QUANTUM PROBABILISTIC SPACE ANALYSIS FOR ENHANCED TWO-WHEELER TRAFFIC SAFETY: FROM CLASSICAL LIMITATIONS TO ADVANCED QUANTUM CIRCUITS

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Article history:
Received 09.10.2024, Accepted 24.12.2024

Abstract

In the increasing traffic scenario and the presence of automated four-wheeler vehicle-like situations, we need to consider the possible safe driving for the two-wheelers also. As two-wheelers need manual balances, embedding the autopilot systems is a tougher task. To resolve this issue, we attempt to address this problem with probability spaces. Then, these classical probability spaces are promoted into quantum probability spaces. The quantum computing aspects such as quantum gates are embedded in the analysis of the quantum probability space. With those quantum computing protocols, various traffic scenarios are calculated, such as the probability of accidents and the avoidance probability of accidents. A set of mathematical lemmas with certain conditions and their proof for mapping classical to quantum probability space and the singularities in this quantum probability space are also discussed in this work.

Key words

Quantum computing application, classical probability, quantum probability space, two-wheeler traffic safety, accident risk modeling, Usable quantum computer

1 Introduction

Urbanization and economic growth have substantial increase in the number of vehicles on roads, with two-wheelers face the steepest incline. These vehicles, due to their smaller size, reduced stability, and direct exposure of riders to external hazards, are highly prone to accidents. The increased density of two-wheelers in various regions has amplified the traffic concerns, particularly the risk of collisions. As a result, the development of effective collision avoidance systems (CAS) has become the need of the hour.

Conventional CAS models focus on calculating safe distances based on parameters such as vehicle velocity, driver reaction time, and braking capabilities [Hasarinda et al., 2023; Luo et al., 2022; Wu and Fu, 2023; Shen et al., 2020]. These approaches, however, used to consider that vehicles move independently and at constant speeds, that leads to safe distances to be calculated in isolation.

While these assumptions might rely for four-wheelers, they address the dynamics of two-wheelers limitedly. Two-wheelers consist unique behaviors such as lateral movements and higher maneuverability, which can highly affect safe distances. Additionally, real-world traffic scenarios are rarely static, with sudden braking, acceleration, and random maneuvers being a regular occurrence. Classical CAS models often neglect the complex interdependencies that arise in peak traffic conditions, where vehicle interactions are neither isolated nor straightforward.

Recent advancements in automated driving systems have introduced improved models that integrate machine learning and real-time data analysis to predict and diminish accidents [Girija and Divya, 2024; Debbarma et al., 2024; Arjaria et al., 2023]. These methods poweered by behavioral data from surrounding vehicles to calculate safe distances dynamically, to provide a promising improvement over traditional models. However, despite their potential, these systems remain constrained by the classical probabilistic frameworks within the regime they operate. Such frameworks are limited in capturing the detailed interdependencies and probabilistic states inherent in dense traffic scenarios, especially in two-wheelers. To address these issues, this paper explores the integration of quantum probability spaces and quantum computing as a novel approach to increase the predictive accuracy and applicability of CAS for two-wheelers in complex traffic environments.

The existing direction of research on CAS has several critical gaps that need to be addressed to improve traffic safety for two-wheeler vehicles. In the existing classical probablistic modeling of traffic prediction, one major gap is the lack of consideration for quantum probablistic space, which would offer more detailed information related to the vehicle dynamics. In dense traffic scenarios, the probable state of one vehicle can influence the probable state of another. This will lead to complex probablistic space, about which the classical models struggle to predict accurately.

The transition from classical probability space to quantum probability space, which is discussed in this paper, is not just a theoretical enhancement but also a practical one. The power of quantum computing algorithms will support the processing of complex, multidimensional interactions in traffic scenarios far beyond what classical systems can do. This paper discussed the possible model by integrating quantum probability spaces with traffic data, we can create more accurate and real-time collision avoidance systems, especially for vulnerable two-wheelers.

The mathematical analysis will help to build better traffic analysis systems. Earlier multiagent systems like traffic systems are discussed mathematically in [García Planas, 2023]. The multiple classical systems with interactions can be seen as a social interactions model based on the Langevin equation reported in [Petukhov, 2019].

Various methods are attempting to solve classical problems using the quantum mechanical approach. For example travvelling salesman problem [Melnikov et al.,]. Quantum computing has numerous proposed applications [Cho et al., 2021]. It has been used in quantum machine learning [Sharma and Renugadevi, 2024] , medical applications [Flöther, 2023] and quantum cryptography [Easttom, 2022] etc., The present work utilizes quantum computing solutions for the two-wheeler traffic management scenario.

The present work attempts to address existing research gaps by proposing a novel approach that integrates quantum computing aspects [Nielsen and Chuang, 2010; Wilde, 2013] into the simulation of safe distances for two-wheelers. In addition to the quantum probability framework, this paper also proposes a quantum circuit model that simulates the interactions between vehicles in a traffic scenario. By considering each vehicle's evolution state as a qubit and using quantum gates to simulate interactions, this model provides a different method for analyzing and predicting road accident probabilities.

This paper is structured as follows: Section 2 outlines the problem statement and a comprehensive review of the existing literature. Section 3 details the theoretical framework of the proposed approach, including its mathematical foundation and assumptions. Sections 4 delve into the novel application of quantum probability spaces. Sections 5 and 6 discuss the transitions from classical to quantum probability space. Section 7 underlines the novel lemmas and theorems for this research work. Section 8 discusses the quantum computing circuits for the bike accident problem. Section 9 and 10 addresses geometric spaces and the accident happening scenareo in dense streets woth density matrix and singularities. Section 11 discusses the intersting results and discussion of this work.

2 Problem Statememt

In the scenareo of heavy traffic there are various models available that discuss the possible avoidance of accidents in road traffic. One of the solutions suggested is automated cars [Nyholm and Smids, 2020; Pütz et al., 2019]. The implementation of automated vehicles might be helpful. But for case two wheelers, the automation requires lots of research on the modification of its design. We narrow down the problem as the avoidance of accidents technology, supposed to be implemented in bikes with existing technology, as well as the solution must be like an add-on instead of completely changing the design of the bike. Due to the complexity and unpredictability of traffic scenarios that involve two-wheelers, particularly in dense urban environments, existing methods of modeling and predicting safe distances may fall short.

Classical models, including both deterministic and stochastic approaches, have been widely used to discuss the traffic dynamics. While deterministic models depend on fixed parameters and predefined relationships, stochastic methods incorporate randomness and conditional probabilities. However, both approaches face limitations when applied to dense traffic scenarios that consists of a large number of interacting bikes. These limitations include the computational complexity of explaining all possible interactions and the need for explicit assumptions about probabilities. To address these challenges, the present study explores a quantum probability framework, which gives a more efficient and adaptable approach to explain the interdependencies.

This limitation lets everyone think of a more advanced approach that can account for the uncertainties and interactions within such systems. To address these challenges, we propose implementation of quantum computing principles, especially quantum probability spaces, to calculate and predict safe distances for bikes. Unlike classical probability, which treats the possible events as independent and definite, the discussed quantum probability allows for the consideration of superposition and entanglement and shows the true complexity of interactions in a multi-bike system in such probability space. By mapping the classical bike dynamics problem into the quantum probability space, we can understand better the probabilistic nature of accidents and interactions. This mapping offers a more nuanced and more accurate solution for accident avoidance in two-wheeler traffic scenarios. Regarding this problem, we create a safer distance solution from the bike. Within this distance from the bike, in which the biker can ride without facing accidents. The same solution is extended to the multiple bike system. The solutions are then projected to the classical probability space and the quantum probability space. In those quantum probability scenarios, the quantum computing solutions are analyzed by constructing quantum computing circuits. The longitudinal safe distances and lateral safe distances for each sought-after bike system, such as one bike system, two bike systems, and three bike systems, are calculated.

The quantum formulation discussed in this study is stochastic, that reflects the probabilistic nature of quantum mechanics. It explains the traffic dynamics by representing the system's state as a superposition of possible outcomes, with probabilities determined by the squared magnitudes of quantum amplitudes. This stochastic framework shows the uncertainty and complexity of dense traffic scenarios, which may not be fully addressed by deterministic approaches.

