HYBRID ADAPTIVE RESONANCE CONTROL OF VIBRATION MACHINES: THE DOUBLE MASS CASE

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***Abstract*—The problem of resonance regime stabilization is considered for the case of unknown natural frequency of vibration machine. A solution for fourth order linear model of the machine with uncertain parameters, external disturbances and partial noisy measurements is proposed.**

I. INTRODUCTION

IN resonance regime an oscillating systems can be forced to oscillations by any sufficiently small input with frequency closed to natural frequency of the system. This is why resonant machines have the best performance since they use resonance oscillations of actuating mechanism as the main operating regime. In resonance regime the drive energy is consumed with the best performance [2].

These facts explain the efficiency of resonant machines, however their practical implementation meets obstacles dealing with necessity of exact tuning to resonance regime. In the papers [4], [5] two solutions of adaptive resonance control are presented for simplified model of vibration machine (vibrational crusher). The first solution is based on adaptive observer from works [3], [7], [12]. The advantage of this solutions consists in estimation of unknown values of all model parameters, that results to high dimension of the regulator, that is the negative feature of the solution. The second solution is based on speed gradient approach. This solution does not provide parameters values estimation, however dimension of regulator in this case equals to the plant dimension. Both solutions are obtained under assumptions that displacement coordinate is measurable with noise, and unknown bounded disturbance affects on the crusher.

In this paper the problem of crusher resonance control is considered for the double mass model case. In such situation the solving problem becomes more complex, since for one mass it is necessary to provide resonant oscillations with desired amplitude, while for another mass it is required to ensure the absence of movements. A solution of the problem is presented.

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II. PROBLEM STATEMENT

The double mass scheme for a vibration crusher is shown in Fig. 1. Comparing with [4] here another one moving platform E with mass m is added, the platform is located on moving platform A of mass M. The foundation B is stationary, values of spring stickinesses c and c_0 are unknown. For simplicity of consideration let us assume, that vibroexciters C and D are rotating synchronously and they form harmonic exciting input on platform A. Let us assume, that movements of platforms A и E are possible in vertical plane only, denote these movements as x_A and x_E correspondingly. It is necessary to design a controller adjusting frequency of motors rotating and amplitude of their total harmonic influence, which provides minimum of movement x_A for the platform A or minimum of the platform energy (the problem of vibrations suppressions). In such situations all energy supplied by vibroexciters C and D are accumulated in platform E, which performs oscillations with maximum amplitude (which is possible for given amplitude of excitation force). It is required to adjust amplitude of control input ensuring oscillations of variable x_E with desired amplitude (the problem of vibrations excitation). Let us stress, that excitation input is applied to the platform A, which movements it is necessary to minimize. Such problem is also called as the problem of an active vibration absorber design [10], [11].

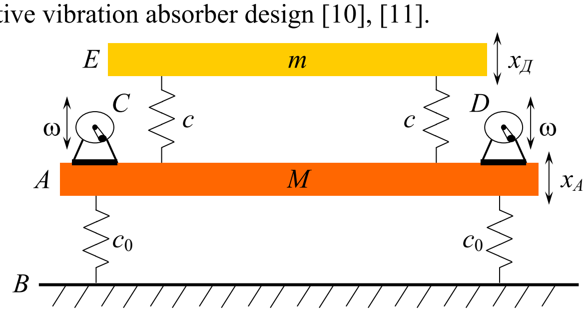

Fig. 1. Double mass scheme of vibration crusher.

The mathematical model of the system has form:

$$\dot{x}_1 = x_2\,;\ y_1 = x_1 + \phi_1\,; \qquad (1)$$
$$\dot{x}_2 = -\beta_1/m\,x_2 - c/m\,(x_1 - x_3) + c\,f/m + g + d_1\,;$$

$$\dot{x}_3 = x_4\,;\ y_2 = x_3 + \phi_2\,;$$
$$\dot{x}_4 = -\beta_2/M\,x_4 + c/M\,(x_1 - x_3) - c_0/M\,x_3 - \qquad (2)$$
$$- c\,f/M - m\,g/M + \varepsilon/M \sin(\omega t) + d_2\,,$$

where $x_1 \in R$, $x_3 \in R$ are displacements of platforms E and

A correspondingly ($x_1 = x_E$, $x_3 = x_A$), $\dot{x}_1 \in R$, $\dot{x}_3 \in R$ are velocities of the platforms; $y_1 \in R$, $y_2 \in R$ are measurements of variables x_1, x_3 available with noises $\phi_1 \in \mathcal{M}_R$, $\phi_2 \in \mathcal{M}_R$; $d_1 \in \mathcal{M}_R$, $d_2 \in \mathcal{M}_R$ are external disturbances acting on the platforms; β_1, β_2 are small unknown friction coefficients; f is unknown initial springs tension between the springs; g is free fall acceleration; ε is adjusting control amplitude of the force formed by motors C and D; ω is adjusting frequency. Values of masses m and M are assumed unknown and constant.

III. Main result

The full energy function for the platform A has form:

$$E = 0.5 M x_4^2 + 0.5(c + c_0) x_3^2,$$

$$\dot{E} = -\beta_2 x_4^2 + x_4(-cf - mg - \varepsilon \sin(\omega t) - x_1 + d),$$

that implies that the most suitable input providing the minimization of energy E is state feedback $x_1 + cf + mg$ (not the harmonic control $\varepsilon \sin(\omega t)$ chosen in the paper). However, the harmonic control can be easily implemented, that is its main advantage.

