with a period T_q that is multiple of the main sampling period T_{o}. Here the algorithms of discrete filtering and polynomial approximation are applied over the period T_p inside of the period T_{o}. Consequently, by the IB discrete signals one can obtain the SC angular rate vector as a *continuous* time function. The integration is performed according to the precise differential equation for the Rodrigues vector $\boldsymbol{\sigma} = \mathrm{tg}(\theta/4)\,\mathbf{e}$. As a result of the integration procedure, the quaternion value is obtained at the right-hand end of the next integration step with a period T_{o}.

The problem of the SINS in-flight recurrent calibration is solved owing to the astronomical correction of the IB signals with the period T_{o}. Moreover, the IB "distort" vector $\mathbf{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ and the scale coefficient m with obtaining $\hat{\mathbf{\Delta}}_k$ and $\hat{m}_k$ estimations are identified by comparing the SC angular rate vector's values which are restored in off-line mode by the IB signals and the AS signals. The identification of the vector value $\mathbf{b}^g$ with obtaining an estimation $\hat{\mathbf{b}}^g$ is ensured by Luenberger linear stationary observer.

4 Filtering the IB noise

The polynomial smoothing Savitsky - Goley discrete filter is applied for suppressing the IB discrete noise $\boldsymbol{\delta}^n_{s+1}$ in (1). This filter is a modification of the least squares method. Here approximation is carried out with a small "sliding" window for a consequence of the discrete IB measurements presented in the Rodrigues vectors form. Such a filtering method does not lead to a phase displacement for an output signal with respect to an input signal.

5 Extrapolation of the angular rate vector

After carrying out the filtering mentioned, we obtain the values of four vectors $\mathbf{i}^{\omega}_1, \mathbf{i}^{\omega}_2, \mathbf{i}^{\omega}_3, \mathbf{i}^{\omega}_4$ over the time intervals with a period T_{o}. On each such an interval the estimation $\hat{\boldsymbol{\omega}}(t)$ of the vector $\boldsymbol{\omega}(t)$ is presented by a vector third degree polynomial. This polynomial is smoothly "pasted together" with the corresponding polynomials on the next intervals. The sets of coefficients for such polynomials are defined by the explicit relations in Somov (2009).

6 Estimation of the SC attitude

The estimation $\hat{\boldsymbol{\omega}}(t)$ obtained in the form of a *continuous* time function permits the SC attitude be estimated. With this purpose, we use the direct differential equation for the Rodrigues vector $\boldsymbol{\sigma} = \mathrm{tg}(\theta/4)\,\mathbf{e}$. This equation is numerically integrated by well-known ODE45 method with a fixed step. The computing drift of such a scheme was studied in Somov (2009), for the period $T_q = 0.015625$ s (frequency 64 Hz) it was obtained that the position computing drift of this procedure is no more than 0.00025 arc sec along any motion axis in a period of 720 sec, e.g. it is very small. As a result, we have the *coordinated* estimations $\hat{\mathbf{\Lambda}}_l$ and $\hat{\boldsymbol{\omega}}^g_l, l \in \mathbb{N}_0$ with the period T_p.

7 The SINS calibration

For the IB situational calibration with respect to the bias $\mathbf{b}^g$ a discrete Luenberger observer is applied with the period T_{o}. Such a calibration is fulfilled during the time intervals, where the module of the SC angular rate vector does not exceed 0.5 deg/s and the AS measurements are perfectly precise. At time moments t_{k+1} the Rodrigues vector $\boldsymbol{\sigma}^g_{k+1}$ is recalculated to the quaternion $\mathbf{\Lambda}^g_{k+1}$, the position error vector $\boldsymbol{\delta}_{k+1} = 2\,\mathrm{vect}(\mathbf{\Lambda}^a_{m\,k+1} \circ \tilde{\mathbf{\Lambda}}^g_{k+1})$ is formed and the estimation $\hat{\mathbf{b}}^g_{k+1} = \hat{\mathbf{b}}^g_k + g^{\mathrm{o}}_2\boldsymbol{\delta}_{k+1}$ is calculated for the IB bias