3 Safer Distances

Building on the challenges identified in Section 2, we propose a novel framework with the advantage of quantum probability spaces to address the limitations of classical models. For the bike which travels in a traffic, we can calculate the safer distances with the mathematical perspectives. The safer distances can be coined as $x_1(v, m), x_2(v, m), y_1(v, m)$, and $y_2(v, m)$, which corresponds to the distances in front, back, and sides of the bike. The following assumptions can be introduced to find out the solution of safer distances. As the velocity of the bike increases, the required safe distances x_1 and x_2 will also increase. This assumption is considered due to the fact that higher velocities will require more time and distance to stop or avoid obstacles. Similarly, for y_1 and y_2, higher velocity consequences for a greater potential for lateral movement due to swerving or slipping. Hence we assume that the safer distance is a function of velovity. As the mass increases, the inertia of the bike increases. This change will make the bike harder to stop or change the moving direction. Hence, higher mass would require larger safety distances such as x_1, x_2, y_1, and y_2. For the case of safety distances, there may be other negligible parameters such as reaction time, road conditions, and tire grip. The safe distance function is then calculated as,

$$x_1(v, m) = k_1 \cdot v^a \cdot m^b \tag{1}$$
$$x_2(v, m) = k_2 \cdot v^c \cdot m^d \tag{2}$$
$$y_1(v, m) = k_3 \cdot v^e \cdot m^f \tag{3}$$
$$y_2(v, m) = k_4 \cdot v^g \cdot m^h \tag{4}$$

Here, k_1, k_2, k_3, k_4 are constants that can be related to the specific characteristics of the bike, such as the braking system and design. a, b, c, d, e, f, g, h are the exponents that describe how strongly each factor affects the safe distance function. Let's determine the exponents for $x_1(v, m)$ and $x_2(v, m)$. The distance required to stop the bike is proportional to the velocity squared, as well as proportional to mass due to the fact that greater the mass of the bike requires more momentum for the bike to be stopped. In the lateral distance, which depends on how the bike might swerve or drift, the influence of mass might be less direct, so for that case we can assume a weaker dependence on mass. Hence we can assume,

$$a = 2, \quad b = 1 \quad c = 2, \quad d = 1 \tag{5}$$
$$e = 2, \quad f = 0.5 \quad g = 2, \quad h = 0.5 \tag{6}$$

The longitudinal distances x_1 and x_2 are strongly influenced by velocity due to the quadratic dependence of stopping distance on speed. This is consistent with the equation for stopping distance:

$$d_{\text{stopping}} \propto v^2. \tag{7}$$

Hence, the velocity exponents a and c are taken as 2.

The linear dependence of longitudinal stopping distance on mass reflects the relationship between momentum and mass ($p = mv$) as well as the energy required to stop the vehicle. This results the choice of $b = 1$ and $d = 1$."

Lateral distances y_1 and y_2 also show a quadratic dependence on velocity. As velocity become higher and that increases the risk of lateral drift or swerving. The values $e = 2$ and $g = 2$ align with this observation.

As inertia plays a less dominant role in side-to-side motion compared to forward or backward motion, the influence of mass on lateral distances is weaker than in the longitudinal direction, A fractional value ($f = h = 0.5$) represents this reduced dependence.

3.1 y < x

As α and β are less than 1, y_1 and y_2 will be less than x_1 and x_2. This results in the lateral distances being smaller than the longitudinal distances. With the condition $y < x$, longitudinal distances are obtained as,

$$x_1(v, m) = x_2(v, m) = k_1 \cdot v^2 \cdot m \tag{8}$$
$$x_2(v, m) = k_2 \cdot v^2 \cdot m \tag{9}$$

Lateral distances are obtained as,

$$y_1(v, m) = \alpha \cdot k_1 \cdot v^2 \cdot m^{0.5} \quad \text{with } \alpha < 1 \tag{10}$$
$$y_2(v, m) = \beta \cdot k_2 \cdot v^2 \cdot m^{0.5} \quad \text{with } \beta < 1 \tag{11}$$

These safety distances are the functions of velocity and ensure that the lateral safe distances are smaller than the longitudinal ones.

3.2 2 Bike case

While expanding the scenario for the bike system, the interaction between the two vehicles in terms of safer distance is also to be considered. The difference in the velocities between the two bikes will affect the calculated safe distances. If the bikes are moving at different velocities, the longitudinal distances x_1 and x_2 have to be considered for the possibility of one bike catching up to or being overtaken by the other. The lateral distances y_1 and y_2 should be higher to avoid side collisions, especially when both bikes travel close. We can denote the parameters for the first bike as v_1, m_1, and for the second bike as v_2, m_2. The safer distances will then depend on the parameters of both bikes.

Longitudinal Safe Distances

The Lateral Safe Distances are calculated for the two bike systems as, For the front ir rear Bike a (relative to Bike b),

$$x_i^{(a)}(v_a, m_a, v_b, m_b) = k_i \cdot (v_a + v_b)^2 \cdot (m_a + m_b) \quad (12)$$

Here, $x_i(a)$ are the longitudinal distances for Bike a concerning Bike b with $a, b \in \{1, 2\}$. Also $i \in \{1, 2\}$

Lateral Safe Distances

similarly the lateral distances are calculated for the below conditions. For the left of Bike a (relative to Bike b),

$$y_i^{(a)}(v_a, m_a, v_2, m_2) = \alpha \cdot k_1 \cdot (v_1 + v_2)^2 \cdot (m_1^{0.5} + m_2^{0.5}) \quad (13)$$

We can find out that, the presence of a second bike increases the required safe distances due to the need to avoid possible collisions. Also, we can understand the existence of possible symmetry in the safe distances, which means the safe distance from Bike 1 to Bike 2 is the same as from Bike 2 to Bike 1. As the condition $y < x$ holds, the lateral distances will remain smaller than the longitudinal ones.

3.3 3 Bikes scenareo

In most traffic, the 3-bike system often results in more accidents. To avoid that, we can expand the two-bike accidental avoidance scenario into three-bike scenarios. The safe distance problem can be solved for both same-direction and opposite-direction scenarios. Initially, we focused on three bikes in the same direction. Let us consider these three bikes a, b, c have velocities v_a, v_b, v_c and masses m_a, m_b, m_c.

Longitudinal Safe Distances

For Bike a travels relative to Bikes b and c,

$$x_i^{(a)} = k_i \cdot (v_a + \max(v_b, v_c))^2 \cdot (m_a + m_b + m_c) \quad (14)$$

Where a, b, c corresponds to bikes $1, 2, 3$ respectively and i reprecents the ith bike's perspective of the accident about to occur.

Lateral Safe Distances

For Bike a travels relative to Bikes b and c:

$$y_i^{(a)} = \alpha \cdot k_a \cdot (v_a + \max(v_b, v_c))^2 \cdot (m_1^{0.5} + m_2^{0.5} + m_3^{0.5}) \quad (15)$$

$$y_j^{(a)} = \beta \cdot k_b \cdot (v_a + \max(v_b, v_c))^2 \cdot (m_1^{0.5} + m_2^{0.5} + m_3^{0.5}) \quad (16)$$

Here $i\&j$ reprecents the i, jth bike's perspective of the accident about to occur.

3.4 Three bikes in opposite directions

Then the safe distance calculations can be extended for opposite directions of the three-bike system. When three bikes in a road are traveling in opposite directions, the safe distances, especially the lateral distances, are important to avoid head-on or side-swipe collisions. It is safer to consider that for bikes traveling in opposite directions, the effective velocity determining the safe distances is the sum of the individual velocities.

Longitudinal Safe Distances

For opposite directions, the terms longitudinal distances preserve enough space for bikes to avoid a head-on collision. The following scenarios can be considered for that distance. If Bike a and Bike b are relative to each other in opposite directions, the longitudinal distance is determined as,

$$x_i^{(a)} = k_i \cdot (v_a + v_b)^2 \cdot (m_a + m_b) \quad (17)$$

Lateral Safe Distances

Lateral safe distances are supposed to be notably increased to avoid collisions when bikes are nearby but moving in opposite directions. For this case, we can consider the following scenarios. If Bike a moves relative to Bike b in the opposite direction, then

$$y_i^{(a)} = \alpha \cdot k_i \cdot (v_a + v_b)^2 \cdot \left(m_a^{0.5} + m_b^{0.5}\right) \quad (18)$$

$$y_j^{(a)} = \beta \cdot k_j \cdot (v_a + v_b)^2 \cdot \left(m_a^{0.5} + m_b^{0.5}\right) \quad (19)$$

The equations for x_i^j where $i, j \in 1, 2, 3$ look similar because they are based on the same fundamentals that govern the interaction between the velocities and masses of two moving bikes. In these equations, the variables are considered to show the relative velocity and combined mass of the bikes in each scenario. In these equations, whether the bikes are moving in opposite directions or the same direction, the equations include the sum of the velocities of the bikes will be like $(v_1 + v_2)^2, \quad (v_1 + v_3)^2, \quad (v_2 + v_3)^2$

This indicates the relative motion between the bikes. When bikes move in opposite directions, the sum of their velocities determines how rapidly they approach each other, which consequences the longitudinal distance between them.

3.5 Safety velocity

For these safer distances, we can calculate the safety velocity as an addition. To calculate the safety velocity we need to consider the functional relationship between velocity, safety distance, and the physical parameters of the bikes. The safety velocity v_s is the maximum velocity at which a motorcycle can travel while maintaining a safe distance from other bikes or obstacles. Safety Velocity for a single bike v_{s1} is related to the safety distance x_1 and x_2 and other physical parameters such as reaction time t_r, braking distance d_b, and deceleration a. Then the safety velocity for a single bike can be calculated using the relation,

$$x_1 = v_{s1} t_r + \frac{v_{s1}^2}{2a} \tag{20}$$

here v_{s1} is the safety velocity for single bike, t_r is the reaction time of the rider and a is deceleration rate.

