STRUCTURAL PROPERTIES OF GUARANTEED CONTROL-ESTIMATION PROBLEMS FOR HIERARCHICAL SYSTEMS

Sergey V. Kruglikov
Department of Optimal Control
Institute of Mathematics and Mechanics UB RAS
Yekaterinburg, Russia
svk@imm.uran.ru

Abstract

The paper deals with axiomatic description of hierarchical systems, the choice of adequate statements of optimization problems under uncertainty [Kurzhanski, 2006] for such systems, the study of analogues of the basic structural properties (duality, separation) that are obtained earlier for the control and estimation problems in the operator form [Kruglikov, 1997]. The mathematical formalism is motivated by applied research, including the simulation of control of objects team motion [Kurzhanski, 2009] and processes of decision making in organizational systems, that are connected by the unity of applied mathematics.

Algorithms of a priori designing routes for group objects to bypass the obstacles, that are described by a system of non-convex, disconnected sets. In this case the problems of guaranteed observation and a priori preparation of cartographic information are analogues. The optimal control problem solution depends on the particular dynamics of objects of practical interest. Examples simulating classes of systems, of obstacles and admissible routes are presented.

Another realm of interest includes simulating procedures for managing the modernization of the production of high-tech engineering enterprizes. The adequacy of research in terms of hierarchical systems is provided by the multilevel structure of managed object.

Key words

hierarchical systems, control and estimation under uncertainty, duality, separation, algorithms

1 Introduction

Recently the problems of control and decision-making in organizational systems are of great significance and value. Wide range of problems includes a real-time modelling of teams interaction. Analysis shows that various interactions may be considered as a sequence of individual or common actions provided with restricted resources under hierarchically organized control. The motion of participants may be treated in terms of system trajectories reflecting state, spatial, conceptual and organizational structures, results of observation and control. Participants may change their positions in accordance with consequent decisions, formed step-wise in accordance with positions on discrete grid.

Hence a common interaction may be split into multiple layers of respectively independent processes for couples of symmetrical systems.

To describe adequately effects, that arise in such problems, mathematical notions have to satisfy a number of key assumptions.

1) Situational scheme describing the interaction of open systems with relatively constant properties of environment and participation.

2) Description of systems interaction in discrete time, which implies a concerted shifts of an action of the participants in the situation.

3) Hierarchy in description of the participation, structure and behavior of the systems leads to the absorption of the description of a lower layer subsystems that are out of the observability boundaries.

4) Internal information model, describing data available, is constructed via inverse scheme and reflects shifts of perception with the center on the image of system.

Several mathematical concepts are sufficiently coordinated with assumptions given above. In particular, problems of control and simulation of team behavior have significant importance in modern game theory and theory of extremal networks.

Our discussion is based on the notions of the theory of guaranteed control and estimation for systems with uncertainty [Kurzhanski, 2006]. In several cases the uncertainties in dynamics and location of object is possible to imbed in description of space state. The approach may be efficiently applied due to well-known structural properties of guaranteed problems and corresponding solutions. Structural symmetry and duality for conjugate systems with uncertain parameters are essential results in the optimal control theory [Kurzhanski, 2009]. Then structure of admissible trajectories may be investigated in terms of guaranteed tubes. In [Kruglikov, 1997] was shown that problems of a priory choice of optimal tube and approximation of obstacle are symmetrical.

The operator formalization of a priori problems of guaranteed control and observation that correspond to the problems of planning optimal routes and preprocessing of geographic information can be viewed as a dual pairs with respect to structured set of criteria. The duality and separation properties for control problems with set-membership description of disturbances and integral/extremal performance index [Kruglikov, 1997] have been investigated on the base of operator formalization. A system of algorithms for describing current situations and the set of possible solutions may be realized in accordance with these properties. At the same time, to be efficient in problems of decision-making, the approach has to to be adjusted to meet above given assumptions. Several versions to develop the approach are possible [Kurzhanski, 2006].

Possibility to improve the approach to meet effects of hierarchy is in the center of discussion. The concept of hierarchical system has a natural capacity to change the level of complexity in accordance with circumstances. That allows to get a unified techniques to simulate the route, set of multiple obstacles and passages between them. The possibility to use results on duality of guaranteed estimation and control problems for the route planning is considered below.

To verify theoretical speculations, examples of applications may be considered. The report considers axiomatic definition of hierarchical system in two cases. The main example is related to navigation and net tracing. The original is a problem of planning motion for a team of objects that are constrained in dynamics and are overcoming obstacles under coordinated control. Recently problems of route planning for the object formation overcoming obstacles in the common motion are of great interest. The central point for designing algorithms of route planning is a choice of adequate information structure. To reflect appropriately a coordinated motion of objects with constrained dynamics in complex circumstances information structure has to meet the assumptions given above.