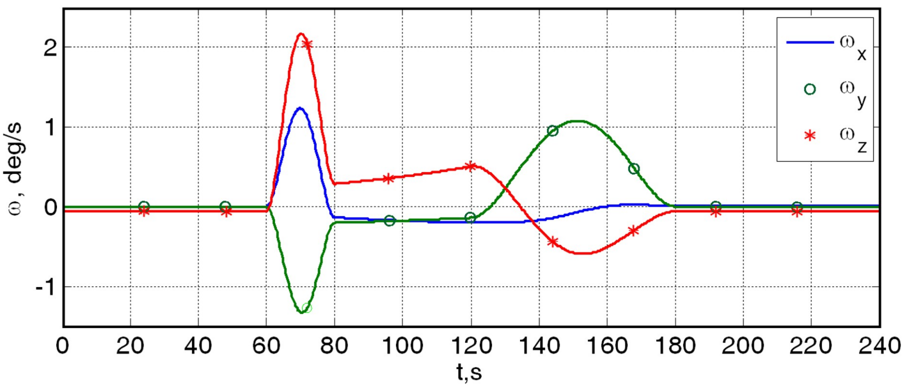

Figure 6. The SC angular rates at consequence of the RMs

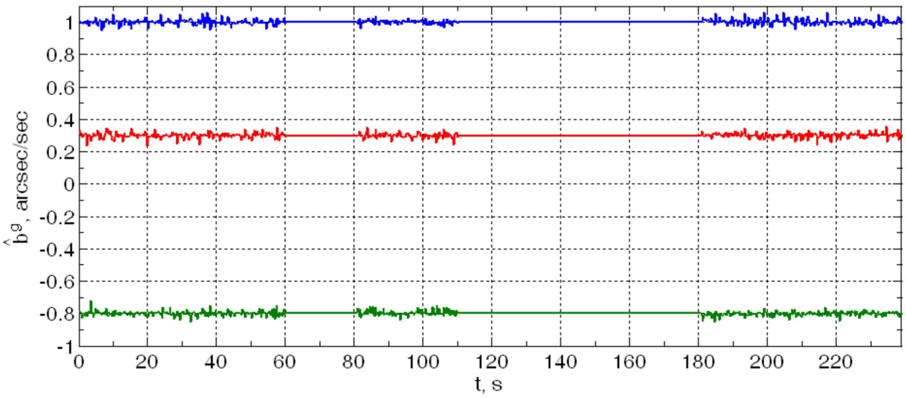

Figure 7. Estimations for components of the IB bias vector

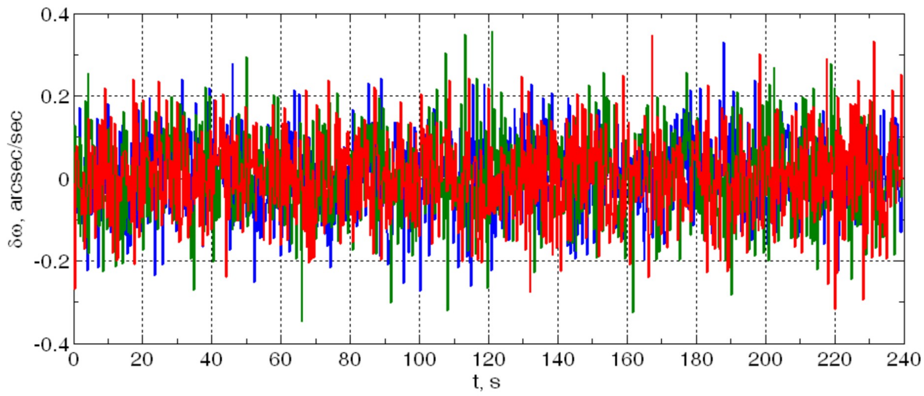

Figure 8. Errors of estimating the angular rate vector

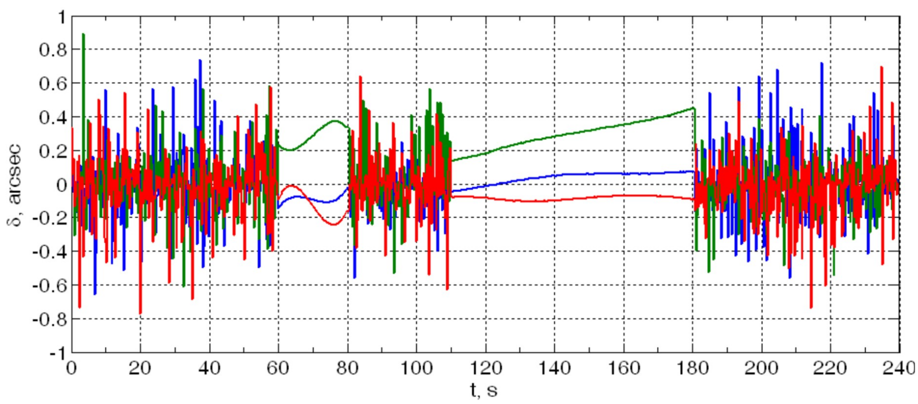

Figure 9. Errors of estimating the angular position vector

vector on the next $(k+1)$ time semi-interval $t \in [t_{k+1}, t_{k+1}+T_{\rm o})$. Here $g_2^{\rm o}$ is a constant scalar coefficient of the Luenberger observer. At the SC fast RMs with the angular rate module from 0.5 deg/s up to 5 deg/s in the time intervals with the duration of up to 120 s, the astronomical correction is switched off and estimation of the SC attitude is fulfilled by using a forecast of the IB drift variation. Such a forecast is formed based on the analysis of the drift trend at some preceding time intervals when the SINS is operated with the mentioned correction.