The safe stopping distance for a bike is affetced by several factors, such as the rider's reaction time, braking capacity, and deceleration rate. The relationship between these parameters leads to derive equation 20. The equation is derived using the basic principles of kinematics, with the combination of reaction and braking distances. This is introduced to compute the safety velocity required for collision avoidance.

Solving equation 20, for safety velocity term leads to

$$v_{s1} = \frac{-t_r a + \sqrt{(t_r a)^2 + 2ax_1}}{1} \tag{21}$$

The equation 21 gives the safety velocity for a single bike which depends upon the reaction time, deceleration, and safety distance. To find the safety velocity between the two bikes, we need to consider the interaction between the two bikes. let v_{s1} and v_{s2} are safety velocities of them. If the bikes are traveling in the same direction, then the safety distance x_{12} between them will be enough to avoid a possible collision. If both bikes are in the same direction, the safety distance x_{12} between Bike 1 and Bike 2 can be written as,

$$x_{12} = v_{s1} t_r + \frac{v_{s1}^2}{2a} - v_{s2} t_r - \frac{v_{s2}^2}{2a} \tag{22}$$

Then the safety velocities should satisfy the following conditions for Bike 1 and Bike 2 not to collide

$$v_{s1} > v_{s2} \tag{23}$$

If both bikes move in opposite directions the safety distance x_{12} will be

$$x_{12} = (v_{s1} + v_{s2}) t_r + \frac{v_{s1}^2}{2a} + \frac{v_{s2}^2}{2a} \tag{24}$$

For a required safety distance x_{12}, we can solve for v_{s1} and v_{s2}. If the bikes are identical, then we can fix $v_{s1} = v_{s2}$, then equation is written as,

$$v_s = \frac{-2t_r a + \sqrt{(2t_r a)^2 + 4ax_{12}}}{2} \tag{25}$$

3.6 Dealing the exponents

The safe distance equation can be formed as,

$$d_{\text{safe}} = kv^{\alpha} m^{\beta} \tag{26}$$

Here, d_{safe} is the safe distance, v is the velocity of the two-wheeler, m is the mass of the vehicle (or combined mass of the rider and vehicle) k is a constant, α and β are the exponents for velocity and mass, respectively. The velocity term v^{α} in the equation shows how changes in velocity affect the required safe distance. Safe distance increases as the velocity of the bike increases because the stopping distance depends on both the reaction time of the rider and the braking capacity of the two-wheeler. A higher exponent α indicates a stronger dependence of the equation on velocity. Even small changes in velocity will result in larger changes in the safe distance. If velocity doubles, more than double the stopping distance is required. Hence an exponent α greater than 1 is necessary to exist. The mass term m^{β} explains the fact that heavier vehicles will have greater momentum and thus require more force as well as more distance to stop safely. Also, it shows that the effect of mass is nonlinear. The exponent β shows the relationship for how much more stopping distance is required as mass increases. Also, it is important to consider that doubling the mass might not double the stopping distance. The friction and the braking system's efficiency may induce nonlinear effects on how rapidly a vehicle can stop.

In classical dynamics, stopping distance d_{stop} is often derived as [Leff and Mallinckrodt, 1993; Awrejcewicz, 2012]:

$$d_{\text{stop}} = \frac{v^2}{2a} \tag{27}$$

where a is the deceleration,

The braking force F_{brake} can be obtained as,

$$F_{\text{brake}} = \mu mg \tag{28}$$

where μ is the friction coefficient.

m is the mass, and g is the gravitational acceleration. The deceleration a then becomes,

$$a = \frac{F_{\text{brake}}}{m} = \mu g \tag{29}$$

Substituting equation 29 into 27 gives,

$$d_{\text{stop}} = \frac{v^2}{2\mu g} \tag{30}$$

This solution suggests a velocity exponent of $\alpha = 2$. Consideration of the rider's reaction time or non-ideal braking conditions may induce more nonlinear solutions. The exponents a, b, c, d, e, f, g, and h in the safe distance equations are the critical aspects of modeling the accidental behavior. These exponents are initially based on basic assumptions about how velocity and mass influence safe distances.

In Lateral safe distances, a weaker velocity dependence is assumed with $e = 2$ but a reduced mass exponent $f = 0.5$ indicates the less direct effect of mass on lateral swerving than stopping. The exponents for mass (b, d, f, and h) indicate the relationship between a two-wheeler's mass and its inertia. A heavier vehicle needs more force (and distance) to stop or maneuver. The exponents $b = 1$ and $d = 1$ for longitudinal distances show a linearity with mass, while the exponents $f = 0.5$ and $h = 0.5$ for lateral distances suggest that mass has a less direct impact on side-to-side movement.

Having established the theoretical framework, we next review foundational concepts of probability spaces to set the stage for the proposed quantum extension in the sections 4 and 5

4 Probability Space

Even though safer distances are been derived, in order to obtain the probability of accidents and their possible avoidance, the problem yet to be promoted into the probability space. The safer distances for three bikes (B1, B2, B3) are written as $X_1^{(i)}$ and $X_2^{(i)}$ They are the longitudinal safe distances (front and rear) for bike i and $Y_1^{(i)}$ and $Y_2^{(i)}$ are the lateral safe distances (left and right) for bike i.

The probability space is represented as the possible safe distances overlapping, which could lead to an increased risk of an accident. We can define random variables $D_x^{(i,j)}$ and $D_y^{(i,j)}$ that represent the distances between the bikes i and j in the longitudinal and lateral directions respectively. The probability of an accident due to the overlap of safer distances can be described as the probability that the actual distances between the bikes i and j are less than the required safe distances.

For bikes i and j, the longitudinal overlap probability is determined as,

$$P_{acc}^{(i,j)}(x) = P\left(D_x^{(i,j)} < X_1^{(i)} + X_2^{(j)}\right) \tag{31}$$

here $D_x^{(i,j)}$ is the actual longitudinal distance between bikes i and j.

Similarly, lateral overlap probability is determined as,

$$P_{acc}^{(i,j)}(y) = P\left(D_y^{(i,j)} < Y_1^{(i)} + Y_2^{(j)}\right) \tag{32}$$

here $D_y^{(i,j)}$ is the actual lateral distance between bikes i and j.

For the three bikes case, the overall probability for an accident to occur is the probability that any pair of bikes i and j would cause a dangerous overlap. For the simpler case, we can assume the events of overlap between different pairs are independent. The total probability of an accident can be calculated with the union of the pairwise overlaps.

$$P_{acc}^{(total)} = 1 - \prod_{i<j}\left(1 - P_{acc}^{(i,j)}(x)\right) \times \left(1 - P_{acc}^{(i,j)}(y)\right) \tag{33}$$

Here $P_{acc}^{(i,j)}(x)$ and $P_{acc}^{(i,j)}(y)$ are the probabilities of longitudinal and lateral overlaps for each pair of bikes (i, j). By assuming the distances $D_x^{(i,j)}$ and $D_y^{(i,j)}$ are random variables and they follow normal distributions, they can be written as,

$$D_x^{(i,j)} \sim \mathcal{N}(\mu_x^{(i,j)}, \sigma_x^{(i,j)}) \tag{34}$$

$$D_y^{(i,j)} \sim \mathcal{N}(\mu_y^{(i,j)}, \sigma_y^{(i,j)}) \tag{35}$$

Where $\mu_x^{(i,j)}$ and $\mu_y^{(i,j)}$ are the expected distances between the bikes in the longitudinal and lateral directions, respectively. $\sigma_x^{(i,j)}$, $\sigma_y^{(i,j)}$ are the corresponding standard deviations.

The probability of overlap, which expresses the probability of an accident, can be computed using the cumulative distribution function (CDF) of the normal distribution [Severini, 2005; lodzimierz Bryc, 1995].

$$P_{acc}^{(i,j)}(x) = \Phi\left(\frac{X_1^{(i)} + X_2^{(j)} - \mu_x^{(i,j)}}{\sigma_x^{(i,j)}}\right) \tag{36}$$

$$P_{acc}^{(i,j)}(y) = \Phi\left(\frac{Y_1^{(i)} + Y_2^{(j)} - \mu_y^{(i,j)}}{\sigma_y^{(i,j)}}\right) \tag{37}$$

Where $\Phi(\cdot)$ is the CDF of the standard normal distribution. As we calculated the probability of accidents in classical probability space, we need to promote it into quantum probability space in order to construct a more accurate accidental avoiding algorithm.

5 Promoting Classical Probability Space into Quantum Probability Space

Even though the classical probability space promises useful solutions, real-world traffic scenarios which consist of multiple bikes been affected by various parameters such as sudden changes in speed, unexpected obstacles, and varying road conditions. These introduce large uncertainties in the classical models and as a result, require further advanced models. To address these complexities, we extend our analysis into the realm of quantum probability spaces.