System (1), (2) is a linear one, under imposed conditions its characteristic polynomial has two pares of complex conjugate roots with strictly negative real parts: $\lambda_{12} = -\alpha \pm \beta i$, $\lambda_{34} = -\gamma \pm \delta i$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\delta > 0$. Thus, the analytical expression of the system solutions can be written as follows:

$$x_1(t) = e^{-\alpha t}(A\sin(\beta t) + B\cos(\beta t)) + e^{-\gamma t}(C\sin(\delta t) + D\cos(\delta t)) + U + P\sin(\omega t) + Q\cos(\omega t) + x_1^d(t);$$

$$x_2(t) = -\alpha e^{-\alpha t}(A\sin(\beta t) + B\cos(\beta t)) + \beta e^{-\alpha t}(A\cos(\beta t) + B\sin(\beta t)) + \dot{x}_1^d(t) - \gamma e^{-\gamma t}(C\sin(\delta t) + D\cos(\delta t)) + \delta e^{-\gamma t}(C\cos(\delta t) + D\sin(\delta t)) + \omega(P\cos(\omega t) - Q\sin(\omega t));$$

$$x_3(t) = e^{-\alpha t}(E\sin(\beta t) + G\cos(\beta t)) + e^{-\gamma t}(K\sin(\delta t) + L\cos(\delta t)) + R + S\sin(\omega t) + T\cos(\omega t) + x_3^d(t);$$

$$x_4(t) = -\alpha e^{-\alpha t}(E\sin(\beta t) + G\cos(\beta t)) + \beta e^{-\alpha t}(E\cos(\beta t) + G\sin(\beta t)) + \dot{x}_3^d(t) - \gamma e^{-\gamma t}(K\sin(\delta t) + L\cos(\delta t)) + \delta e^{-\gamma t}(K\cos(\delta t) + L\sin(\delta t)) + \omega(S\cos(\omega t) - T\sin(\omega t));$$

where A, B, C, D, E, G, K, L, U, P, Q, R, S, T are the solutions parameters dependent on initial conditions and the system coefficients; $x_1^d(t)$ and $x_3^d(t)$ are forced parts of the solutions originated by disturbances d_1 and d_2 presence. Since all eigen-values have negative real parts, then normal parts of solutions converge to zero asymptotically and for the problem solution the analytical expressions for parameters U, P, Q, R, S and T should be derived only. Basing on the kind of their dependence on adjusting amplitude ε and frequency ω it is possible to determine the optimal values of these parameters providing minimum of displacements of the platform A with oscillations of the platform E with desired amplitude. To find the expressions let us use differential equations for velocities $\dot{x}_2$ and $\dot{x}_4$ from (1), (2). Substituting in these equations expressions for the system solutions and equating the coefficients with the same multipliers we obtain:

$$c(U - R) = cf + mg;$$
$$cf + mg + c_0 R = c(U - R);$$
$$c(Q - T) + \beta_1 P\omega = Qm\omega^2;$$
$$c(S - P) - \beta_2 T\omega - S\omega^2 M + c_0 S = \varepsilon;$$
$$P\omega^2 m + \beta_1 Q\omega = c(P - S);$$
$$\beta_2 S\omega + c_0 T = T\omega^2 M + c(Q - T).$$

Solutions of this system of linear equations have form:

$$R = 0;\ S(\omega) = \varepsilon a(\omega)/z(\omega);$$
$$T(\omega) = \varepsilon b(\omega)/z(\omega),\ U = f + mgc^{-1},$$
$$P(\omega) = S(\omega) + [(c_0 - \omega^2 M)S(\omega) - \varepsilon - \beta_2 \omega T(\omega)]c^{-1}, \quad (3)$$
$$Q(\omega) = (\beta_1 \omega P(\omega) - cT(\omega))(m\omega^2 - c)^{-1},$$

where

$$a(\omega) = -m^2 M\omega^6 + ([2cm - \beta_1^2]M + m^2[c_0 + c])\omega^4 + (\beta_1^2[c + c_0] - c^2[m + M] - 2cc_0 m)\omega^2 + c_0 c^2;$$

$$b(\omega) = \omega[-\beta_2 m^2 \omega^4 + (2cm - \beta_1^2)\beta_2 \omega^2 - c^2(\beta_1 + \beta_2)];$$

$$z(\omega) = M^2 m^2 \omega^8 + (\beta_1^2 M^2 - 2[m + M]Mmc + [\beta_2^2 - 2c_0 M]m^2)\omega^6 + \{(m + M)^2 c^2 + 2(c_0 m^2 - \beta_1^2 M + [2c_0 M - \beta_2^2]m)c + c_0^2 m^2 - 2\beta_1^2 c_0 M + \beta_1^2 \beta_2^2\}\omega^4 + \{([\beta_1 + \beta_2]^2 - 2c_0[m + M])c^2 + c_0^2 \beta_1^2 + 2c_0(\beta_1^2 - c_0 m)\}\omega^2 + c_0^2 c^2.$$

For the posed problem solution it is necessary to find frequency ω^*, which provides minimum absolute values for polynomials $S(\omega)$ and $T(\omega)$, for example, the fulfillment of the equality:

$$S(\omega^*) = T(\omega^*) = 0.$$

Polynomials a, b and z have complex form of dependence on frequency, that prevents to analytical solution of the problem without additional assumptions. As such additional requirements let us suppose, that values of friction coefficients β_1, β_2 are negligible small comparing with all other parameters values. In this case we can skip these terms in equations (1), (2), then expressions (3) for S and T can be reduced to the following:

$$\widetilde{S}(\omega)=\frac{\varepsilon(c-m\omega^2)}{mM\,\omega^4-(cM+m[c+c_0])\omega^2+c_0\,c},\ \widetilde{T}(\omega)=0.$$

It is clear, that in such situation the desired frequency has the form:

$$\omega^*=\sqrt{c/m},$$

that is the anti-resonance frequency between a force applied on platform A and the corresponding displacement of platform A, and values

$$\begin{bmatrix} S(\omega^*) \\ T(\omega^*) \end{bmatrix}=\begin{bmatrix} -\varepsilon m\beta_1^2(cM-m[c+c_0]) \\ -\varepsilon\, m^2\beta_1\omega^*(cm+\beta_1\beta_2) \end{bmatrix}\times$$

$$\times\{c^3m^3+2\beta_1\beta_2\,c^2m^2+\beta_1^2(cm\beta_2^2+[cM-m(c+c_0)]^2)\}^{-1}$$

become small for small β_1, β_2. If values of the friction coefficients are rather big, then one can try to derive numerical solution ω^* providing a minimum of polynomials $S(\omega)$ and $T(\omega)$.

