

PARAMETRIC STABILITY OF TWO INDUCTION MOTOR MODELS WITH UNSYMMETRICAL CONNECTION TO THE SUPPLY-LINE.

Abstract: The parametric stability regions of two induction machines mathematical models with the unsymmetrical connection to the supply-line were compared in the paper. The first model was formed by only Ohm's and Kirchhoff's laws use. The second one was formed through the transformations, which are used usually in the classical theory of the electrical machines. As the research the parameter combinations, at which the model stability can not be treated one-to-one, are revealed. The algorithm of Lyapunov's high-order characteristic index calculation is used for the formation of the induction machine model parametrical stability domains.

I. INTRODUCTION

Operating of electrical machines is connected with appearance of unsymmetrical regimes (UR) in their dynamics. Such regimes can appear as a result of either emergencies for example the phase disconnect, short circuit, etc. or operating modes of some converters of adjustable electric drive UR.

It is necessary to make a series of the equivalent transformations [1-3] to form the mathematical model regarding the particular UR.

But this equivalent transformation usage can change the parametric stability regions of the formed model, for example [4].

In particular, the absence of the parametric stability analysis while control system designing caused to the several major accidents.

With the purpose of emergency prevention, these transformation verification for the model of 3-phase IM for the regime of 2-

phase connection to the supply-line with infinite power is presented in the paper.

Thereupon the parametric stability regions of two models are compared.

The first model is formed through three sequential transformations. This model is conventional (section 2).

The second one is formed by only Kirchhoff's and Ohm law use. ally, without mentioned above transformations (section 3).

The parametric stability regions of both models were determined in accordance with the algorithm [6] of the Lyapunov's high-order characteristic index calculation method (section 4).

II. PROBLEM STATEMENT

In the most cases [1-3] the following transformation sequence (Fig. 1) is used while forming the IM model for unsymmetrical regime:

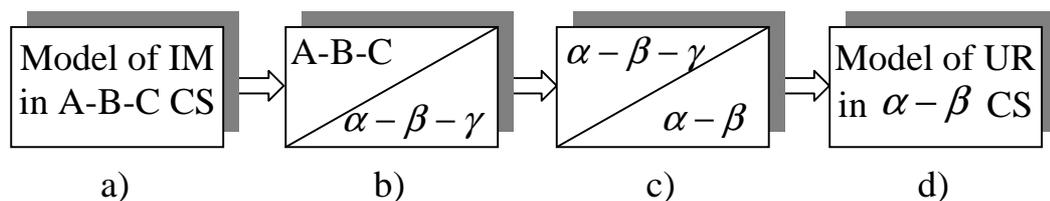


Figure 1. – The mainly used transformation sequence while making the model of IM.

- a) Formation of three-phase IM model in ABC coordinate system on the basis of Kirchhoff's and Ohm laws.
- b) Transformation of the engine rotor windings equations from rotating A-B-C to fixed $\alpha-\beta-\gamma$ coordinate system relatively to the stator by rotation EMF including in rotor circuits.
- c) Formation of the IM two-phase model generalized in the fixed $\alpha-\beta-\gamma$ coordinate system relatively to the stator.
- d) Transformation of the generalized model by including of the additional conditions characterized the particular asymmetry kind.

On the basis of the conclusions presented in [4,5] one can classify all mathematical transformations into two groups:

- 1 The equivalent transformations.
- 2 The transformations that equal in expanded sense.

The first group transformations from the first group are mathematically equivalent, but they can change the model parametric stability regions.

So, emergences can appear within working parameter variation regions, than was shown in [4].

The second group transformations don't change both placement and size of parametric stability regions.

Thereby, it is recommended to verify each transformation for equivalence in expanded sense.

III. MATHEMATICAL MODEL OF IM WITH THE UNSYMMETRICAL CONNECTION TO THE SUPPLY-LINE

Let us form IM mathematical model by of Kirchhoff's and Ohm law use.

