OPTIMIZATION METHODS FOR SPUR GEAR DYNAMICS

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Abstract
In the present paper two different approaches for spur gear noise reduction using micro-geometrical modifications are compared. The two approaches are based respectively on the reduction of Static Transmission Error (STE) and Dynamic Transmission Error (DTE) fluctuations. The dynamic behaviour of the system is computed through a simple lumped parameter model. A genetic algorithm is proposed to find the best solutions inside the parameters space because the evaluation of the objective functions requires finite elements calculations and numerical ODE integrations. A reliability analysis is afterwards performed to evaluate the effect of manufacturing errors on the dynamic performance of the achieved optimum.

Key words
Optimization, gears, NVH, genetic algorithms, manufacturing errors.

1 Introduction

The reduction of vibration and noise is one of the main issue in spur gear design. Literature offers many approaches to evaluate the dynamic behaviour of such systems and their design optimization. Most of the design methods are based on the use of tip an root relief, which consist in a removal of material from the tooth flank in order to reduce the fluctuation of the static transmission error (STE) at a specific torque level. Kahraman and Blankenship [1] proved the influence of profile modifications on the dynamic response with experiments.

In the last century, optimization approaches were mainly based on simplified analytical model. For example, rigid models of gear teeth were used to evaluate the static transmission error, assuming that low excursions of the STE correspond to low vibrations, i.e. low fluctuations of the dynamic transmission error (DTE) [2]. Gear tooth deformations were also taken into account using a simplified cantilever beam model [3]. Beghini et al. [4] proposed an iterative method to reduce the peak to peak of the STE based on FEM models. In their paper, a sequential approach, spanning couples of profile modification parameters, is described. Despite the accuracy in stiffness evaluation, Finite Element Methods do not allow to bring out a search in the parameter space using common optimization techniques, e.g. steepest descent, because the objective function is not known in an explicit formulation. It is to note that all approaches found in literature are not global in nature, because only one load value is considered, while, due to nonlinearities, both STE and DTE depend on the applied torque.

The relationship between static and dynamic transmission error can be investigated using lumped parameter models, which permits to find the dynamic behaviour of the gear pair for varying speed starting from a series of static FEM calculations. On this topic, previous works from Parker et al. [5] were considered to develop a tool for studying the dynamic effectiveness of profile modifications. In this paper STE is interpolated by means of Discrete Fourier Transform (DFT) starting from fifteen reciprocal positions within a mesh cycle.

This work focuses on the comparison between two different objective functions in gear profile optimization: the first one takes into account the harmonic content of the STE, the second considers mean amplitudes of vibration of the system. Both approaches perform simulations at three different torque levels, so that the minima, found by the optimization processes, keep their effectiveness despite the applied load condition.

Since both objective functions can be estimated only for a finite number of configurations, a genetic algorithm is developed and applied to both static and dynamic optimization. The effectiveness of the optimum is afterwards checked by means of the method proposed in [10]: a random profile is created starting from the optimal one and the dynamic behaviour of the new gear pair is compared to the unperturbed case. It is to stress that usual tolerances in the profile manufacturing are larger than the magnitudes of profile modifications, for this reason such a comparison is quite important to understand whether or not profile modifications are suitable for technological application.

2 Dynamic model

The dynamic model used in this paper is the simple lumped mass model showed in Fig. 1. Such model considers spur gears as rigid disks, coupled along the line of action through a time varying mesh stiffness $k(t)$ and a constant mesh damping c; $\theta_{g1}(t)$ is the angular position of the driver wheel (pinion), $\theta_{g2}(t)$ is the angular position of the driven wheel (gear); $T_{g1}(t)$ is the driving torque, $T_{g2}(t)$ is the braking torque; I_{g1} and I_{g2} are the rotary inertias; d_{g1} and d_{g2} are the base diameters.

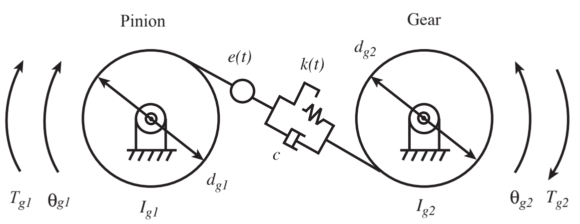

Figure 1. Dynamic model of a spur gear pair

According to literature [8], the relative dynamics of gears along the line of action can be represented by the following equation of motion:

$$m_e\ddot{x}(t)+c(\dot{x}(t))+k(t)f_1(x(t))+k_{bs}(t)f_2(x(t)) = T_g(t) \tag{1}$$

where $\dot{(\cdot)} = \frac{d(\cdot)}{dt}$, m_e is the equivalent mass:

$$m_e = \left(\frac{d_{g1}^2}{4I_{g1}} + \frac{d_{g2}^2}{4I_{g2}}\right)^{-1} \tag{2}$$