For desired frequency ω^* realization it is necessary to calculate estimates for values of unknown parameters c and m. As in [4] let us design adaptive observer with the filter, which provides estimation of unmeasured velocity of variable x_1:

$$\dot{q}=-\rho(q+y_1),\ \hat{x}_2=\rho(q+y_1). \tag{4}$$

$$\begin{aligned} \dot{z}_1&=z_2+K_1(y-z_1); \\ \dot{z}_2&=K_2(y-z_1)+g; \end{aligned} \tag{5}$$

$$\begin{aligned} &\begin{bmatrix} \dot{\Omega}_{11} & \dot{\Omega}_{12} & \dot{\Omega}_{13} \\ \dot{\Omega}_{21} & \dot{\Omega}_{22} & \dot{\Omega}_{23} \end{bmatrix}= \\ &=\begin{bmatrix} -K_1\Omega_{11}+\Omega_{21} & -K_1\Omega_{12}+\Omega_{22} & -K_1\Omega_{13}+\Omega_{23} \\ -K_2\Omega_{11}+y_1-y_2 & -K_2\Omega_{12}+\hat{x}_2 & -K_2\Omega_{13}-1 \end{bmatrix}; \end{aligned} \tag{6}$$

$$\begin{aligned} \dot{\hat{\theta}}_1&=-\gamma\Omega_{11}(y_1-z_1+\Omega_{11}\hat{\theta}_1); \\ \dot{\hat{\theta}}_2&=-\gamma\Omega_{12}(y_1-z_1+\Omega_{12}\hat{\theta}_2); \\ \dot{\hat{\theta}}_3&=-\gamma\Omega_{13}(y_1-z_1+\Omega_{13}\hat{\theta}_3), \end{aligned} \tag{7}$$

where $\mathbf{z}=[z_1\ z_2]\in R^2$ is vector of variables x_1 and x_2 estimation; $\mathbf{\Omega}=[\Omega_{11}\ \Omega_{12}\ \Omega_{13};\Omega_{21}\ \Omega_{22}\ \Omega_{23}]\in R^{2\times3}$ is auxiliary matrix variable; $\hat{\mathbf{\theta}}=[\hat{\theta}_1\ \hat{\theta}_2\ \hat{\theta}_3]\in R^3$ is vector of estimates for parameters $\mathbf{\theta}=[c/m\quad \beta_1/m\quad cf/m]$; parameters K_1, K_2, γ, ρ are strictly positive. Variable $\hat{x}_2\in R$ is the estimation of unmeasured variable x_2. Differential equation for estimation error $\mathbf{e}=[e_1\ e_2]=[x_1-z_1\ x_2-z_2]$ has form ($\iota=\hat{x}_2-x_2$):

$$\begin{aligned} \dot{e}_1&=e_2-K_1e_1-K_1\phi_1; \\ \dot{e}_2&=-K_2e_1-\theta_1(y_1-y_2)-\theta_2\hat{x}_2+\theta_3- \\ &\quad-\theta_1(\phi_1-\phi_2)-\theta_2\iota+d_1, \end{aligned} \tag{8}$$

with auxiliary variable $\mathbf{\delta}=\mathbf{e}+\mathbf{\Omega\theta}$:

$$\dot{\mathbf{\delta}}=\mathbf{G}\mathbf{\delta}+\mathbf{r}(t),\ \mathbf{G}=\begin{bmatrix} -K_1 & 1 \\ -K_2 & 0 \end{bmatrix}, \tag{9}$$

$$\mathbf{r}(t)=\begin{bmatrix} -K_1\phi_1(t) \\ -\theta_1(\phi_1(t)-\phi_2(t))-\theta_2\iota(t)+d_1(t) \end{bmatrix}.$$

Tacking in mind equations (8), (9) it is possible to prove the following result (the analog of Lemma 2 from [4]).

L e m m a 1. *Let* $\phi_1(t)$ *be differentiable and* $\|\dot{\phi}_1\|<+\infty$, $\|\phi_1\|<+\infty$, $\|\phi_2\|<+\infty$, $\|d_1\|<+\infty$, $\|d_2\|<+\infty$; *there exist* $r>0$, $\Delta>0$ *such, that signals* $\Omega_{11}(t)$, $\Omega_{12}(t)$, $\Omega_{13}(t)$ *admit* (Δ,r)*–persistency of excitation (PE) condition. Then for any* $K_1>0$, $K_2>0$, $\gamma>0$, $\rho>0$ *all solutions of the system (1), (2), (4)–(7) are bounded and the estimate holds*:

$$\lim_{t\to+\infty}|\mathbf{\theta}-\hat{\mathbf{\theta}}(t)|\le\sqrt{(\gamma+2r^{-1}e^{-0.5r\gamma})\Delta}\,[\|\phi_1\|+$$

$$+8\lambda_1^{-1}\lambda_2\,\vartheta_{\max}(K_1K_2)^{-1}\times((K_1+c/m)\|\phi_1\|+c/m\|\phi_2\|+$$

$$+2\sqrt{2}\,\beta_1/m(\|\dot{\phi}_1\|+\rho^{-1}\|\dot{x}_2\|)+\|d_1\|)],$$

$$\lambda_{1,2}=0.25\begin{bmatrix} (K_2+1)^2+K_1^2\pm \\ \pm\sqrt{(K_2+1)^2+K_1^2}\sqrt{(K_2-1)^2+K_1^2} \end{bmatrix},\lambda_1<\lambda_2,$$

$$\vartheta_{\max}=0.125\,((K_2+1)^2+K_1^2)\times$$

$$\times\left[K_1^2+1+K_2^2+\sqrt{(K_2+1)^2+K_1^2}\sqrt{(K_2-1)^2+K_1^2}\right]. \blacksquare$$