The problem is considered in the plane formulation. For some particular obstacles the structure of hierarchical systems, algorithms based on them and the simulation results are considered. The aim is to develop general foundation for algorithms of a priori planning of motion of objects team. The notion of hierarchical system may provide the unified description of organizational structure, routes and geography, which is in proper correspondence with the well-known fractal presentation of coastline. In this case, the accuracy and adequacy of mathematical models can be interpreted depending on the characteristics of the object.

The special attention to this problem reflects the fact that assumptions, arising out of considerations related to the organizational systems, here may be directly tested on objects having a clear physical nature, allowing for an adequate description in terms of theoretical mechanics if not planar geometry.

Different mathematical approaches are examined for problems of motion planning. The reference presented below does not even sketch the variety of ideas but outlines some restrictions to realize them. Interval analysis techniques [Jaulin, 2001] based on the description of the obstacles as the union of rectangles proves to be efficient. In the papers, based on the use of interval analysis [Kostousova, 2011], it is proposed to represent complex charts information as a disjoint union of rectangles with sides parallel to coordinate axes. There is a positive experience in practical application, but corresponding constructions do not depend on the level of investigation regularly. That complicates the hierarchy analysis. Variation techniques developed in [**?**] allow full scale route modelling based on contingent constructions under requirements of significant manoeuvrability. Well-known techniques of the route choice are based on the description of obstacles as union of cells various in complexity. The trajectory turns nearby contour vertices. Further it is convenient to discuss the obstacles as islands.

Hierarchical system, that describes an admissible route, is a regular combination of elementary subsystems such as trajectory tubes, which reflects the change in cross-section accumulated errors. Characteristic features of a particular object are taken into account by selecting the number of hierarchical levels and restrictions on the considered units of length and radius. For groups of objects that have different technical specifications, restrictions on mobility are formulated through the angles and lengths of linear segments. Note that logics of business planning is the same and decision-making tree techniques are evident examples.

Presented models, based on a notion of hierarchical $(i)-$system, below may provide the unified description of organizational structure, routes and geography. Then the problem of shoreline description is dual with the problem of admissible route design. Main constructions are based on finite combination of chains with cylindrical branches. Complex shape of shoreline may be described in advance in the form of sea graph and in the consequence the admissible routes may be chosen on the graph. The admissible route corresponds to the tube of trajectories, sections of which involve accumulated errors.

The presented algorithms are illustrated by a prototype software that implements the simulation of a priori routing bypass nonconvex obstacles of the different topology (an arbitrary star, system of stars). Practical design, based on mathematical results and implement applied research, carried out during several years to develop algorithms and software complex routing products, bypassing the land formations. The results of applied research prospects mathematical and software design creation of ship management information systems.

2 PROBLEM STATEMENT

Let U be an arbitrary bounded set, $U \subseteq R^2$; and G, S are nonintersecting correspondingly closed and open subsets of U, $U = G \cup S$. There exists a convex partition of G, $\quad G = \bigcup_{m \leq Mm} G.m.\quad$ Sets $G.m$ present the basic zero-level information for description of islands and S describes sea.

Suppose there are data on original and final position of the moving system. The problem is to find the tube of trajectories corresponding to the properties of dynamical object.

The scaling of object properties and circumstances description is arbitrary. Hence the solution may be given in sequence of two procedures.

Under the interaction of hierarchical control systems according to the scheme of duality we understand representation of the solution of the problem of constructing a route with a predetermined precision in the form of two coordinated subtasks: routing problem, satisfying the condition 1, with an accuracy determined by subproblem 2 for preparing an island description in the form of a combination of stars and problem representations of an island in the form of a hierarchical system, matching the accuracy of information in the target detection and the value of $Rmax$.

2.1 Hierarchical (i)-system

The hierarchical system is a set consisting of: bounded region; an ordered set of hierarchical system of lower layer, by level, matching the accuracy of information and value $Rmax$; an ordered set describing elements of the convex hull and the modified passes, reflecting the narrowness and the presence of relatively nearby objects.

Definition 1. Hierarchical $(i)-$system $(i)CG = \{\mathbf{X}G, \mathbf{P}G, \mathbf{Q}G\}$ is the triple including:

1.1) state: $\mathbf{X}G = \{XG|\mathbf{e}0\}$, where XG is a subset of metric space embedding subordinate hierarchical $(i-1)-$systems of lower layer and vector $\mathbf{e0}$ defines the polar coordinate system LC. In particular, if $XG = B(c0, rc0)$ is a ball then $LC = \{c0, rc0|\mathbf{e}0\}$.

1.2) $\mathbf{P}G = \{PG, \Pi G\}$, where PG is an ordered list of subordinate hierarchical $(i-1)-$systems $(i)PG = \{(i-1)CG.mj|1 \leq mj \leq Mm\}$. Note that $B(c0, rc0) \supseteq \cup\{(i-1)XG.mj\}$.