8 The SINS alignment

The problem of aligning the SINS and determining an estimation $\hat{m}$ of the IB scale coefficient error is also solved by the situational approach, but off-line. Here the values of the SC angular rate vector, which are restored autonomously and simultaneously according to the signals of the IB and the AS, are compared. Moreover, a set of quaternions $\mathbf{\Lambda}_{mk}^a$ measured by the AS is recomputed into a set of the Rodrigues vector values $\boldsymbol{\sigma}_k^a$. These values are filtered by the Savitsky - Goley method and are numerically differentiated by a small "sliding" window with obtaining the estimations $\hat{\boldsymbol{\omega}}_k^a$ by the reverse equation for the Rodrigues vector. Further the modules and the units of the vectors $\hat{\boldsymbol{\omega}}_k^a$ and $\hat{\boldsymbol{\omega}}_k^g$ sets are computed. Statistical processing of the modules ratio gives the estimation $\hat{m}$ of the scale coefficient error, and application of the QUEST algorithm (Somov and Butyrin, 2009) for vector units permits an estimation of a quaternion error $\hat{\mathbf{\Lambda}}^{\Delta}$ between the bases $\mathbf{A}$ and $\mathbf{G}$ to be obtained.

9 Some results of numerical simulation

Figure 6 presents a consequence of 3 routes and the components of the SC angular rate vector into the base $\mathbf{B}$ at fulfilling these routes

$$\text{M1: } t \in [0,60) \text{ s}; \quad \text{M2: } t \in [80,120) \text{ s} \quad \text{and} \quad \text{M3: } t \in [180,240] \text{ s}$$

with different survey types and 2 rotational maneuvers

$$\text{RM1: } t \in [60,80) \text{ s}; \quad \text{RM2: } t \in [120,180) \text{ s}$$

between them. For the periods $T_{\rm o} = 0.25$ s (frequency 4 Hz), $T_q = 0.03125$ s (frequency 32 Hz) and the QMD of the AS measuring errors 0.33 arc sec, the given IB bias vector $\mathbf{b}^g = \{1, -0.8, 0.3\}$ arc sec/sec is restored at 60 sec and then it is followed with the QMD $\approx 1\%$, see Fig. 7. The errors of estimating the SC angular rate and attitude are presented in Figs. 8 and 9. For these data the estimation of the "alignment" vector with accuracy ≈ 3 arc sec and the estimation with an accuracy of 0.025%, were obtained.

10 Conclusion

The original multiple discrete algorithms for filtering, integration and calibration of a strapdown inertial navigation system with astronomical correction for precise determining a large-scale spacecraft' attitude were presented. Some presented numeric results prove an efficiency of the new method suggested.

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References

Branetz, V.N. and I.P. Shmyglevsky (1992). *Strapped-down inertial navigation systems: introduction to the theory*. Nauka. Moscow.

Gusinsky, V.Z., M.V. Lesyuchevsky, Yu.A. Litmanovich, H. Mussoff and G.T. Schmidt (1997). New procedure for deriving optimized strapdown attitude algorithms. *Journal of Guidance, Control, and Dynamics* **20**(4), 673–680.

Pittelkau, M.E. (2001). Kalman filtering for spacecraft system alignment calibration. *Journal of Guidance, Control, and Dynamics* **24**(6), 1187–1195.

Somov, Ye. and S. Butyrin (2009). Onboard alignment identification of a space telescope and a star tracker cluster. In: *Proceedings of 4th International Scientific Conference on Physics and Control (PhysCon09), http://lib.physcon.ru/?item=1943*. University of Catania. Catania. pp. 1–6.

Somov, Ye., S. Butyrin and S. Somov (2009). Guidance and control of a land-survey spacecraft with a smooth conjugation of boundary conditions. In: *Proceedings of 4th International Scientific Conference on Physics and Control (PhysCon09), http://lib.physcon.ru/?item=1949*. University of Catania. Catania. pp. 1–6.

Somov, Ye.I. (2009). Multiple algorithms for filtration, integration and calibration of a strapdown inertial system for a spacecraft attitude determination. In: *Proceedings of 16th Saint Petersburg international conference on integrated navigation systems*. CSRI "Elektropribor". St.Petersburg. pp. 110–112.