ON PHYSICAL NATURE OF LOW-FREQUENT BEATINGS IN SYSTEMS WITH PULSE-PHASE SYNCHRONIZATION MECHANISM

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Abstract

In this article the method of continuation over inertia parameter is used for studies of appearance nature of low-frequent beatings in the system with pulse-phase synchronization mechanism. It is shown that the use of pulse-phase control on the one hand ensures global stability of control regime and on the other hand makes possible low-frequent beatings appearance. Even the beatings with quasi-continuous spectrum may appear.

Key words

Control system, pulse-phase control, mathematical model, stability, stationary motion, low-frequent beatings.

1 Introduction

Frequency synthesizer is a key component of almost any test end measurement, communication and monitoring system [Chenakin, 2007]. Today frequency synthesizers need regular improvement of technical characteristics, functional possibilities widening, size diminution, power requirement reduction and retuning acceleration [Chenakin, 2007], [Ryabov, Yuriev, Nemcev, 2011], [Izmailova, 2009], [Antonovskaya, Goryunov, 2007]. Moreover the most perspective elaborations in the very near future apparently must be connected with the synthesizers based on phase synchronization use [Chenakin, 2007], for such synthesizers are of lesser size and complexity level and have more pure spectrum of input signal.

The principle of pulse-phase control based on number counting and phase position of control pulses is widely used in electro-mechanics and communications equipment [Levin, Malinovsky and Romanov, 1989]. Besides for the purposes of spectral characteristics improvement for stationary signal, as a rule, inertial filters are used. But also it is of utmost interest the study of processes taking place in that sort of systems over low-inertial filter circuits [Antonovskaya, Goryunov, Palochkin, 2010]. The work [Goryunov, 2003] represents the attempt to obtain some qualitative knowledge of non-local character on dynamic processes peculiarities in synchronization systems using pulse-width modulation of control signal. As the example the scheme of frequency synthesizer (FS) with pulse frequent-phase detector and astatic filter was studied [Levin, Malinovsky and Romanov, 1989]. But in spite of general methodological approach use for processes description on the base of point mappings method, point mappings functions construction in correlation with engineer view did not allow to divide the parameter space into the regions of motions of different complexity. It was the result of great difficulties over analysis.

In current paper the results of qualitative analysis of the system from the work [Goryunov, 2003] are described. The obtaining of qualitative results became possible due to the method of continuation over inertia parameter use [Neimark, 2004] (it was formally approved in [Antonovskaya, Goryunov, 2008] and [Antonovskaya, Goryunov, 2009]) for the systems with broken limit cycle.

2 Mathematical Model of System and Its Qualitative Analysis

The equations of FS mathematical model (MM) with guided coordinate x over arbitrary period of control pulses follow have a form: over switched detector output

$$\begin{gathered} \alpha\dot{\theta} = g(x(\tau)), \\ \mu\dot{x} = u - x, \\ (0 \le \theta \le 1, 0 \le \tau \le 1, u = \pm 1) \end{gathered} \tag{1}$$

and over not switched detector output

$$\begin{gathered}\alpha\dot{\theta} = g(x(\tau)),\\ x(\tau) = x_0.\\ (0 \le \theta \le 1, 0 \le \tau \le 1)\end{gathered} \tag{2}$$

Here dot means differentiation over indifferent time τ changing within the limits of standard generator (SG) signal period, $0 < \mu << 1$ – a-static parameter, α – control parameter of counter (C), u – output detector signal, x – output filter coordinate, θ – C-coordinate ($\theta = 0$ when C is empty and $\theta = 1$ when it is full), $g(x)$ – standardized over 1 ($g(0) = 1$) guided generator (GG) characteristics.

Since the equations (1),(2) corresponds to the system with variable structure with two-positional value of control signal u, it is useful to look at three subspaces for phase trajectories $\theta(\tau), x(\tau)$ study. That is the subspace Π_1 where the system (1) with $u = +1$ is defined, the subspace Π_3 where the system (1) with $u = -1$ is defined and the subspace Π_2 where the system (2) is defined.

As in (1),(2) $\dot{\theta} > 0$, there is no equilibrium state in each subspace. That is why there may exist stationary motions of cyclic type only. Namely stationary motions with representing motion points (RMP) crossing subspaces partly or fully may exist. In the case of ideal a-statism ($\mu = +0$) and the stability in Π_1 and Π_3 area of subspace $x = u = const$ stability the phase trajectory of each cyclic motion for small μ values is located in $O(\sqrt{\mu})$-neighborhood of non-disgraced motion [Andronov, Vitt and Haykin, 1959]. This fact gives the possibility reason for the method of method of continuation over inertia parameter μ.