A definition of PE property and some useful lemmas are presented in Appendix 1. Proofs of the lemma and all other results are postponed to Appendix 3. Posed PE properties of signals $\Omega_{11}(t)$, $\Omega_{12}(t)$, $\Omega_{13}(t)$ are satisfied if vector signal $(y_1-y_2,\hat{x}_2,1)$ (input to the system (6)) has the same property, that is a mild requirement for any operating regime of vibrational crusher.

Further it is necessary to design adaptation algorithm for amplitude ε, ensuring the desired amplitude of oscillations a for variable x_1 with fixed control frequency ω. Asymptotically the behavior of variable x_1 is defined by constants U, P, Q. According to (3) constant U does not depend on regulating variables ω and ε, i.e. the relative displacements of the platform E with respect to the platform A can not be modified with the control produced by vibroexciters C and D. On the other hand the polynomials $P(\omega)$ and $Q(\omega)$ depend on amplitude ε in linear fashion. These coefficients P and Q define the amplitude of variable x_1 oscillations asymptotically. Output $y_1(t)$ amplitude on j^{th} half-period of the control input can be defined as follows:

$$\begin{aligned} a_{j+1}&=\sup_{\pi\omega^{-1}j\le t<\pi\omega^{-1}(j+1)}\{|y_1(t)|\}= \\ &=\sup_{\pi\omega^{-1}j\le t<\pi\omega^{-1}(j+1)}\{|x_1(t)+\phi_1(t)|\}, \end{aligned}\quad j=0,1,2,\ldots.$$

It is available for measurements piecewise constant signal. It is necessary to ensure convergence of amplitudes a_j to desired a with index j increasing. From (3) it follows that:

$$a_j = \varepsilon\beta_j + w_j, \ j = 0,1,2,...,$$

where values of $w_j \geq 0$ corresponds to external disturbances presence and they depend on initial conditions, values of $\beta_j > 0$ depend on initial conditions only (both β_j and w_j depend on system (1) parameters of course). According to problem statement values of β_j and w_j are bounded, positive and unknown. As in [4] in the same situation, applying algorithm from book [6] (formulated in Appendix 2), it is possible to solve this problem.

Lemma 2. *Let $u(t) = \varepsilon\sin(\omega t + \varphi)$, $\varepsilon, \omega, \varphi \in R$. If for system (1), (2) and given ϕ_1, ϕ_2, d_1, d_2 there exist constants $0 < \delta < 1$ and $\varepsilon^* \in R$ such, that for all $j = 0,1,2,...$ the following recurrent inequalities are satisfied*

$$|\varepsilon^*\beta_j + w_j - a| \leq \delta^2, \tag{10}$$

then there exist $\mu > 0$ and $\mathrm{E} > 0$ such, that algorithm:

$$\eta_j = a_j - a, \ \nu_j = sign(\delta - |\eta_j|), \ |\varepsilon_0| \leq \mathrm{E},$$

$$\varepsilon_{j+1} = \begin{cases} \varepsilon_j, & \text{if } \nu_j = 1 \text{ or } |\varepsilon_j| > \mathrm{E}; \\ \varepsilon_j - \mu\eta_j, & \text{if } \nu_j = -1, \end{cases} \tag{11}$$

for some value of index $J > 0$ provides for all $j \geq J$ the following properties:

$$\varepsilon_j = \varepsilon_J, \ |a_j - a| \leq \delta.$$

Let $u(t) = \varepsilon\sin(\omega t + \varphi)$, $\varepsilon, \omega, \varphi \in R$ and $\phi_1(t) = \phi_2(t) = d_1(t) = d_2(t) = 0$, $t \geq 0$. Then for any $0 < \delta < 1$ there exist $\mu > 0$, $\mathrm{E} > 0$ such, that algorithm (11) for some index $J > 0$ provides for all $j \geq J$ the following properties:

$$\varepsilon_j = \varepsilon_J, \ |a_j - a| \leq \delta. \qquad \blacksquare$$

The inequalities (10) are the single verified condition of the lemma. The complexity of condition (10) verifying consists in its dependence on unknown disturbances, however for the case $\phi_1 = \phi_2 = d_1 = d_2 = 0$ the inequalities (10) are satisfied for arbitrary δ.

Adaptive observer (4)–(7) for any bounded control provides the estimation of system (1) unknown parameters and identification of its natural frequency ω^* (control input should possess some excitation properties to ensure PE conditions imposed in Lemma 1). Adaptation algorithm (11) from Lemma 2 for any control $\varepsilon\sin(\omega t + \varphi)$ with fixed frequency ω guarantees the desired amplitude of plant (1) oscillations via ε adjusting. Let us combine both results.