$T_g(t)$ is the equivalent applied preload:

$$T_g(t) = m_e\left(\frac{d_{g1}T_{g1}(t)}{2I_{g1}} + \frac{d_{g2}T_{g2}(t)}{2I_{g2}}\right) \tag{3}$$

$T_{g2}(t) = T_{g1}(t)\frac{d_{g2}}{d_{g1}}$ and $T_{g1}(t) = T_{g1}$ are assumed to be constant.
The dynamic transmission error $x(t)$ along the line of action is defined as:

$$x(t) = \frac{d_{g1}}{2}\theta_{g1}(t) - \frac{d_{g2}}{2}\theta_{g2}(t) \tag{4}$$

$k_{bs}(t)$ is the back side contact stiffness.
Smoothing backlash functions are considered in order to simulate clearances:

$$\begin{aligned} f_1(t) &= \tfrac{1}{2}\left[(x(t)-b)\left\{1+\tanh[\lambda(x(t)-b)]\right\}\right] \\ f_2(t) &= \tfrac{1}{2}\left[(x(t)+b)\left\{1+\tanh[-\lambda(x(t)+b)]\right\}\right] \end{aligned} \tag{5}$$

where $2\,b$ is the backlash along the line of action and λ is the shape parameter ($\lambda = 10^8$). The gear pair mesh stiffness along the line of action is given by:

$$k(t) = \frac{2T_{g1}}{d_{g1}STE(t)} = \frac{4T_{g1}}{d_{g1}^2\delta(t)} \tag{6}$$

where

$$\delta(t) = \theta_1(t) - \frac{d_{g2}}{d_{g1}}\theta_2(t) \quad STE(t) = \frac{d_{g1}\delta(t)}{2} \tag{7}$$

$\delta(t)$ is the difference between the nominal position of the wheel 1 (pinion) given by the exact kinematics and the actual position influenced by the teeth flexibility; $STE(t)$ is the static transmission error along the line of action, it depends on time because, during meshing, both the contact point and the number of teeth in contact can change.
Parker et al. [5] described a methodology to compute $\theta_1(t)$ and $\theta_2(t)$, referred as the rotational degrees of freedom of the pinion and the gear, for small "rigid-body" motions; this approach is followed here for static analyses.
Since no manufacturing errors are included, the mesh stiffness is periodic within a mesh cycle; therefore, it is expanded in terms of Fourier series:

$$k(t) = k_0 + \sum_{j=1}^{N} k_j\cos(j\omega_m t - \varphi_j) \tag{8}$$

where ω_m is the mesh circular frequency, amplitudes k_j and phases φ_j are obtained using the Discrete Fourier Transform (DFT); the number of samples n is related to the number of harmonics $N = (n-1)/2$; in the following, $n = 15$ is considered to ensure enough accuracy in the expansion.
Similarly we have:

$$k_{bs}(t) = k_0 + \sum_{j=1}^{N} k_j\cos(-(j\omega_m t - \varphi_j + j\frac{s_{ts,1}}{d_{g1}})) \tag{9}$$

where $s_{ts,1}$ is the thickness of the pinion tooth space at the pitch operating diameter, see equation (3.2.32) of Ref. [9] for details.
A dimensionless form of equation is obtained by letting:

$$\omega_n = \sqrt{\frac{k_0}{m_e}}\ ;\ \ \zeta = \frac{c}{2m_e\omega_n}\ ;\ \ \tau = \omega_n t\ ;$$
$$\bar{T}_g = \frac{T_g}{bm_e\omega_n^2}\ ;\ \ \bar{x} = \frac{x}{b} \tag{10}$$

and:

$$\bar{k}_j = \frac{k_j}{m_e\omega_n^2}\ ; \qquad \bar{k}(\tau) = 1 + \sum_{j=1}^{N} \bar{k}_j \cos\left(j\frac{\omega_m}{\omega_n}\tau - \varphi_j\right);$$
$$\bar{k}_{bs}(\tau) = 1 + \sum_{j=1}^{N} \bar{k}_j \cos\left(-j\frac{\omega_m}{\omega_n}\tau + \varphi_j - j\frac{s_{ts,1}}{d_{g1}}\right) \tag{11}$$

Dynamic transmission error is computed using an implicit Runge-Kutta numerical integrator.