Unlike classical probability, which treats events as binary and independent, quantum probability allows for more sophisticated modeling of events where multiple outcomes can coexist in a superposition. This is particularly relevant for multi-bike systems, where the presence and actions of one bike can directly influence the safe distances of others. As per the theorem and proof to be discussed in Section 7, relevant to the classical quantum probability space, further mapping can be done. The classical probability of this bike-traveling event can be mapped into the probability space. In consists of sample space (Ω), event ($\mathcal{F}$), and probability measure (P). The probability measures P are supposed to satisfy non-negativity, normalization, and additivity axioms [Grinstead and Snell, 2012; Pollard, 2002].

To know the exact probabilistic distribution of the bike dynamics problem, we need to promote it into quantum probability space. Such promotion must address the following aspects such as Hilbert space $\mathcal{H}$ which is analogous to the sample space Ω, operators, and quantum states. In general, the probability of a quantum event for a quantum state ρ is calculated as,

$$\mathbb{P}(P) = \text{Tr}(\rho P) \tag{38}$$

For example the quantum probability of measuring the qubit in the state $|0\rangle$ is written as,

$$\mathbb{P}(P_0) = \text{Tr}(\rho P_0) = \text{Tr}(|\psi\rangle\langle\psi||0\rangle\langle 0|) = |\langle 0|\psi\rangle|^2 = |\alpha|^2 \tag{39}$$

The probability discussed in equation 39 leads to the measurement of the qubit that will give the output as the state $|0\rangle$.

5.1 Projected Space

Regrding the projection from classical probability space $(\Omega, \mathcal{F}, P)$ to quantum probability space $(\mathcal{H}, \mathcal{P}(\mathcal{H}), \rho)$, we can write

$$P(\{\omega_1\}) = \text{Tr}(\rho P_{\omega_1}) = p_1, \quad P(\{\omega_2\}) = \text{Tr}(\rho P_{\omega_2}) = p_2 \tag{40}$$

Initially, the quantum probability space is constructed for two bike systems. The classical binary probabilities can be promoted into quantum probability space. In the classical probability space, we define a binary random variable A as if $A = 1$, an accident can occur, which means overlapping the safer distance. If $A = 0$ then no accident occurs.

The classical probabilities can be obtained for these events as,

$$P(A = 1) = p_{acc} \tag{41}$$

$$P(A = 0) = 1 - p_{acc} \tag{42}$$

where p_{acc} is the classical probability of an accident. In the quantum probability space, we substituite the classical probabilities with quantum probability amplitudes. In general, a quantum system is described by a state vector $|\psi\rangle$ in a Hilbert space. We can define two quantum states corresponding to the two possible outcomes for the events in quantum probability space as, $|acc\rangle$ and $|no\_acc\rangle$ as quantum state corresponding to an accident occurring and quantum state corresponding to no accident.

The quantum state of the system is a superposition of these two states are obtained as,

$$|\psi\rangle = \alpha|acc\rangle + \beta|no\_acc\rangle \tag{43}$$

where α and β are probability amplitudes. For the normalization, the total probability must sum to 1. Hence the amplitudes satisfy the following relation.

$$|\alpha|^2 + |\beta|^2 = 1 \tag{44}$$

In the quantum probability space, the probability of an outcome is derived from the probability of an accident to occur as,

$$P_{quantum}(A = 1) = |\alpha|^2 \tag{45}$$

The probability of no accidents is written as,

$$P_{quantum}(A = 0) = |\beta|^2 \tag{46}$$

Then the normalization condition becomes

$$|\beta|^2 = 1 - |\alpha|^2 \tag{47}$$

To avoid an accident, the quantum probability of the system is supposed to be in the $|no\_acc\rangle$ state after measurement

$$P_{quantum}(A = 0) = |\beta|^2 = 1 - |\alpha|^2 \tag{48}$$

For example, let us assume a simpler state.

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|acc\rangle + \frac{1}{\sqrt{2}}|no\_acc\rangle \tag{49}$$

Here, the amplitudes are written as,

$$\alpha = \frac{1}{\sqrt{2}}, \quad \beta = \frac{1}{\sqrt{2}} \tag{50}$$

The probabilities of the accident, in the quantum probability space, is calculated as,

$$P_{quantum}(A = 1) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \tag{51}$$

Then probability of no accident is calculated as

$$P_{quantum}(A = 0) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \tag{52}$$

Using this example we can identify that there exists 50% of chance for both accidents to happen and not happen.

In general, if the initial state is not evenly distributed, like,

$$|\psi(0)\rangle = \alpha|acc\rangle + \beta|no\_acc\rangle \tag{53}$$

with $\alpha = a + ib$ and $\beta = c + id$, then the probability of an accident in quantum probability space is obtained as,

$$P_{quantum}(A = 1) = a^2 + b^2 \tag{54}$$

Similarly, the quantum probability of avoiding an accident is calculated

$$P_{quantum}(A = 0) = c^2 + d^2 \tag{55}$$

For the quantum probability space, if the two bikes in the system are in a balanced superposition state, then

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|acc\rangle + \frac{1}{\sqrt{2}}|no\_acc\rangle \tag{56}$$

Here, the amplitudes are written as,

$$\alpha = \frac{1}{\sqrt{2}}, \quad \beta = \frac{1}{\sqrt{2}} \tag{57}$$

The probability of an accident is calculated as,

$$P_{quantum}(A = 1) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} = 0.5 \tag{58}$$

The probability of no accident is obtained as

$$P_{quantum}(A = 0) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} = 0.5 \tag{59}$$

Due to normalization, we can have, $a^2 + b^2 + c^2 + d^2 = 1$. Let us apply these quantum probability space ideas to the bike kinetics problem.

5.2 Quantum Probability in 3 Bike Scenario

The quantum probability calculation can be applied to a three-bike scenario. For the three bike systems, B1, B2, and B3, the quantum probability space has the following quantum states.

$|acc_{12}\rangle$: Accident between B1 and B2.
$|acc_{13}\rangle$: Accident between B1 and B3.
$|acc_{23}\rangle$: Accident between B2 and B3.
$|no\_acc\rangle$: No accident between any of the bikes.

The quantum state in this probabilistic space $|\psi\rangle$ can be considered as a superposition of states as

$$|\psi\rangle = \alpha_{12}|acc_{12}\rangle + \alpha_{13}|acc_{13}\rangle + \alpha_{23}|acc_{23}\rangle + \beta|no\_acc\rangle \tag{60}$$

where α_{12}, α_{13}, α_{23}, and β are complex probability amplitudes. The normalization condition is then written as,

$$|\alpha_{12}|^2 + |\alpha_{13}|^2 + |\alpha_{23}|^2 + |\beta|^2 = 1 \tag{61}$$

For each probabilistic event (accident or no accident), the quantum probability is obtained for an accident between B1 and B2, an accident between B1 and B3, an accident between B2 and B3, and no accident are calculated respectively as,

$$P_{quantum}(A_{12} = 1) = |\alpha_{12}|^2 \tag{62}$$
$$P_{quantum}(A_{13} = 1) = |\alpha_{13}|^2 \tag{63}$$
$$P_{quantum}(A_{23} = 1) = |\alpha_{23}|^2 \tag{64}$$
$$P_{quantum}(A = 0) = |\beta|^2 \tag{65}$$

The quantum amplitudes α_{12}, α_{13}, and α_{23} can interfere each other due to the quantum superposition. This interference can be either constructive or destructive, which will affect the overall quantum probabilities.

If the initial state is not evenly distributed, then there will be unequal probabilities.

$$|\psi(0)\rangle = \alpha_{12}|acc_{12}\rangle + \alpha_{13}|acc_{13}\rangle + \alpha_{23}|acc_{23}\rangle + \beta|no\_acc\rangle \tag{66}$$

Here $\alpha_{12} = a + ib$, $\alpha_{13} = c + id$, $\alpha_{23} = e + if$, and $\beta = g + ih$. Then the probabilities can be distinguished in the following scenario

Probability of Accident between the bikes B1 and B2

$$P_{quantum}(A_{12} = 1) = a^2 + b^2 \tag{67}$$

Probability of Accident between the bikes B1 and B3

$$P_{quantum}(A_{13} = 1) = c^2 + d^2 \tag{68}$$

Probability of Accident between the bikes B2 and B3

$$P_{quantum}(A_{23} = 1) = e^2 + f^2 \tag{69}$$

The probability for no accident scenario in quantum probability space is calculated as,

$$P_{quantum}(A = 0) = |\beta|^2 = g^2 + h^2 \tag{70}$$

The condition is supposed to be satisfied as,

$$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 = 1 \tag{71}$$

It is important to consider the output in this quantum probability space will depend upon the initial quantum state configuration as well as the quantum interferences.