As the transfer from one subspace to another is defined by the moment of RMP coming to one of the sections $\theta = 1$ or $\tau = 1$ and is simply found from (1),(2), it may be proved that stationary motions are realized by means of RMP consequent passing over the circuit $\Pi_1 \leftrightarrow \Pi_2 \rightarrow \Pi_3 \leftrightarrow \Pi_2 \rightarrow \Pi_1$ and may be studied with the help of point mapping T properties study. Point mapping T is the composition of point mappings T_+ and T_-.

The point mapping T_+ sets up the connection between the initial point (x_0, θ_0) of the section $\tau = 0$ in Π_1-subspace and the following point $(\bar{x}, \bar{\tau})$ of the section $\theta = 0$ in Π_3-subspace and is described by correspondence function of the type:

$$\begin{gathered}\bar{x} = +1,\\ \bar{\tau} = (\alpha/g(1))(m_1 + 2 - \theta_0) - m_1,\end{gathered} \tag{3}$$

where

$$\begin{gathered}0 \le 1 - ((g(1)/\alpha) - 1)(m_1 + 1) \le \theta_0 \le\\ \le 1 - ((g(1)/\alpha) - 1)m_1,\\ m_1 = ceil[(1 - \theta_0)/((g(1)/\alpha) - 1)].\end{gathered} \tag{4}$$

Here the operation "$ceil$" is used, it means carrying over the value to the bigger nearest whole number.

The value m_1 in (3) defines the number of return motions from Π_2 to Π_1 arising in the course of phase recount for the pulses controlling of comparison device state until the situation when RMP in Π_2 comes from the section $\theta = 0$ to $\theta = 1$ and then leave for the section $\theta = 0$ of Π_3-subspace.

The point mapping T_- sets up the connection between the initial point (x_0, τ_0) of the section $\theta = 0$ in Π_3-subspace and the following point $(\bar{x}, \bar{\theta})$ of the section $\tau = 0$ in Π_1-subspace and is described by correspondence function of the type:

$$\begin{gathered}\bar{x} = -1,\\ \bar{\tau} = (g(-1)/\alpha)(m_2 + 2 - \tau_0) - m_2,\end{gathered} \tag{5}$$

where

$$\begin{gathered}0 \le 1 - ((\alpha/g(-1)) - 1)(m_2 + 1) \le \tau_0 \le\\ \le 1 - ((\alpha/g(-1))m_2,\\ m_2 = ceil[(1 - \tau_0)/((\alpha/g(-1)) - 1)].\end{gathered} \tag{6}$$

The value m_2 in (5) defines the number of return motions from Π_2 to Π_3 arising in the course of phase recount for the pulses controlling of comparison device state until the situation when RMP in Π_2 comes from the section $\tau = 0$ to $\tau = 1$ and then leave for the section $\tau = 0$ of Π_1-subspace.

It is necessary to mention, that for $m_1, m_2 \neq 0$ due to return motions the graphs of correspondence functions (3),(5) are non-continuous mappings of straight line to straight line of saw form. Those mappings may be characterized by gap number. That is due to (3),(5) by the values $M_1 = \max_{\theta_0} m_1(\theta_0)$ and $M_2 = \max_{\tau_0} m_1(\tau_0)$. Corresponding values of α are

$$\begin{gathered}\alpha(M_1) = (M_1 + 1)g(+1)/(M_1 + 2),\\ \alpha(M_2) = (M_2 + 2)g(-1)/(M_2 + 1),\\ (M_1, M_2 = 1, 2, ...).\end{gathered} \tag{7}$$

As $d\alpha(M_1)/dM_1 > 0$, the values $\alpha(M_1)$ are situated between the minimal value $\alpha(M_1 = 0) = g(1)/2$ and the maximal value $\alpha(M_1 = \infty) = g(1)$. $d\alpha(M_2)/dM_2 < 0$, so the values $\alpha(M_2)$ are situated between the maximal value $\alpha(M_2 = 0) = 2g(-1)$ and the minimal value $\alpha(M_2 = \infty) = g(-1)$, which is also controllability boundary of system for $\alpha < 1$.

Since $T-$mapping is composed by the mappings T_+ and T_- it may be characterized by a pair of numbers which regulative correlation arises when taking into account the properties of characteristic $g(x)$ of control object.Thus for the characteristics $g(x) = 1 + Sx$, where S is characteristics slope, the boundaries $\alpha = \alpha(M_1)$ and $\alpha = \alpha(M_2)$ for different M_1, M_2 values form two "fans" on the plain α, S. The "fan" for parameter M_1 is unrolled about the point $(0, -1)$ in the limits of straight lines $\alpha = 1 + S$ and $\alpha = (1 + S)/2$. The "fan" for parameter M_2 is unrolled about the point $(0, 1)$ in the limits of straight lines $\alpha = 1 - S$ and $\alpha = 2(1 - S)$.