3 Profile modifications

Profile modifications are micro-geometrical removals of material both from the tip and from the root of the tooth. The parameters that define these profile modifications are the roll angle of start and magnitude relief at the tip, the roll angles of start and magnitudes of relief at the root; this way the parameter space is 8-dimensional. Fig 2 shows the parameters defining tooth profile modifications. The type of the modifications can be linear or parabolic with respect to the roll angle. Since parabolic modifications give worst results in optimization [6], linear modifications are considered. Ranges spanned by each parameter are reported in Tab. 1.

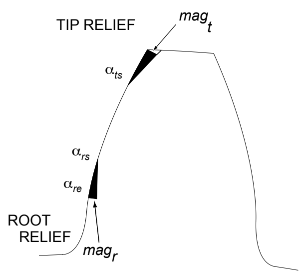

Figure 2. Representation of profile modification parameters

4 Optimization

Genetic algorithms are optimization methods that start from an initial population of points and improve it through the iterative application of three transformations: selection, crossover and mutation. Although discrete in nature, since genetic algorithms work on bit strings, it is possible to use them to find minima of multivariable functions through a discretization of the domain. The three transformations resemble the evolution of biological systems. At the first stage, selection, the objective function is evaluated for each member of the actual population and a new set of points is extracted from the actual one by means of a stochastical sampling in which better elements have higher probability of extraction. Crossover, which acts like chromosome recombination, combines the extracted solutions in order to search different areas of the parameter space. Since some regions of the space could not be reached by crossover, mutation adds a random variation of points inside the population.
An optimization strategy based on genetic algorithms is proposed and applied to an actual gear pair. Tab. 2 shows data of the case study.

Parameter	Start	End
α_{ts}	Roll angle at operating pitch diameter	Roll angle at tip diameter
mag_t	0	40 μm
α_{rs}	Roll angle at operating pitch diameter	Roll angle at initial point of contact diameter
mag_r	0	40 μm

Table 1. Parameter ranges for the optimization

Data	Pinion	Gear
Number of teeth	28	43
Module $[mm]$	3	3
Pressure angle $[Deg]$	20	20
Base radius $[mm]$	39.467	60.610
Theoretical pitch radius $[mm]$	42	64.5
Thickness on theoretical pitch circle $[mm]$	6.1151	6.7128
Addendum modification $[mm]$	1.927	2.748
Face width $[mm]$	20	20
Hob tip radius $[mm]$	0.9	0.9
Outer diameter $[mm]$	93.1	139.7
Root diameter $[mm]$	79.1	126.2
Inner diameter $[mm]$	40	40
Mass $[kg]$	0.71681	1.9823
Inertia $[kg\, m^2]$	0.0008076	0.0047762
Young's modulus $[MPa]$	206000	206000
Poisson's coefficient	0.3	0.3
Center distance $[mm]$	111	
Backlash $[mm]$	0.3461	
Backlash ($2b$) on the line of action $[mm]$	0.312	
Backside stiffness phase $[rad]$	1.594232	
Transmission ratio	0.6511	
Contact ratio	1.28565	
Torque (T_{g1}) $[Nmm]$	470000	
Damping coefficient (ζ)	0.01	

Table 2. Geometrical data for the case study (courtesy of CNH Case New Holland)

4.1 Effectiveness of optimization

At a first stage, the proposed algorithm is used to find an optimum set of profile modifications for a specific torque value, i.e. the nominal torque. Two different objective functions, peak to peak of STE (CASE B) and mean value of the first seven harmonics of STE (CASE C) are applied and their dynamic effectiveness is compared to that of a pure involute profile (CASE A). Fig 3 shows the dynamic results: the amplitude-frequency diagram is obtained starting from strongly perturbed initial conditions. This approach, even though produces lower vibration levels, is considered to be more realistic because the operating environment is subjected to strong perturbations. Minimizations at the nominal torque value of peak to peak of STE produces the best results in terms of vibration reduction. In the following, we will consider the harmonic content of STE, instead of the peak to peak, because it is the direct source of excitation and for this reason can be related to the dynamic behaviour.