5.3 Safety Velocity and Probability Space

Mapping safety velocity into the probability space can offer a more rigorous framework for assessing the possibility of accidents. The safety velocity is the quantum probability space that can be derived with, $P(v \leq v_s)$ for the probability that the bike is traveling at or below the safety velocity and $P(v > v_s)$ is the probability that the bike is traveling above the safety velocity. In the quantum probability space, the probability amplitudes α_{ijk} can now be functions of both the distance and the velocity. For multiple bikes, the quantum state can be written as,

$$|\psi\rangle = \alpha_{000}(v_1 \leq v_{s1}, v_2 \leq v_{s2}, v_3 \leq v_{s3})\,|000\rangle + \dots + \alpha_{111}(v_1 > v_{s1}, v_2 > v_{s2}, v_3 > v_{s3})\,|111\rangle \quad (72)$$

Here, each amplitude α_{ijk} shows the probability of certain bikes either exceeding or staying within their corresponding safety velocities.

The quantum states in terms of qubits can show a binary condition (safe/unsafe) for both distance and velocity in the probability space. In this aspect, a qubit d_i represents the safety distance for bike i. Here $|0\rangle$ indicates the distance is safe in probability space and $|1\rangle$ shows the distance is unsafe. Similarly, a qubit v_i represents the safety velocity for bike I, with $|0\rangle$ showing the velocity is within the safe limit and $|1\rangle$ represents the velocity exceeding the safe limit. For n bikes, we would have $2n$ qubits with n for distances and n for velocities.

The combined quantum state for n bikes in the quantum probability space can be represented as,

$$|\psi\rangle = \sum_{i,j=0}^{1} \alpha_{ij} |d_1 d_2 \dots d_n\rangle \otimes |v_1 v_2 \dots v_n\rangle \quad (73)$$

here, $d_1 d_2 \dots d_n$ represents the possible distance state, $v_1 v_2 \dots v_n$ represents the possible velocity state and α_{ij} reprecent the probability amplitudes corresponding to these quantum states.

6 Why Solving with Quantum Probability Space is Important?

Consider three bikes (A, B, and C) moving nearby in a dense traffic environment. If any one bike attempts braking instantly or faces an obstacle, the probability of an accident increases for the other bikes due to the limited reaction time and space to maneuver. In a classical probabilistic approach, consider probabilities $P(A)$, $P(B)$, and $P(C)$ for each bike to crash.

$$P(A) = 0.1, \quad P(B) = 0.1, \quad P(C) = 0.1 \quad (74)$$

In a classical probability space, the total probability of an accident consisting of any of the three bikes can be computed by summing the individual probabilities.

$$P(\text{Accident}_{\text{classical}}) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \quad (75)$$

If the accidents are independent (no correlation between the bikes), the probability is written as,

$$P(\text{Accident}_{\text{classical}}) = 0.1 + 0.1 + 0.1 = 0.3 \quad (76)$$

The classical probability scenario assumes that each accident event is completely independent, without considering the increased risk caused by one bike's crash influencing the others.

Thus, it underestimates the various possible risks in dense traffic. In the quantum probability space, independent events can be even considered using superposition and entanglement phenomena. These features of quantum probability space correlate with fine-tuned probabilities of accidents between bikes.

The classical probability space the dependencies can be discussed through conditional probabilities, which describe how the likelihood of one event depends on another. However, this approach becomes increasingly complex and computationally expensive as the number of interacting systems increase. In contrast, quantum probability spaces explain interdependencies through principles such as superposition and entanglement. This make quantum probablity space to be esier for computing complex, dynamic interactions without the need for explicit conditional definitions. Hence this method offer a more efficient and scalable alternative to classical methods

The quantum probability framework gives a natural way to explain complex interactions, especially in systems with dynamic and multi-agent interactions. Unlike classical methods, which require detailed definitions for conditional probabilities or distributions, quantum probabilities encode these relationships detailly through superposition and entanglement. The quantum probablity space approach has shown numerous advantages in high-dimensional, dense systems [Melucci, 2018; Ciaglia et al., 2017]. In quantum probability, each bike's accident probability can be represented as a superposition of accident ($|1\rangle$) and no accident ($|0\rangle$) states. For example, Bike A's quantum probability state might be a superposition of both outcomes as,

$$|\psi_A\rangle = \alpha_A\,|0\rangle + \beta_A\,|1\rangle \quad (77)$$

Similarly for Bike B and Bike C:

$$|\psi_B\rangle = \alpha_B\,|0\rangle + \beta_B\,|1\rangle\,, \quad |\psi_C\rangle = \alpha_C\,|0\rangle + \beta_C\,|1\rangle \quad (78)$$

The total state of the quantum probabilistic system can be expressed as an entangled state as,

$$|\Psi_{\text{total}}\rangle = \alpha\,|000\rangle + \beta\,|111\rangle + \gamma\,|110\rangle + \delta\,|101\rangle + \epsilon\,|011\rangle \tag{79}$$

In this entangled state shown in equation 79, the outcome of one bike's accident ($|1\rangle$) can affect the others, which shows the correlation between them.

Using the quantum state amplitudes from the entangled system, the probability of an accident is then calculated as,

$$P(\text{Accident}_{\text{quantum}}) = |\alpha|^2 + |\beta|^2 + 2 \cdot \text{Re}(\alpha\beta^*) \tag{80}$$

This quantum probability shows the interdependence between the bike states. This which would increase the total accident probability when one bike's state consequences the others. For example, if α and β are non-zero and the interference term is positive, the total accident probability will be higher than the simple sum of individual classical accident probabilities.

The application of quantum probabilities to traffic modeling is a new area of study. While preliminary results demonstrate numerous advantages, further research and numerical validation are needed to fully assess its applicability across different traffic scenarios.

7 Mathematical Discussion on Probability Spaces

We now present the mathematical formulations developed as part of this research, including new lemmas and a theorem that explain the transition from classical to quantum probability spaces.

As the interaction happens within the probability space, the outcome only accounts for two possible states. We can interpret this into quantum probability space as,

$$\mathcal{R} = \mathcal{W}(\psi) \tag{81}$$

Where $\mathcal{W}$ is the projected quantum space from classical probability space. The transition from a classical probability problem to a quantum probability problem requires a transformation of the original problem into one that can be addressed using quantum computing techniques.

For mathematical considerations lets consider classical probability space $(\Omega, \mathcal{F}, P)$ with, Ω as the sample space, $\mathcal{F} \subseteq 2^{\Omega}$ as the event space and $P : \mathcal{F} \to [0, 1]$ as the probability measure which satisfies $P(\Omega) = 1$. For a classical system with n possible states, the number of possible outcomes should be x^n, where x is the number of possible outcomes for each state. Similarly, we can consider a Hilbert space $\mathcal{H}$ with $\mathcal{H}$ as an x-dimensional complex vector space and the quantum state of the system as represented by a density operator ρ on $\mathcal{H}$ which satisfies $\text{Tr}(\rho) = 1$. With these considerations, we can construct the following lemmas.

7.1 Lemma 1

For any classical system with n possible states, each with x outcomes, the corresponding quantum state can be constructed in an x-dimensional Hilbert space $\mathcal{H}$. For every classical probability distribution P over n the classical states can be mapped to a quantum density operator ρ provided the following conditions are satisfied.

Non-negativity Condition: The classical probabilities P_i must be non-negative such as $P_i \geq 0$ for all i.
Normalization Condition: The sum of all classical probabilities must be equal to one, such as $\sum_{i=1}^{x^n} P_i = 1$.
Orthogonality Condition: The classical states must correspond to orthogonal quantum states in $\mathcal{H}$.

7.1.1 Proof for Lemma 1

For each classical state $\omega_i \in \Omega$, associate a quantum basis state $|\psi_i\rangle$ in $\mathcal{H}$. Hence,

$$P(\omega_i) = |\langle\psi_i|\psi_i\rangle|^2 \tag{82}$$

Considering that the classical probabilities satisfy the normalization and non-negativity conditions, the quantum state corresponding to the classical distribution is written as,

$$\rho = \sum_{i=1}^{x^n} P(\omega_i)|\psi_i\rangle\langle\psi_i| \tag{83}$$

This density operator ρ is supposed to satisfy the quantum normalization condition $\text{Tr}(\rho) = 1$, which will complete the proof.

7.2 Lemma 2

For the probabilistic outcomes of the classical system to be accurately mapped in the quantum space, the quantum states $|\psi_i\rangle$ should be able to form superpositions and entanglement by satisfying the following conditions.

Superposition Condition: Any classical mixed state must correspond to a superposition of quantum states.
Entanglement Condition: If the classical states are not independent, the corresponding quantum states must exhibit quantum entanglement.

7.2.1 Proof for Lemma 2

lets consider two classical states ω_i and ω_j with corresponding quantum states $|\psi_i\rangle$ and $|\psi_j\rangle$. A classical mixed state $\omega_k = \alpha\omega_i + \beta\omega_j$ (where $\alpha + \beta = 1$) corresponds to the quantum state is calculated as,

$$|\psi_k\rangle = \alpha|\psi_i\rangle + \beta|\psi_j\rangle \tag{84}$$

The quantum state $|\psi_k\rangle$ is a superposition of $|\psi_i\rangle$ and $|\psi_j\rangle$, which satisfies the superposition condition.