Theorem 1. *Let*

$$u(t) = \varepsilon_j \sin(\hat{\omega}^*(t)t), \ \hat{\omega}^*(t) = \sqrt{|\hat{\theta}_1(t)|}, \tag{12}$$

and for any ϕ_1, ϕ_2, d_1, d_2 ($\phi_1(t)$ is differentiable and $\|\phi_1\| < +\infty$, $\|\phi_2\| < +\infty$, $\|\dot{\phi}_1\| < +\infty$, $\|d_1\| < +\infty$, $\|d_2\| < +\infty$) there exist $r > 0$, $\Delta > 0$ such, that signals $\Omega_{11}(t)$, $\Omega_{12}(t)$, $\Omega_{13}(t)$ admit (Δ, r)–PE condition. Then for any $\varepsilon_x > 0$, $K_1 > 0$, $K_2 > 0$ and initial conditions $|\mathbf{x}(0)| \leq \varepsilon_x$ there exist $\rho > 0$, $\gamma > 0$, $0 < \delta < 1$, $\mu > 0$, $\mathrm{E} > 0$ such, that all solutions of the system (1), (2), (4)–(7), (11), (12) are bounded and for the case $\phi_1(t) = \phi_2(t) = d_1(t) = d_2(t) = 0$, $t \geq 0$ there exist index $J > 0$ and time instant $T_\omega > 0$ which for all $j \geq J$ and $t \geq T_\omega$ provide the following properties:

$$\varepsilon_j = \varepsilon_J, \ |a_j - a| \leq \delta, \ |\omega^* - \hat{\omega}^*(t)| \leq \delta. \qquad \blacksquare$$

Thus, hybrid adaptive regulator (4)–(7), (11), (12) provides for any given compact set of initial conditions the solution of the posed problem with some accuracy δ. If variable x_2 is available for the direct measurements, then as it will be proven in the following result, the accuracy can be taken arbitrary.

Corollary 1. *Let $\hat{x}_2(t) = x_2(t) + \phi_3(t)$, $t \geq 0$ (variable x_2 is available for noisy measurements) and for any functions ϕ_1, ϕ_2, ϕ_3, d_1, d_2 ($\|\phi_1\| < +\infty$, $\|\phi_2\| < +\infty$, $\|\phi_3\| < +\infty$, $\|d_1\| < +\infty$, $\|d_2\| < +\infty$) there exist $r > 0$, $\Delta > 0$ such, that signals $\Omega_{11}(t)$, $\Omega_{12}(t)$, $\Omega_{13}(t)$ admit (Δ, r)–PE condition. Then for any $K_1 > 0$, $K_2 > 0$, $\gamma > 0$, $0 < \delta < 1$ and initial conditions $\mathbf{x}(0) \in R^2$ there exist $\mu > 0$, $\mathrm{E} > 0$ such, that all solutions of the system (1), (2), (5)–(7), (11), (12) are bounded and for $\phi_1(t) = \phi_2(t) = \phi_3(t) = d_1(t) = d_2(t) = 0$, $t \geq 0$ the relation holds*

$$\lim_{t \to +\infty} \hat{\omega}^*(t) = \omega^*$$

and there exists index $J > 0$ such, that for all $j \geq J$ properties hold

$$\varepsilon_j = \varepsilon_J, \ |a_j - a| \leq \delta. \qquad \blacksquare$$

Example. Graphics of transient processes for the system (1), (2), (4)–(7), (11), (12) with parameters values $m = 2.5$ kg, $c = c_0 = 5300$ N/m, $\beta_1 = \beta_2 = 5$, $f = -0.05$ m, $M = 11$ kg, $\gamma = [5000 \ 200 \ 100]$, $K_1 = 40$, $K_2 = 400$, $\rho = 500$, $a = 0.05$, $\delta = 0.01$, $\mu = 100$ and disturbances $d_1(t) = 0.005\sin(0.5t)$, $d_2(t) = 0.005\sin(t)$, $\phi_1(t) = 0.1d_1(t)$, $\phi_2(t) = 0.1d_2(t)$ are shown in Fig. 2. At time instant $t = 500$ sec values of mass m for the platform *E* and initial springs tension f are changed, they become $m = 2$ kg, $f = -0.01$ m. In Fig. 2,*a* graphic of frequency $\hat{\omega}^*$ estimation is shown, in Fig. 2,*b* and 2,*c* graphics for variables y_1 and y_2 are plotted, in Fig. 2,*d* the control amplitude graphic is presented. Here, as in the first example from [4], the deviation of estimated frequency $\hat{\omega}^*$ from true value ω^* results to serious increasing of control amplitude. When frequency estimation error has converged to zero, the control amplitude decreases. After that the deflections of regulated variables y_1 and y_2 from their desired values are originated by disturbances presence in the system. □

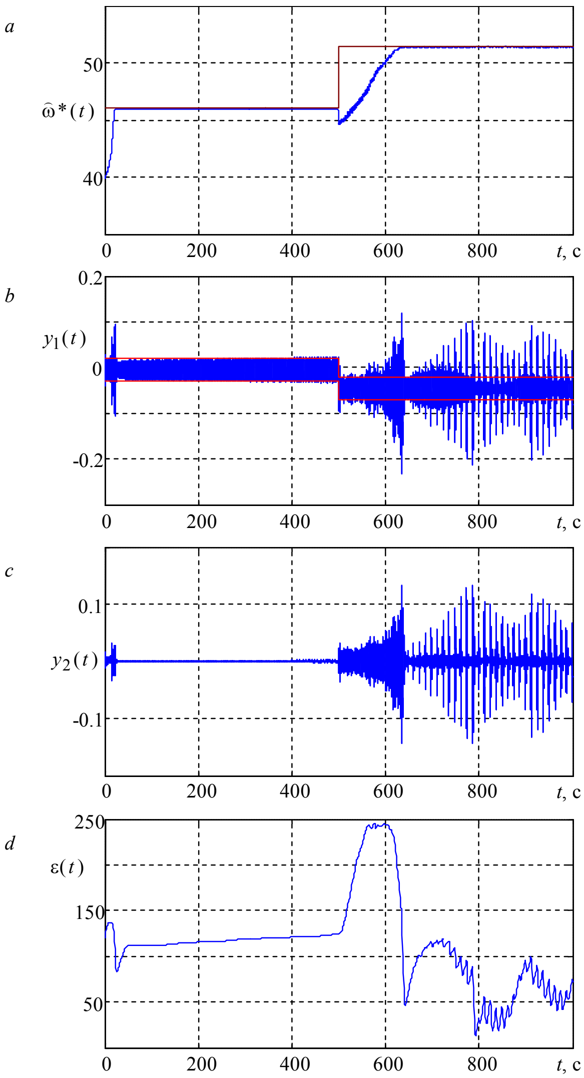

Fig. 2. Trajectories for the system (1), (2), (4)–(7), (11), (12).