In each region formed by pointed boundaries correspondence functions (3),(5) graphs are of a concrete type and allow full qualitative study of the mapping T by Lamerey diagram (by correspondence functions graphs representation over one diagram [Andronov, Vitt and Haykin, 1959]).

Taking into account (3),(5) the composition of T_+ and T_- mappings has press coefficient $g(-1)g(+1) < 1$. That is globally stable cycle consisting of fixed points of the mapping T always exists. When $g(x) = 1 + Sx$ the plain α, S is divided into countable number of subregions and in each the cycle of definite multiplicity exists. If correspondence functions (3),(5) graphs are intersected on the sector of continuity then the simple fixed point of the mapping T exists. On the plain α, S it exists everywhere except the regions with the cycles of multiplicity 2 and higher which are given by non-equalities: for parameter m_1

$$\begin{gathered} \underline{\alpha}_p(m_1) \le \alpha \le \bar{\alpha}_p(m_1), \\ \underline{\alpha}_p(m_1) = \frac{(1+m_1)g(1)+g(-1)}{2+m_1}, \\ \bar{\alpha}_p(m_1) = g(1)\frac{(1+m_1)g(1)+2g(-1)}{(2+m_1)g(1)+g(-1)}, \end{gathered} \tag{8}$$

for parameter m_2 ($\alpha \le 1$)

$$\begin{gathered} \underline{\alpha}_p(m_2) \le \alpha \le \bar{\alpha}_p(m_2), \\ \underline{\alpha}_p(m_1) = g(-1)\frac{(1+m_2)g(1)+g(-1)}{(2+m_2)+2g(-1)}, \\ \bar{\alpha}_p(m_1) = \frac{(2+m_2)g(-1)g(+1)}{(1+m_2)g(-1)g(+1)}. \end{gathered} \tag{9}$$

Over intersection of such intervals located on the plain α, S between corresponding boundaries (7) consequently the cycles of high, low (T^2) and once again high multiplicity appear.

Also it is necessary to mention that for $S = 0$ $\underline{\alpha}_p(m_1) = \bar{\alpha}_p(m_1) = \underline{\alpha}_p(m_2) = \bar{\alpha}_p(m_2) = 1$. So on the plain α, S all the existence intervals for the cycles of high multiplicity press to a point $(0, 1)$ when $S \to 0$. When $S = 1$ $\underline{\alpha}_p(m_2) = \bar{\alpha}_p(m_2) = 0$ and $\underline{\alpha}_p(m_1) = \bar{\alpha}_p(m_1) = 2(1 + m_1)/(2 + m_1)$, so for $m = 0, 1, 2, ...$ $\underline{\alpha}_p(m_1) = \bar{\alpha}_p(m_1)$ form the consequence $1; 4/3; 3/2; ...$ with the limit $\alpha = 2$. And because of the fact that $sign(\underline{\alpha}_p(m_1) - 1) = signS, sign(1 - \bar{\alpha}_p(m_2)) = signS$, existence regions for multiple cycles with m_1-index are situated on the plain α, S over $\alpha > 1$ (except the case of$m_1 = 0$, when the left boundary of the region is $\alpha = 1, 0 \le S \le 1$) and existence regions for multiple cycles with m_2-index are situated on the plain α, S over $\alpha < 1$. Also $\partial\underline{\alpha}_p(m_1)/\partial m_1, \partial\bar{\alpha}_p(m_1)/\partial m_1 > 0, \partial\underline{\alpha}_p(m_2)/\partial m_2, \partial\bar{\alpha}_p(m_2)/\partial m_2 < 0$ and for each S the boundaries of existence regions for the cycles move to the right when m_1 increase and to then left when m_2 increase.

3 Conclusion

Qualitative system analysis shows that on the one hand the use of pulse-phase control principle guarantees global stability of control regime and on the other hand makes possible low-frequent beatings appearance [Leonov, 1959]. Even the beatings with quasi-continuous spectrum may appear. Thus as in FS dynamics the countable number of sub-ranges exists where the phase of control pulses appearance is modulated by the cycles of low-frequent reiteration the use of frequent-phase detection ensures the uniqueness and stability of limit trajectory in wide frequency range, but for definite sub-ranges of the interval of conrollability makes of strict demands on filtration of low-frequency specter components for input signal of GG.

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