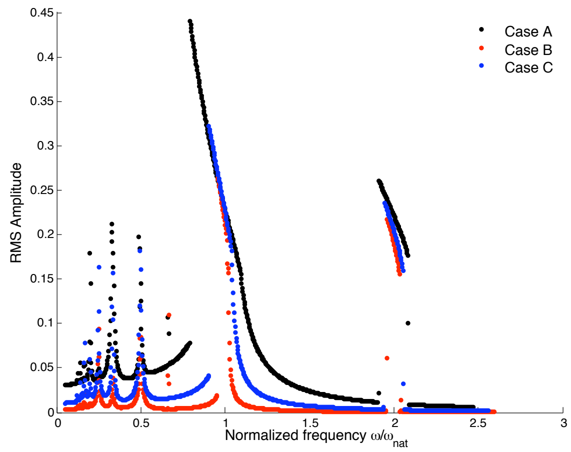

Figure 3. Amplitude-Frequency diagrams: CASE A - no profile modifications; CASE B - peak to peak of STE; CASE C - mean value of the first seven harmonics of STE

4.2 Static optimization

In the present section, the same algorithm has been applied to minimize the mean amplitude of the first seven harmonics of STE at three different torque levels (CASE D): nominal load, 66% and 33% of nominal load. This way, for each point in the parameter space, 3x15 FEM simulations are performed, so that the first seven harmonics of STE could be evaluated by means of DFT. With reference to the algorithm described in [6], the parameters used in the present simulations are collected in Tab. 3:

The optimal set of profile modification, which should be capable to reduce vibrations at different loads, is tested to find out its dynamic behaviour. Figure 4 summarizes the dynamic scenarios obtained for the three different torque values.

Number of strings in the population n_{pop}	50
Crossover probability p_c	1
Mutation rate p_m	0.4
Multiplier for the fitness scaling c_{mult}	1.5
Number of iterations n_{iter}	60

Table 3. Parameters of the genetic optimization

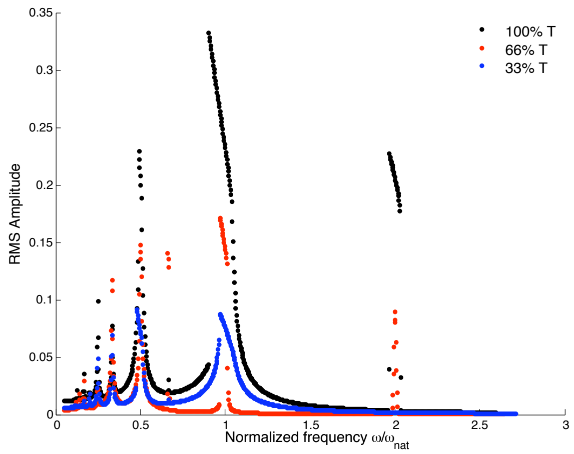

Figure 4. Amplitude frequency diagrams: CASE D - Static optimization

4.3 Dynamic optimization

In this case for each solution three numerical integrations are performed for three different torque values. In this way, the Root Mean Square (RMS) of the solution is evaluated in the range between 500 to 25000 rpm. The target of optimization is the mean value of the RMS for the three different torque levels. Fig 5 shows the dynamic scenario for the dynamically optimized gears. The dynamic optimization approach is applied to the same case study (Case E) and the optimal solution shows the same behaviour of the static one: vibration levels are higher at the maximum torque, lower at 33% of the nominal torque.

Tab 4 presents the sets of optimized profile modifications for CASE D and CASE E: it is to note that both solutions present very small modifications at root. Since root modifications are more difficult to manufacture than tip modifications, this observation suggests that tip only modifications could be considered in designing gear transmissions.

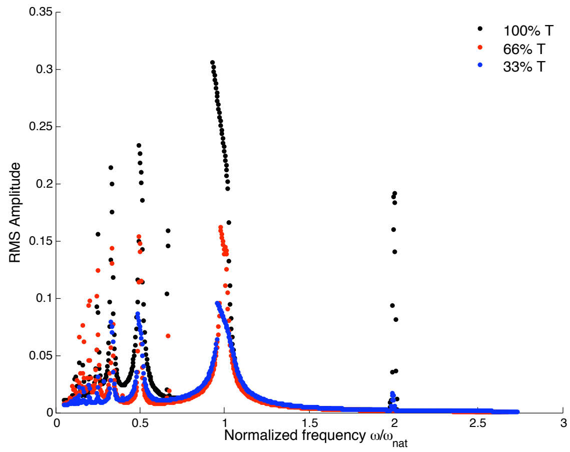

Figure 5. Amplitude frequency diagrams: CASE E - Dynamic optimization

	Case D		Case E	
	Pinion	Gear	Pinion	Gear
Tip Relief				
α_{ts} [deg]	34.488	30.006	32.715	30.886
mag_t [mm]	0.039	0.037	0.022	0.020
Root Relief				
α_{rs} [deg]	23.455	25.168	21.947	22.545
α_{re} [deg]	14.433	20.576	14.433	20.576
mag_r [mm]	0.010	0.000	0.006	0.002

Table 4. Optimal profile modifications according to optimizations: cases D and E

4.4 Reliability analysis

Considering the set of optimal profile modifications proposed in Tab. 4 Case D , it can be seen that the magnitudes lie in the range $0 - 40\mu m$, which is usually the same size of the allowable manufacturing error. To simulate the profile error, a random profile is considered. This profile is defined by means of random normally distributed points with standard deviation set up considering prescribed tolerances. In gear profile, tolerances are defined using a "k-chart", that is a sketch of the domain of allowable profile, see Fig. 6.