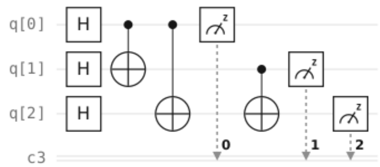

Figure 1. evolution of all three bikes dynamics in the quantum probability space, shown as quantum circuit

For entanglement, let us consider two classical subsystems ω_A and ω_B. The corresponding quantum subsystems are $|\psi_A\rangle$ and $|\psi_B\rangle$. If ω_A and ω_B are not independent, their combination state in the quantum space must be entangled.

$$|\psi_{AB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \tag{85}$$

Instead, it must satisfy:

$$|\psi_{AB}\rangle = \sum_{i,j} \alpha_{ij} |\psi_A^i\rangle \otimes |\psi_B^j\rangle \tag{86}$$

where $\sum_{i,j} |\alpha_{ij}|^2 = 1$, that provides the entanglement condition.

7.3 Theorem

Let $(\Omega, \mathcal{F}, P)$ be a classical probabilistic space with x^n possible outcomes. There exists a mapping $\phi : (\Omega, \mathcal{F}, P) \to (\mathcal{H}, \rho)$ to a quantum probabilistic space $(\mathcal{H}, \rho)$ if and only if the following conditions are satisfied:

The classical probabilities $P(\omega)$ should satisfy non-negativity and normalization.
The quantum states corresponding to each classical probabilistic outcome are supposed to be orthogonal and can form superpositions.
The quantum states should possess entanglement if the classical states exhibit dependencies.

7.3.1 Proof of Theorem Given the conditions driven by Lemmas 1 and 2, the mapping ϕ can be done on each classical state $\omega_i \in \Omega$ with a quantum state $|\psi_i\rangle \in \mathcal{H}$ such that,

$$\phi(\omega_i) = |\psi_i\rangle \quad \text{and} \quad \phi(P(\omega_i)) = |\langle\psi_i|\psi_i\rangle|^2 \tag{87}$$

The resulting quantum state ρ is expressed as,

$$\rho = \sum_{i=1}^{x^n} P(\omega_i) |\psi_i\rangle\langle\psi_i| \tag{88}$$

This proof satisfies the quantum normalization condition $\text{Tr}(\rho) = 1$. The superposition and entanglement properties ensure that the quantum space exposes the full probabilistic structure of the classical space, and this completes the proof.

8 Quantum Computing Circuits

We can promote the three-bike classical probability space into a quantum probability space we can use three qubits, and we can map the possible quantum states associated with accidents or no accidents onto qubits states. The classical probability space can be mapped with the possible accident and no accident scenario as qubit's two possible states. As with classical probability space, we can represent each bike's accident scenario in quantum probability space using qubits as $|0\rangle$ for no accident for the corresponding bike pairs, and $|1\rangle$, for the possible accident for the corresponding bike pairs. For the bike scenario, in quantum probability space, the qubit states represent the accidents between bike pairs. These quantum circuits are made run in the IBM quantum computing backend [Javadi-Abhari et al., 2024].

The state of the system is written as a superposition of the above-suggested states.

$$\begin{aligned}|\psi\rangle &= \alpha_{000}\,|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}\,|011\rangle \\ &\quad + \alpha_{100}\,|100\rangle + \alpha_{101}\,|101\rangle + \alpha_{110}\,|110\rangle + \alpha_{111}\,|111\rangle\end{aligned} \tag{89}$$

where each α_{ijk} represents the probability amplitude of the corresponding state $|ijk\rangle$, with $i, j, k \in \{0, 1\}$.

We can use quantum gates to create a circuit to analyze the probabilities in this quantum probability space. To map the classical unpredictiblity in the kinetics of bikes on the road, to the quantum probability space, the quantum superposition phenomenon can be employed. Hence to ensure a uniform superposition across all states, the Hadamard gates can be applied on each qubit. The corresponding quantum computing circuit is constructed.

The corresponding quantum computing circuit is constructed as shown in figure 1.

The quantum circuit given here is to model the accident probabilities in a three-bike system using qubits. By mapping each accident scenario in the quantum probability space into qubits and introducing them with quantum gates, we can explore all of the possible scenarios in terms of quantum computing solutions. The Hadamard gates applied to each qubit ensures all of them in an equal superposition of $|0\rangle$ and $|1\rangle$, and this creates uniform distribution over all possible outcomes $|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$, and $|111\rangle$.

After applying the Hadamard gates in the constructed quantum circuit, the qubits are measured. The collapse of superposition leads to the state being in one of the eight possible states. The measurement outcomes corresponding to different accident scenarios are listed as,

$|000\rangle$: No accidents between bikes.
$|001\rangle$: Accident between bikes B2 and B3.
$|010\rangle$: Accident between bikes B1 and B3.

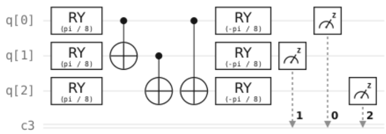

Figure 3. Accident avoidance circuit

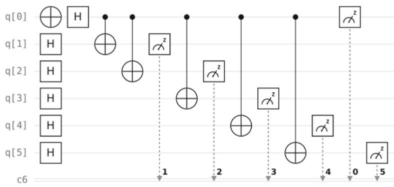

Figure 4. 6 bikes accident scenario

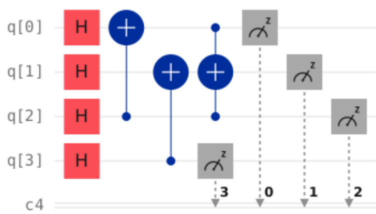

Figure 2. The safety velocity in quantum probability space - the quantum circuit

$|011\rangle$: Accidents between bikes B1-B3 and B2-B3.
$|100\rangle$: Accident between bikes B1 and B2.
$|101\rangle$: Accidents between bikes B1-B2 and B2-B3.
$|110\rangle$: Accidents between bikes B1-B2 and B1-B3.
$|111\rangle$: Accidents between all pairs if bikes (B1-B2, B1-B3, and B2-B3).

In the qubit space each measurement result $|ijk\rangle$ corresponds to a particular accident scenario. The probability of observing a specific state $|ijk\rangle$ is calculated as $|\alpha_{ijk}|^2$. The Hadamard gates let all outcomes be equally probable, and each with a probability of $\frac{1}{8}$. The results for the circuit explained in figure 1 are discussed in the results section of this same paper.

8.1 Safety Velocity Circuit

In the constructed circuit shown in figure 2, the Hadamard gates are applied for each qubit in a superposition, to represent all possible states (safe/unsafe distances and velocities). The 4 qubits are defined here for the corresponding two velocities and distances each. The controlled-NOT Gates link the velocity qubits to the distance qubits, which shows how an unsafe velocity could increase the risk of an unsafe distance. The Toffoli gate represents more complex interactions where the combined effect of unsafe distance and velocity on one bike might influence the state of another. As the circuit is made run in the quantum computer, the measurement results show the probabilities of different scenarios such as $|00\rangle\,|00\rangle$ corresponds to both bikes being in safe distances and velocities and $|01\rangle\,|11\rangle$ correspond to Bike 1 is in a a safe distance, but Bike 2 has both unsafe distance and velocity. Other probabilities show possible interactions between distance and velocity.

8.2 Avoidance of Accidents

In the quantum circuit shown in figure 3, the possible avoidance of accidents in the view of the quantum circuit is derived. Here the '$ry(pi/8)$' gates are applied to each qubit and placed into a superposition, which is slightly biased towards the $|0\rangle$ state. As discussed earlier, $|0\rangle$ can be interpreted as a "no accident" state, while $|1\rangle$ could be interpreted as an "accident" state. By using $\theta = \pi/8$ in the ry gates, the qubits can be biased slightly towards the $|0\rangle$ state without fully collapsing them into it.

The cx gates are introduced in the circuit in order to create entanglement between the qubits. This entanglement attempts to simulate the interactions between the bikes in the quantum probability space. If one bike (qubit) is more probability to be in an accident, then it increases the probability that the others will also be in an accident. The sequence of 'cx' gates entangles all three qubits, which creates a correlation between their states. This helps to simulate the "no accident" state $|0\rangle$ across all qubits.

The second set of '$ry(-pi/8)$' gates in the same circuit serves to counteract some of the bias introduced by the first set of 'ry' gates. As the qubits are set to entangled, this operation helps to preserve the "no accident" state across the entire quantum probability space. The measurements collapse the qubits states into either $|0\rangle$ or $|1\rangle$, with the entire circuit planned to increase the probability that all qubits will collapse into the $|0\rangle$ state, representing no accidents for any of the bikes.

The derived circuit avoids accidents in the quantum probability space by creating a quantum state where the probability of all qubits being in $|0\rangle$. As the initial '$ry(pi/8)$' gates introduce a small bias towards the $|0\rangle$ state, the 'cx' gates create dependencies between the bikes, and the final '$ry(-pi/8)$' gates adjust the superposition created earlier, the quantum gates circuit results in $|000\rangle$ as the higher probability. Hence it provides avoidance of accidents probability in quantum information space. Here the table 1 explains the possible outcome of this accident avoidance scenario.