IV. Conclusion

For the double mass model of vibrational crusher the hybrid adaptive regulator is proposed, which ensures for upper mass the resonant oscillations stabilization with desired amplitude and minimum movements of lower mass. The model uncertain parameters with external disturbances and measurement noises are taking into account. The identification of all system parameters and the estimation of natural plant frequency is ensured. The adaptive regulator contains one part operating in discrete time and another continuous one, that explains term “hybrid” placed in the title. Computer simulation results show workability of the proposed solution.

Appendix 1

The next property is frequently used in adaptive control systems theory to establish identification ability of adaptation algorithms [1], [6], [8], [9].

Definition A1. *It is said, that Lebesgue measurable and square integrable matrix function* $\mathbf{R}: R_+ \to R^{l_1 \times l_2}$ *with dimension* $l_1 \times l_2$ *admits* (L, ϑ)*–persistency of excitation (*PE*) condition, if there exist strictly positive constants* L *and* ϑ *such, that for any* $t \ge 0$

$$\int_t^{t+L} \mathbf{R}(s)\mathbf{R}(s)^T\, ds \ge \vartheta \mathbf{I}_{l_1},$$

where $\mathbf{I}_{l_1}$ *denotes identity matrix of dimension* $l_1 \times l_1$.

The following lemma introduces an equivalent characterization of PE property used in the sequel.

Lemma A1. *Let Lebesgue measurable and square integrable matrix function* $\mathbf{R}: R_+ \to R^{l_1 \times l_2}$ *with dimension* $l_1 \times l_2$ *be* (L, ϑ)*–PE. Then for any* $\ell \ge L$ *and* $t \ge 0$ *inequality is satisfied:*

$$\int_t^{t+\ell} \mathbf{R}(s)\mathbf{R}(s)^T\, ds \ge \frac{\vartheta}{2L}\ell \mathbf{I}_{l_1}. \qquad \text{(A1)}$$

The converse statement is obvious, if for matrix function $\mathbf{R}$ inequality (A1) is satisfied for all $\ell \ge L$, then $\mathbf{R}$ is $(L, \vartheta/2)$–PE. Let us stress, that inequality (A1) means, that positive semidefinite matrix $\mathbf{R}(t)\mathbf{R}(t)^T$ has positive definite averaged matrix for large enough time interval (the length of the interval should be bigger than L). Such positivity in average was used in paper [3]. The importance of PE or positivity in average properties are explained in the following lemma [1], [6], [8], [9].

Lemma A2. *Let us consider time-varying linear dynamical system*

$$\dot{\mathbf{p}} = -\Gamma \mathbf{R}(t)\mathbf{R}(t)^T \mathbf{p} + \mathbf{b}(t),\ t_0 \ge 0, \qquad \text{(A2)}$$

where $\mathbf{p} \in R^{l_1}$, Γ *is a positive definite matrix of dimension* $l_1 \times l_1$ *and functions* $\mathbf{R}: R_+ \to R^{l_1 \times l_2}$, $\mathbf{b}: R_+ \to R^{l_1}$ *are Lebesgue measurable,* $\mathbf{b}$ *is essentially bounded, function* $\mathbf{R}$ *is* (L, ϑ)*–PE for some* $L > 0$*,* $\vartheta > 0$. *Then for any initial condition* $\mathbf{p}(t_0) \in R^{l_1}$ *solution of system (A2) is defined for all* $t \ge t_0$ *and it admits estimate*

$$|\mathbf{p}(t)| \le |\mathbf{p}(t_0)|\, e^{-0.5\gamma\vartheta L^{-1}(t - t_0 - L)} +$$
$$+ (1 + 2\vartheta^{-1}\gamma^{-1} e^{-0.5\vartheta\gamma L^{-1}(L + t_0)}) L \|\mathbf{b}\|. \qquad ■$$

Appendix 2

Let recurrent inequalities be given:

$$|\beta_j \mathbf{v}_j^T \boldsymbol{\tau} + \alpha_j| \le \delta_j,\ j = 0,1,2,\ldots, \qquad \text{(A3)}$$

where β_j, α_j and δ_j are real numbers, $\mathbf{v}_j$ and $\boldsymbol{\tau}$ are vectors. It is required basing on values $\mathbf{v}_j$, α_j, δ_j, $\eta_j = \beta_j \mathbf{v}_j^T \boldsymbol{\tau} + \alpha_j$, $\nu_j = sign(\delta_j - |\eta_j|)$ and on sign of β_j to design an algorithm of vectors $\boldsymbol{\tau}_0$, $\boldsymbol{\tau}_1,\ldots$, $\boldsymbol{\tau}_j$ adjusting, providing solution of recurrent inequalities (A3). Here

$$sign(\xi) = \begin{cases} 1, & \text{if } \xi \geq 0; \\ -1, & \text{if } \xi < 0. \end{cases}$$

For solution of this problem let us use the result of Theorem 2.1.2 from book [6].