The equation system describes the voltage balance in the stator and rotor windings of the three-phase IM with the squirrel-cage rotor has the following matrix form [7]:

$$\mathbf{u}_{ABC}^s = R_s \mathbf{i}_{ABC}^s + \frac{d\mathbf{\Psi}_{ABC}^s}{dt}, \quad (1)$$

$$0 = R_r \mathbf{i}_{abc}^r + \frac{d\mathbf{\Psi}_{abc}^r}{dt} \quad (2)$$

where $\mathbf{u}_{ABC}^s, \mathbf{i}_{ABC}^s (\mathbf{i}_{abc}^r)$ – 3×3 vectors of stator voltages and stator (rotor) currents accordingly.

$R_s (R_r)$ – resistance of stator (rotor) windings. It is assumed that the stator and rotor resistances are distributed symmetrically.

$\mathbf{\Psi}_{ABC}^s (\mathbf{\Psi}_{abc}^r)$ – 3×3 vectors of the stator (rotor) flux-linkages presented in according with [7].

For example, the stator flux-linkage equation for “A” winding has the following view:

$$\begin{aligned} \Psi_{sA} = & i_{sA} (L_{sA,l} + M_{AA}) + i_{sB} M_{AB} \cos\left(\frac{2\pi}{3}\right) + \\ & i_{sC} M_{AC} \cos\left(-\frac{2\pi}{3}\right) + i_{ra} M_{Aa} \cos(\theta) + \\ & + i_{rb} M_{Ab} \cos\left(\theta + \frac{2\pi}{3}\right) + i_{rc} M_{Ac} \cos\left(\theta - \frac{2\pi}{3}\right), \end{aligned} \quad (3)$$

where $i_{sA}, i_{sB}, i_{sC} (i_{ra}, i_{rb}, i_{rc})$ – the stator (rotor) current instantaneous values in the corresponding windings “A”(“a”), “B”(“b”) and “C”(“c”);

$L_{sA,l}$ – leakage inductance of the stator winding “A”;

θ – solid angle between the stator and rotor windingse.

$M_{AA}, M_{AB}, M_{AC}, M_{Aa}, M_{Ab}, M_{Ac}$ – mutual inductances between the stator winding “A” and other stator and rotor windings.

At that, the maximum value (M) of these mutual inductances is accepted as a value for each of them for the IM ideal model.

Let us describe forming of an unsymmetrical regime equation system by the example of BC-regime when the stator A-phase is open (Fig. 2).

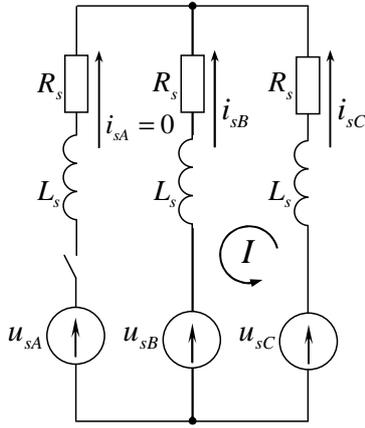


Figure 2. – Scheme of stator circuits of three-phase IM in unsymmetrical regime “BC”.

In this case: L_s – a total stator winding inductance taking into account that different stator winding inductances are accepted equal to $L_s = L_{sA,l} + M$ for symmetrical ideal engine.

Due to the Kirchhoff's two laws (in the anticlockwise path-tracing «I»):

$$i_{sA}(t) = 0; \quad (4)$$

$$i_{sB}(t) = -i_{sC}(t); \quad (5)$$

$$u_{sC} - u_{sB} = R_s i_{sC}(t) + \frac{d\psi_{sC}(t)}{dt} - R_s i_{sB}(t) - \frac{d\psi_{sB}(t)}{dt} \quad (6)$$

By equation (5) substituting into the equation (6):

$$u_{sC} - u_{sB} = 2R_s i_{sC}(t) + \frac{d\psi_{sC}(t)}{dt} - \frac{d\psi_{sB}(t)}{dt} \quad (7)$$

Then it is necessary to add the equation (7) by three equations of the rotor winding voltage balance and the equation of the moment balance on the engine shaft [7] building of the “BC”-regime complete equation system.

These equations are not changed and are coincide with symmetrical regime equations (2).

However, this approach to IM modeling has not used for a long time in cause of the great labour while final equation system

reduction to Cauchy normal form. Onrush development of the modern symbolic mathematic applied systems leads to possibility such reduction realizing.