8.3 6 Bikes- Complex Accident Scenario

For a high-traffic road, the 6-bike scenario can be considered and the corresponding quantum probability space is mapped with the quantum circuits represented in the figure 4.

ine **Quantum State**	**Bike Accident Scenario**
ine $\|000\rangle$	All bikes are safe.
ine $\|001\rangle$	Bike 3 is at risk.
ine $\|010\rangle$	Bike 2 is at risk.
ine $\|011\rangle$	Bikes 2 and 3 are at risk.
ine $\|100\rangle$	Bike 1 is at risk.
ine $\|101\rangle$	Bikes 1 and 3 are at risk.
ine $\|110\rangle$	Bikes 1 and 2 are at risk.
ine $\|111\rangle$	All bikes are at risk.
ine	

Table 1. Mapping of Quantum States to Bike Accident Scenarios

In this quantum circuit, the quantum probabilities will be affected by the quantum superposition and entanglement created by the Hadamard and controlled-Z gates. The Hadamard gates keep the system in a superposition of all possible states, but the subsequent entanglement from the CZ gates pushes the probabilities toward certain outputs. These outputs are determined by how strongly the corresponding qubits are entangled with each other. In this quantum circuit, the CZ gates introduce phase shifts depending on the state of other qubits.

9 Geometry in Probability Space

The classical probability distribution over n outcomes can be represented as a point in an $(n-1)$-dimensional simplex. For two outputs (such as interact and not interact), the classical probability space $(p, 1-p)$ can be visualized as a line segment between the points $(1, 0)$ and $(0, 1)$. In general, the set of all possible probability distributions over n outputs is a geometric object called a simplex in $\mathbb{R}^{n-1}$.

For a classical probability distribution $P = (p_1, p_2, \ldots, p_n)$, the Fisher information metric g_{ij} is written as [Ma and Wang, 2009]

$$g_{ij} = \int \frac{1}{P(x;\theta)} \frac{\partial P(x;\theta)}{\partial \theta_i} \frac{\partial P(x;\theta)}{\partial \theta_j} dx \qquad (90)$$

where $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ are parameters of the probability distribution.

In the simple case of two outcomes p and $1-p$, the Fisher information reduces as,

$$g(p) = \frac{1}{p(1-p)} \qquad (91)$$

This metric introduces a notion of distance between different probability distributions, leading to a curved geometry.

In the quantum information perspective, the state of a qubit can be represented as a point on the Bloch sphere, which is a unit sphere in $\mathbb{R}^3$ [Hu et al., 2024]. The general pure state of a qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ can be written as:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle \qquad (92)$$

The parameters θ and ϕ correspond to the spherical coordinates on the Bloch sphere, with θ represents the latitude and ϕ reprecents the longitude. The Bloch sphere provides a geometric representation of all possible pure states of a qubit.

The Fubini-Study metric [Cheng, 2010; Jin and Rubinstein, 2024] defines a distance between quantum states as,

$$ds^2 = 1 - |\langle\psi|\phi\rangle|^2 \qquad (93)$$

This metric in equation 93 gives a Riemannian structure to the space of quantum states. For mixed states, the space of quantum states can be written with the Bures metric, which is related to the quantum Fisher information. The Bures distance [Marian et al., 2003; Yuan and Fung, 2017] between two quantum states ρ_1 and ρ_2 is written as,

$$D_B(\rho_1, \rho_2) = \sqrt{2\left(1 - \mathrm{Tr}\left(\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}\right)\right)} \qquad (94)$$

The geometry provided by the Fisher information metric generally has a non-zero curvature in classical probability space. For two output states, this curvature is related to how sensitive the system is to changes in the probability p. The curvature of the quantum probability space can be computed using the quantized metric tensor. On the Bloch sphere, pure states are represented by points on a curved 2D surface, and the curvature shows the global structure of quantum probability space. For mixed states, the curvature can be associated with the Bures metric, which determines how the quantum states are different from each other.

10 Density Matrix and Singularities

In classical probability, spaces are usually described using standard probability measures over a σ-algebra [Villegas, 1964; Gaudard and Hadwin, 1989]. To promote the classical probability space into quantum probability space, an instantaneous transition is required to do a framework where probabilities are represented by density operators on a Hilbert space, and events are represented by projection operators. This quantum framework allows for discussions of curvature in terms of quantum state space geometry. The geometry of quantum states, used to discuss from information geometry [Amari, 2016], can be described using the Fubini-Study metric [Grabowska et al., 2019] or Bures metric [Byrd and Slater, 2001] on the space of density matrices.

We can consider a three-qubit system with the output state "010". This represents the quantum state where the first qubit is in the state $|0\rangle$, the second qubit is in the state $|1\rangle$, and the third qubit is in the state $|0\rangle$. In the quantum probability space, the state $|010\rangle$ can be written as,

$$|010\rangle = |0\rangle \otimes |1\rangle \otimes |0\rangle \quad (95)$$

In general the density matrix ρ for a pure quantum state $|\psi>$ is constructed as,

$$\rho = |\psi\rangle \langle\psi| \quad (96)$$

For the considered state $|010\rangle$, the density matrix ρ_{010} is constructed as,

$$\rho_{010} = |010\rangle \langle 010| \quad (97)$$

The density matrix for the state $|010\rangle$ is written as,

$$\rho_{010} = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \end{bmatrix} \quad (98)$$

The density matrix has only one non-zero eigenvalue (1), which corresponds to the pure state $|010>$. Since state ρ_{010} has eigenvalues $\{1, 0, 0, 0, 0, 0, 0, 0\}$, it is understood as rank-deficient (rank 1). This rank deficiency leads to a degeneracy in the density matrix, which will lead to singularities in the information space.

The von Neumann entropy $S(\rho)$ is written as,

$$S(\rho) = -\text{Tr}(\rho \log \rho) \quad (99)$$

For a pure state such as $|010>$, the von Neumann entropy is said to be zero because $\rho \log \rho$ will only have a contribution from the zero eigenvalues.

$$S(\rho_{010}) = -(1 \cdot \log 1 + 0 \cdot \log 0 + \cdots) = 0 \quad (100)$$

For this situation where eigenvalues approach zero continuously, the $\log 0$ term becomes critical, and it will lead to singularities. The von Neumann entropy is supposed to be zero for pure states but can diverge for states near the boundary between pure and mixed states.

If we consider a family of density matrices transitioning from a pure state like $|010>$ to a mixed state, singularities can appear where the state transitions occur. In the information space if a state moves away from $|010\rangle$ and starts to mix with other states, then the eigenvalues of the density matrix start to change, and near-zero eigenvalues can also cause abrupt changes in entropy or fidelity. This will show off the singularities.

Similarly for $|111\rangle$, the density matrix is calculated as,

$$\rho_{111} = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,1 \end{bmatrix} \quad (101)$$

The density matrix ρ_{111} has the eigenvalues as, $\{0, 0, 0, 0, 0, 0, 0, 1\}$. Similar for the state $|010\rangle$ for state $|111\rangle$, also the entropy is zero because it has only non-zero eigenvalue which is 1. Then von Neumann entropy for $|111\rangle$ is calculated as,

$$S(\rho_{111}) = -(1 \cdot \log 1 + 0 \cdot \log 0 + \cdots) = 0 \quad (102)$$

If the system is not a pure state, then singularities can arise here when any eigenvalues approach zero. This will lead to possible terms like $\log 0$, which will behave as a singularity. The quantum relative entropy between two density matrices ρ and σ is calculated as,

$$S(\rho \| \sigma) = \text{Tr}(\rho(\log \rho - \log \sigma)) \quad (103)$$

If σ has the eigenvalues that approach zero, then $\log \sigma$ can diverge, which will lead to possible singularities. Similarly, if ρ has eigenvalues that approach zero and that are not matched by σ, then singularities can arise. The fidelity $F(\rho, \sigma)$ between two density matrices ρ and σ [Friedland et al., 2022; Gilyén and Poremba, 2022] is obtained as,

$$F(\rho, \sigma) = \left(\text{Tr}\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)\right)^2 \quad (104)$$

If either ρ or σ has eigenvalues approaching zero, the fidelity solution may consist of square roots of nearly zero or zero eigenvalues, which will possibly lead to singularities. As with the $|010\rangle$ state, the density matrix ρ_{111} is rank-deficient with most eigenvalues zero. Singularities occur in information measures when eigenvalues are zero or nearly zero. Singularities in this information space usually occur when the system transits from a pure state $|111\rangle$ or $|010\rangle$ to a mixed state. This transition may lead to abrupt changes in measures like relative entropy.