In the present case study, since profile modification manufacturing is less accurate than pure involute profile, tolerances are set up as in the following: $\Delta_i = 8\mu m$, $\Delta_r = 20\mu m$. Starting from these values, a random profile is defined; the corresponding transmission error is calculated and represented using the first four harmonic components; which are added to the dynamic model as $e(t)$ in Fig. 1.

Figure 7 shows the dynamic behaviour of the perturbed profile: vibration levels are higher than in Case D, but still lower than pure involute profile (Case A).

5 Conclusion

Two optimization strategies are proposed to find optimal sets of profile modifications at different torque levels: the first is based on static finite element calculations, the second involves a numerical ODE integration as well. The main focus is to reduce vibrations in spur gears. The effectiveness in vibration reduction has been

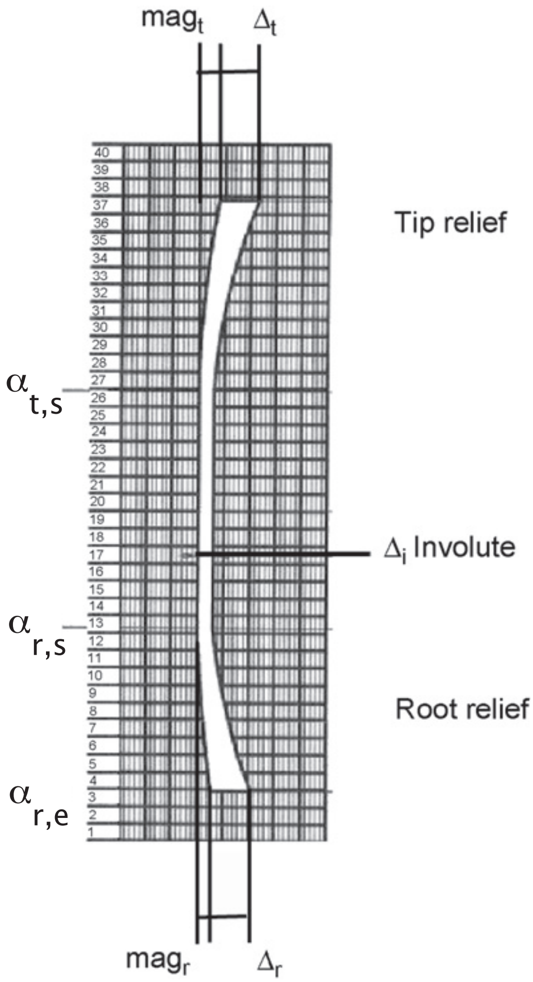

Figure 6. Example of k-chart used to represent tolerances in gear profile

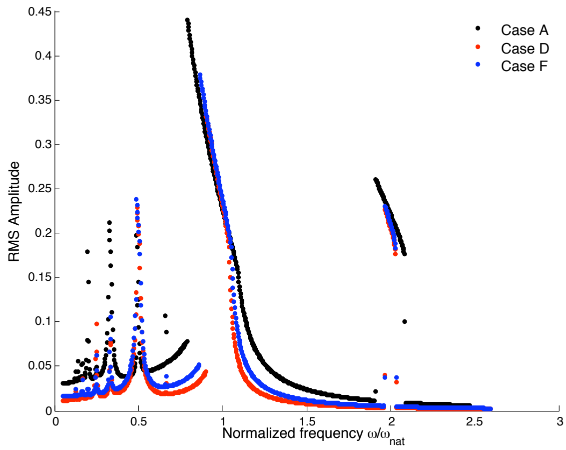

Figure 7. CASE A - no profile modifications; CASE D - Static optimum; CASE F - Effect of manufacturing error on D case

checked for both approaches, showing a good reduction of noise level. The first method gives good results with much lower computational time. For this reason, the optimum obtained on a static basis is assumed as a standard. Since the magnitudes of the modifications are of the same size of the manufacturing error, a perturbed profile is considered in order to prove that vibration level of the optimized profile is still better than pure involute even if profile errors are taken into account.

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