Theorem A1. *Let us assume, that*

a) *there exist numbers* $C_\beta > 0$, $\chi > 0$ *such, that*

$$0 < |\beta_j| \leq C_\beta,\ \delta_j \geq \chi |\mathbf{v}_j|;$$

b) *there exist vector* $\boldsymbol{\tau}^*$ *and number* ϕ ($0 \leq \phi < 1$) *such, that for* $\boldsymbol{\tau} = \boldsymbol{\tau}^*$ *inequalities (A3) are satisfied "with reserve"*:

$$|\beta_j \mathbf{v}_j^T \boldsymbol{\tau}^* + \alpha_j| \leq \phi\, \delta_j;$$

c) *there exist numbers* μ', μ_j, μ'' *providing*:

$$0 < \mu' \leq \mu_j \leq \mu'' < 2(1-\phi) C_\beta^{-1}.$$

Then for any real $\boldsymbol{\tau}_0$ *the following algorithm provides a solution of recurrent inequalities (A3)*:

$$\boldsymbol{\tau}_{j+1} = \begin{cases} \boldsymbol{\tau}_j, & \text{if } \nu_j = 1; \\ \boldsymbol{\tau}_j - \mu_j \eta_j\, sign(\beta_j) |\mathbf{v}_j|^{-2} \mathbf{v}_j, & \text{if } \nu_j = -1. \end{cases} \quad \text{(A4)}$$

For the number of algorithm errors r^0 *the estimate is satisfied:*

$$r^0 \leq |\boldsymbol{\tau}_0 - \boldsymbol{\tau}^*|^2\, C_\beta \{\delta^2 \mu' [2(1-\phi) - C_\beta \mu'']\}^{-1}.$$

Algorithm (A4) has Lyapunov function $V(\boldsymbol{\tau}) = |\boldsymbol{\tau} - \boldsymbol{\tau}^*|^2$. □

Algorithm (A4) was called "Strap-2" in book [6].

APPENDIX 3

Proof of lemma 1. According to conditions of the lemma $\|\mathbf{x}\| < +\infty$, where $\mathbf{x} = [x_1, x_2, x_3, x_4]$ and, hence, $\|\dot{\mathbf{x}}\| < +\infty$. Then all conditions of Lemma 1 from [4] are satisfied and the error of variable x_2 estimation for filter (4) $\iota = x_2 - \hat{x}_2$ is a bounded function of time and the estimate from Lemma 1 from [4] holds. For any $K_1 > 0$, $K_2 > 0$ eigen-values of matrix $\mathbf{G}$ have strictly negative real parts. Vector signal $\mathbf{r}(t)$ is bounded for almost all $t \geq 0$ by construction. Therefore, variable $\boldsymbol{\delta}(t)$ is bounded. Consider the Lyapunov function:

$$V = \boldsymbol{\delta}^T \mathbf{P} \boldsymbol{\delta},$$

$$\mathbf{P} = \frac{K_1 K_2}{2} \begin{bmatrix} K_1^{-1}(K_2+1) & -1 \\ -1 & (K_1 K_2)^{-1}(K_1^2 + K_2 + 1) \end{bmatrix},$$

where λ_1, λ_2 are eigen-values of matrix $\mathbf{P}$, $\vartheta_{\max}$ is the maximum singular value of matrix $\mathbf{P}$. The time derivative for function V have form

$$\dot{V} = -K_1 K_2 |\boldsymbol{\delta}|^2 + 2\boldsymbol{\delta}^T \mathbf{P}\mathbf{r} \leq -K_1 K_2 |\boldsymbol{\delta}|^2 + 2\vartheta_{\max} |\boldsymbol{\delta}||\mathbf{r}| \leq$$
$$\leq -0.5 K_1 K_2 |\boldsymbol{\delta}|^2 + 8\vartheta_{\max}^2 (K_1 K_2)^{-1} |\mathbf{r}|^2,$$

that implies the following asymptotical estimate for the system (9):

$$\lim_{t \to +\infty} |\boldsymbol{\delta}(t)| \leq 8\lambda_1^{-1} \lambda_2\, \vartheta_{\max} (K_1 K_2)^{-1} \|\mathbf{r}\|,$$

$$\|\mathbf{r}\| \leq (K_1 + c/m) \|\phi_1\| + c/m \|\phi_2\| +$$
$$+ 2\sqrt{2}\beta_1/m(\|\dot{\phi}_1\| + \rho^{-1}\|\dot{x}_2\|) + \|d_1\|.$$

Matrix $\mathbf{G}$ also defines dynamics properties for systems (5), (6) and (8), inputs in these systems are bounded for almost all time instants. Thus, variables $\mathbf{z}(t)$, $\boldsymbol{\Omega}(t)$ and $\mathbf{e}(t)$ are bounded too and for $t \to +\infty$ they admit the same estimates.

Let $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}} = [\tilde{\theta}_1\ \tilde{\theta}_2\ \tilde{\theta}_3]$ is unknown parameters identification error vector:

$$\dot{\tilde{\theta}}_i = \gamma \Omega_{1i} (\delta_1 - \Omega_{1i} \tilde{\theta}_i + \phi_1),\ i = 1,2,3.$$

Let us consider for $\tilde{\theta}_i$ the Lyapunov function:

$$W_i = \tilde{\theta}_i^2,\ i = 1,2,3,$$

$$\dot{W}_i = -2\gamma \Omega_{1i}^2 \tilde{\theta}_i^2 + 2\gamma \tilde{\theta}_i \Omega_{1i} (\delta_1 + \phi_1) \leq -\gamma \Omega_{1i}^2 W_i + \gamma |\delta_1 + \phi_1|^2.$$

Expressions for $\dot{W}_i$ coincide with (A2) for $b(t) = \gamma |\delta_1 + \phi_1|^2$ and $\mathbf{R}(t) = \Omega_{1i}$, $\Gamma = \gamma$. According to conditions of the lemma signals $\Omega_{1i}(t)$ are (Δ, r)–PE for some $r > 0$, $\Delta > 0$ for all $i = 1,2,3$. Applying the result of Lemma A2 to the system we obtain estimates for variable $\tilde{\boldsymbol{\theta}}$ behavior:

$$|\tilde{\theta}_i(t)|^2 \leq |\tilde{\theta}_i(0)|^2\, e^{-0.5\gamma r \Delta^{-1}(t-\Delta)} +$$
$$(\gamma + 2r^{-1} e^{-0.5 r \gamma}) \Delta \|\delta_1 + \phi_1\|^2, i = 1,2,3.$$

Substituting in this expression the substantiated before asymptotical estimate for variable $\boldsymbol{\delta}$ we prove the result.