1.3) $\mathbf{Q}G = \{QG, \Theta G\}$ gives the generalized description of nearest boundary region.

Axiomatical description of the two types of $(i)-$systems modelling sea and seashore is considered.

Definition given above allows to formulate operations over $(i)-$system.

2.2 Adjoint hierarchical (i)-system

adjoint and conjugated $(i)-$systems may be introduced by inclusion in the consideration the notion of region and surrounding of the set of hierarchical (i)-system.

2.3 Problems under consideration

Throughout the paper the symmetrical operator representation of extremal a priori problems stated for linear systems with unknown in advance parameters is used.

Quality criteria under consideration are ordered lexicographically.

1) the minimal length of route $Di < Dmax, (i = 1, 2, ...)$ with the number of piecewise linear segments not exceeding K;

2) the minimum number of piece-wise linear segments;

3) minimum length.

2.4 Duality of extremal problems.

Duality of extremal problems means the symmetry in the problem statement and solutions for adjoint hierarchical (i)-systems. In the paper the results on duality of estimation and control problems stated for different types of $(i)-$systems are considered.

2.5 Separation property

The separation principle in the theory of optimal control means that the solution of the general problem under uncertainty may be formed as superposition of solutions of particular separated problems of optimal control and observation in case of fixed controlling procedure. The above definition reserves the meaning for problems formulated in terms of hierarchical $(i)-$systems.

3 Algorithms structure

The problem of planning routes to bypass islands consists of two interdependent problems. First of them is a problem of overall islands description. The result is a chart presentation adequate for forming tubes of alternative ways of bypassing islands. As there is no data on the terminal positions; the tubes are constructed on the base of guaranteed approach. The second one is the problem of route choice if the terminal positions are given. Then the solution is constructed on the base of previously formed chart presentation. An admissible route has to satisfy the condition given below.

Condition. 1) The total route length is not greater than $Dmin$;

2) the distance between adjacent vertices route is not less than $2Rmax$;

3) angle between two adjacent edges of the route not more than that.

Several cases of obstacles description based on the concepts given above are considered.

1) a star with an arbitrary number of rays;

2) a set of bounded balls;

3) a polar organized set of stars with an arbitrary number of rays.

At present the following algorithms are developed.

- The algorithm constructing a set of hierarchical systems.

- The algorithm selecting admissible routes. Further correction and optimization of a number of lexicographically ordered criteria when data on obstacle are in the form (1-2) above.

- The algorithm for the correction of an existing route to ensure the feasibility of the dynamic characteristics of the product.

Initial data for the simulation route for each of the algorithms is the chart information recorded in the standard format.

4 Modernization Management

Chaotic effects of market behavior determine the relevance of research in terms of uncertainty. Interesting that long cycles of designing and manufacturing as a feature of engineering industries allow us to formulate an optimization problem within a guaranteed approach. For high-tech machinery plants is common to work on forward contracts, fixing production volumes and prices of products for quite a long time (3-4 years). At the same time, inflation expectations cause the increase in cost of basic production factors. Rising prices for energy, raw materials, wages, utilities invariably leads to a limitation of profits. An efficient tool at the disposal of production managers is to reduce standard variable costs at the expense of upgrading and providing its administrative modernization. In the paper the technology determine the optimal date of purchase of equipment and design rationale for its composition, based on the coefficients of equivalence. The market competitiveness of machine-building plant depends on the level of productive capacity. However, the market competitiveness is mainly determined by the efficiency of the government and management for the production system and the whole plant.

The proposed model allows us to consider two aspects of performance management systems.

1) Priority measures to modernize the management in comparison with purely technological improvements in the productive capacity of the enterprize.

2) The comparison of two schemes of management organization on engineering plants. The scheme based on the separation of ownership and control is described in terms of game formalism. One consequence may be effects similar to an agency conflict. Model comparison is made with the scheme of charging the costs to top management on fixed costs. Payment for top managers of state corporations set up this way. The presented results are used extensively when working with students and training specialists in the field of production modernization of high-tech innovative engineering plants and preparing the management reserve.

5 Conclusion

Models presented above provide an unified description of organizational structure, trajectories and cartographic information. Symmetry of hierarchical $(i)-$systems allows to describe complex shape of shoreline in advance by solving a guaranteed estimation problem. Quality induces are extremal functionals. A priori procedures of control and estimation are determined by a nonanticipative operator. Then trajectories choice may be interpreted in terms of guaranteed control problem. The separation property for guaranteed control/estimation problems allows us to split algorithmically a coordinated control on a separate procedures.

Acknowledgements

The research was partly supported by Russian Foundation for Basic Research (project 09-01-00223) and the Collaboration Programme in fundamental research of Ural and Siberian Branches of Russian Academy of Sciences "Qualitative and numerical analysis of evolution equations and control systems".

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