With this purpose “Maple 10.0”[®] (Waterloo Maple Inc.) was used in the paper.

IV. PARAMETRIC STABILITY ANALYSIS

The algorithm of Lyapunov's high-order characteristic index calculation [6] is used for the formation of the IM model parametrical stability domains. Thus it is assumed that if this index is positive then the running mode is unstable. The results of two model stability identification in the $E_s f$ -parametric space are presented in Fig. 3a. In this case E_s is supply voltage amplitude, f is a supply voltage frequency.

The grey circles were used for the UR model formed at the section 3, the black circles were used for the UR model obtained by the transformations which are used usually in the classical electrical machines theory.

Equivalent circuit parameters correspond to IM 4A225M4Y3 (rating power $P = 55$ kW).

The parameters (E_s and f) are varied from 10% till 100% regarding their nameplate values.

The rest parameters of the IM equivalent circuit (R_s , R_r , L_s , L_r , M) are varied from 95% till 105% in relation to their nameplate values.

The function *ode45* of the *MATLAB*-kernel was used for the numerical integration of the models under the zero-value initial conditions and local accuracy till $1 \cdot 10^{-4}$.

The $E_s f$ -parameter values, at which the states (stability or instability) are differ for the considered models are shown in Fig. 3b.

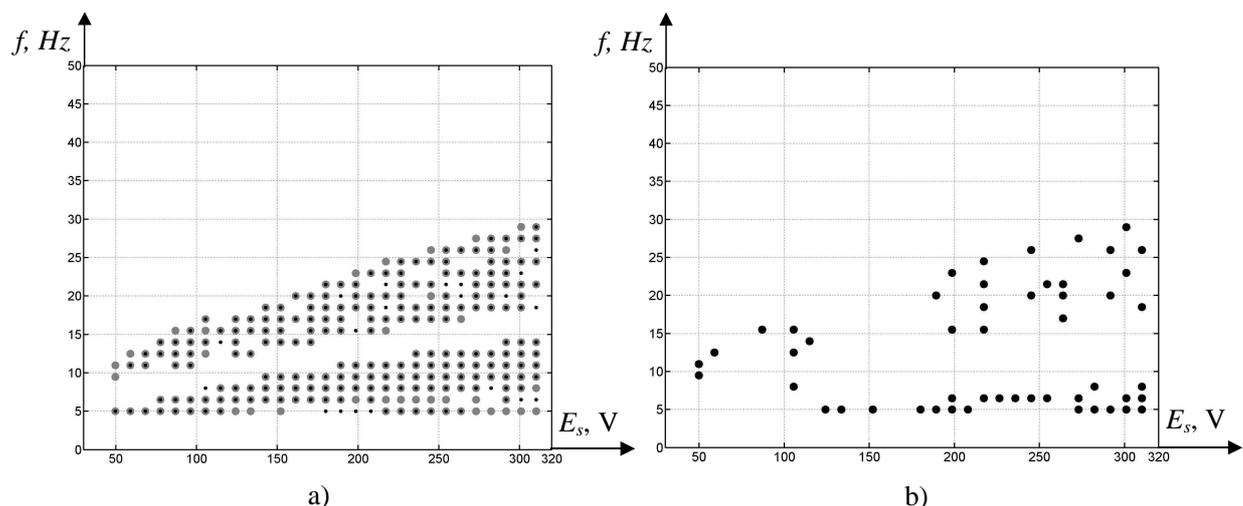


Figure 3. – Diagrams of parametric stability of investigated models.

The diagrams in Fig. 3 allow illustrating the following:

1. The shape of the both models instability regions in the unsymmetrical “BC”-regime looks like the right triangle with the cathetus $E_s \in [30;310]$ and $f \in [5;30]$.
2. There are the parameter combinations at which the model stability can not be treated one-to-one.

V. CONCLUSION

The parametric stability regions of two IM (induction machine) mathematical models with the unsymmetrical connection to the supply-line were compared in the paper. The first model was formed by only Ohm's and Kirchhoff's laws use. The second one was formed through the transformations, which are used usually in the classical theory of the electrical machines.

The comparison results allow illustrating the difference between these model regions.

Therefore, it is necessary to take into account while IM control system designing the uncertainty, which exists in the parametric stability region boundaries.

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