Proof of lemma 2. Let us show, that algorithm (11) is a variant of algorithm (A6) and all conditions of Theorem A1 are satisfied. In this problem $\mathbf{v}_j = 1$, $\delta_j = \delta$ for all $j \geq 0$ and $\alpha_j = w_j - a$. For $\phi = \delta$ from (10) condition (b) of Theorem A1 holds. Expression (3) implies, that:

$$0 < |\beta_j| \leq C_\beta,\ C_\beta > 0.$$

Then condition (a) of Theorem A1 is satisfied for $0 < \chi < \delta$. In such case there exists $\mu > 0$ such, that condition (c) of Theorem A1 holds and algorithm (A6) can be rewritten in form (11). Additional modification of (11) dealing with boundedness of amplitude ε by value E does not change applicability conditions of Theorem A1.

Since for $\phi_1(t) = \phi_2(t) = d_1(t) = d_2(t) = 0$, $t \geq 0$ condition (10) is satisfied for arbitrary $0 < \delta < 1$, then all conditions of Lemma 2 hold. Constant E can be derived from bounds on admissible values of vector $\boldsymbol{\theta}$.

Proof of theorem 1. According to (11) for all $j \ge 0$ property $|\varepsilon_j| \le \mathrm{E}$ holds. Then from (12) $\|u\| < +\infty$ and, hence, $\|\mathbf{x}\| < +\infty$, $\|\dot{x}_2\| < +\infty$. All conditions of Lemma 1 are satisfied and solutions of the system (4)–(7) are bounded.

Let $\phi_1(t) = \phi_2(t) = d_1(t) = d_2(t) = 0$, $t \ge 0$. Then the estimate from Lemma 1 holds:

$$\lim_{t\to+\infty} |\boldsymbol{\theta} - \widehat{\boldsymbol{\theta}}(t)| \le \sqrt{(\gamma + 2r^{-1}e^{-0.5r\gamma})\Delta} \times$$

$$\times 16\lambda_1^{-1}\lambda_2\,\vartheta_{\max}(K_1K_2)^{-1} c/m\,\rho^{-1}\|\dot{x}_2\|,$$

where $\|\dot{x}_2\| \le X = X(\mathbf{x}(0), \boldsymbol{\theta}, \mathrm{E})$. Therefore, for any $\varepsilon_\theta > 0$, $\varepsilon_x > 0$, $K_1 > 0$, $K_2 > 0$ there exist $\rho > 0$, $\gamma > 0$ such, that for some $T_\varepsilon > 0$ it holds that $|\boldsymbol{\theta} - \widehat{\boldsymbol{\theta}}(t)| \le \varepsilon_\theta$ for all $t \ge T_\varepsilon$. For the control (12) the last conclusion means, that for any $\varepsilon_\omega > 0$, $\varepsilon_x > 0$, $K_1 > 0$, $K_2 > 0$ there exist $\rho > 0$, $\gamma > 0$ such, that there exists $T_\omega > 0$ providing $|e_\omega(t)| \le \varepsilon_\omega$ for all $t \ge T_\omega$, where $e_\omega(t) = \omega^* - \widehat{\omega}^*(t)$. For $t \ge T_\omega$ control (12) can be rewritten as follows:

$$u(t) = \varepsilon_j \sin(\omega^* t + e_\omega(t)t) = \varepsilon_j \sin(\omega^* t) + e_u(t),$$

$$\|e_u\| < \mathrm{E}\varepsilon_\omega.$$

Due to continuity of the system solutions with respect to initial conditions and parameters, there exists $\varepsilon_\omega > 0$ such, that for some constants $0 < \delta < 1$ and $|\varepsilon^*| \le \mathrm{E}$ for all $j = 0,1,2,...$ recurrent inequalities (10) are valid. Thus, all conditions of Lemma 2 are satisfied and the result of the theorem follows from Lemma 2.

Proof of corollary 1. From boundedness of discrete time variable ε_j for all $j \ge 0$ the properties $\|u\| < +\infty$ and $\|\mathbf{x}\| < +\infty$ hold. All conditions of Lemma 1 are satisfied and solutions of the system (5)–(7) are bounded.

Let $\phi_1(t) = \phi_2(t) = \phi_3(t) = d_1(t) = d_2(t) = 0$, $t \ge 0$. Then according to the result of Lemma 1:

$$\lim_{t\to+\infty} \widehat{\boldsymbol{\theta}}(t) = \boldsymbol{\theta}.$$

Indeed, in this case the system (4) is redundant and term $\|\dot{\phi}_1\| + \rho^{-1}\|\dot{x}_2\|$ should be replaced with $\|\phi_3\|$. From (12) this conclusion means that $e_\omega(t) \to 0$ for $t \to +\infty$, where $e_\omega(t) = \omega^* - \widehat{\omega}^*(t)$. On ω-limit trajectories of the system the control (12) has form $u(t) = \varepsilon_j \sin(\omega^* t)$. For $\phi_1(t) = \phi_2(t) = \phi_3(t) = d_1(t) = d_2(t) = 0$ all conditions of Lemma 2 hold asymptotically with $\omega = \omega^*$ and $\varphi = 0$, that was necessary to prove.

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