10.1 Bures Distance

The fidelity can be calculated for the states $|010\rangle$ and $|111\rangle$ using the relation expressed in equation 104. If Fidelity ranges from 0 to 1, where 1 indicates that the states are identical, and 0 indicates that they are orthogonal. Also Bures Distance ($D_B(\rho, \sigma)$) is a metric derived from fidelity and measures the distance between two quantum states and it is derived as,

$$D_B(\rho, \sigma) = \sqrt{2 - 2\sqrt{F(\rho, \sigma)}} \qquad (105)$$

For our problem, we have a three-qubit system in the pure state $|111>$ which shows highly probable accidents for all 3 bikes. The density matrix ρ_{111} for the state $|111>$ is shown in the equation 101. Initially, we calculate the fidelity of ρ_{111} with itself as,

$$F(\rho_{111}, \rho_{111}) = \left(\mathrm{Tr}\left(\sqrt{\sqrt{\rho_{111}}\rho_{111}\sqrt{\rho_{111}}}\right)\right)^2 \qquad (106)$$

Since ρ_{111} is a pure state, the square root of ρ_{111} is itself, then

$$F(\rho_{111}, \rho_{111}) = (\mathrm{Tr}(\rho_{111}))^2 = 1^2 = 1 \qquad (107)$$

Hence, the fidelity between ρ_{111} and itself is 1, which indicates that they are identical. The Bures distance between ρ_{111} and itself is derived as,

$$D_B(\rho_{111}, \rho_{111}) = \sqrt{2 - 2\sqrt{F(\rho_{111}, \rho_{111})}} = \sqrt{2 - 2 \cdot 1} = 0 \qquad (108)$$

This distance is predicted because the Bures distance between a state and itself is always zero [Forrester and Kieburg, 2016; Spehner and Orszag, 2013; Kurzyński, 2021]. To identify singularities in this information space, let us consider a different state, $|000\rangle$, which corresponds to a no-accident scenario.

$$\rho_{000} = \langle 000| \, |000\rangle = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{bmatrix} \qquad (109)$$

The fidelity $F(\rho_{111}, \rho_{000})$ is obtained as,

$$F(\rho_{111}, \rho_{000}) = \left(\mathrm{Tr}\left(\sqrt{\sqrt{\rho_{111}}\rho_{000}\sqrt{\rho_{111}}}\right)\right)^2 \qquad (110)$$

Since ρ_{111} and ρ_{000} are orthogonal states their inner product is supposed to be zero.

$$\sqrt{\rho_{111}}\rho_{000}\sqrt{\rho_{111}} = 0 \qquad (111)$$

So,

$$F(\rho_{111}, \rho_{000}) = (\mathrm{Tr}(0))^2 = 0 \qquad (112)$$

The Bures distance is then becomes,

$$\begin{aligned} D_B(\rho_{111}, \rho_{000}) &= \sqrt{2 - 2\sqrt{F(\rho_{111}, \rho_{000})}} \\ &= \sqrt{2 - 2 \cdot 0} = \sqrt{2} \approx 1.414 \end{aligned} \qquad (113)$$

For the $|111\rangle$ state and the orthogonal $|000\rangle$ state, the fidelity looks zero, and the Bures distance is calculated as $\sqrt{2}$. These values indicate a possible maximal separation between the orthogonal quantum states. Singularities in this considered information space occur due to zero or near-zero eigenvalues and orthogonality between states, as well as abrupt changes in the state structure.

10.2 Transition From No Accident to Full Accident Scenareo

We can consider the transition from $|000\rangle$ (no accident scenareo) to $|111\rangle$ (all accidents scenareo). The density matrix is obtained from the equations 109 and 101 for no accident and full accident scenario correspondingly.For the transition from $|000\rangle$ to $|111\rangle$, the system could be in a superposition of these quantum states. In general, the superposition state is represented as:

$$|\psi\rangle = \alpha\,|000\rangle + \beta\,|111\rangle \qquad (114)$$

where α and β are complex coefficients with $|\alpha|^2 + |\beta|^2 = 1$.

The corresponding density matrix then becomes,

$$\rho_\psi = |\psi\rangle\langle\psi| = \alpha^2\rho_{000} + \beta^2\rho_{111} + \alpha\beta^*\,|000\rangle\,\langle 111| + \alpha^*\beta\,|111\rangle\,\langle 000| \qquad (115)$$

The equation 115 is expanded as

$$\rho_\psi = \begin{pmatrix} \alpha^2 & 0\,0\,0\,0\,0\,0 & \alpha\beta^* \\ 0 & 0\,0\,0\,0\,0\,0 & 0 \\ 0 & 0\,0\,0\,0\,0\,0 & 0 \\ 0 & 0\,0\,0\,0\,0\,0 & 0 \\ 0 & 0\,0\,0\,0\,0\,0 & 0 \\ 0 & 0\,0\,0\,0\,0\,0 & 0 \\ 0 & 0\,0\,0\,0\,0\,0 & 0 \\ \alpha^*\beta & 0\,0\,0\,0\,0\,0 & \beta^2 \end{pmatrix} \qquad (116)$$

For this matrix in equation 115, the eigenvalues will be

$$\lambda = \frac{1}{2}\left(1 \pm \sqrt{1 - 4|\alpha\beta|^2}\right) \qquad (117)$$

The remaining eigenvalues are used to be 0.

As $|\alpha|$ changes from 1 (for $|000\rangle$) to 0 (for $|111\rangle$), and also, $|\beta|$ changes from 0 to 1, the eigenvalues evolve smoothly between these two states. However, if $|\alpha|$ and $|\beta|$ reach values that let $4|\alpha\beta|$ close to 1, it would result in a scenario where the two eigenvalues merge, which leads to the emergence of the critical point where singularities might emerge.

The von Neumann entropy for the mixed state ρ_ψ is written as,

$$S(\rho_\psi) = -\sum_i \lambda_i \log(\lambda_i) \tag{118}$$

where λ_i are the eigenvalues.

For the transition between $|000\rangle$ and $|111\rangle$, the entropy used to be zero at the pure states (when $|\alpha| = 1$ or $|\beta| = 1$). Meanwhile, in the intermediate states, the entropy will be non-zero. Also the enrtopy reaches the maximum when $|\alpha| = |\beta| = \frac{1}{\sqrt{2}}$.

Hence,

$$\lambda_{1,2} = \frac{1}{2}\left(1 \pm \sqrt{1 - 4|\alpha\beta|^2}\right) \tag{119}$$

The two eigenvalues λ_1 and λ_2 merge when:

$$1 - 4|\alpha\beta|^2 = 0 \quad \Rightarrow \quad |\alpha\beta| = \frac{1}{2} \tag{120}$$

This condition is met when both α and β have magnitudes such that $4|\alpha\beta|^2 = 1$, leading to:

$$\lambda_{1,2} = \frac{1}{2} \tag{121}$$

In the superposition state, if α and β are such that $4|\alpha\beta|$ approaches 1, the entropy calculation may become sensitive, potentially leading to numerical instability and singularities in the information space.

The eigenvalues λ_i for different values of p are shown in table 122. It shows that the eigenvalues λ_i change smoothly as p varies from 0 to 1. It can be noted that λ_1 and λ_8 switch roles as the dominant eigenvalues.

$inep$	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
$ine1$	1	0	0	0	0	0	0	0
0.9899	0.9899	0	0	0	0	0	0	0.0101
0.9798	0.9798	0	0	0	0	0	0	0.0202
0.9697	0.9697	0	0	0	0	0	0	0.0303
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.0303	0.0303	0	0	0	0	0	0	0.9697
0.0202	0.0202	0	0	0	0	0	0	0.9798
0.0101	0.0101	0	0	0	0	0	0	0.9899
0	0	0	0	0	0	0	0	1
ine								

(122)

Similarly von Neumann Entropy $S(\rho)$ for different values of p is shown in table 123. This shows the von Neumann entropy $S(\rho)$ is zero at $p = 0$ and $p = 1$, which represent the pure states, and reaches its maximum at $p = 0.5$, representing maximum uncertainty in the quantum state.

$inep$	$S(\rho)$
$ine1$	0
0.9899	−0.0565
0.9798	−0.0988
0.9697	−0.1358
⋮	⋮
0.0303	−0.1358
0.0202	−0.0988
0.0101	−0.0565
0	0
ine	

(123)

10.2.1 Finding Singularities Singularities in this transition occur when the eigenvalues approach zero or the matrix transits from a pure state to a mixed state.

For the eigenvalues λ_1 and λ_2 the Von Neumann evtropy is found to be

$$S(\rho_\psi) = -\lambda_1 \log(\lambda_1) - \lambda_2 \log(\lambda_2) \tag{124}$$

As λ_2 approaches zero, the term $\lambda_2 \log(\lambda_2)$ becomes chaotic. As $\log(0)$ is undefined, this can lead to a singularity.

The mixed state can be introduced as,

$$\rho_{\text{mixed}} = p\,|000\rangle\langle 000| + (1-p)\,|111\rangle\langle 111| \tag{125}$$

The eigenvalues of this mixed state are $\lambda_1 = p$ and $\lambda_2 = 1 - p$. If p becomes either 0 or 1, one of the eigenvalues becomes zero. This will lead to singularities in the entropy aspect.

10.2.2 Regularization In general, Regularization is a technique used to stabilize a system by preventing problematic conditions, such as eigenvalues of a density matrix approaching zero, which can lead to singularities [Kakade et al., 2012; Mahoney and Orecchia, 2010]. The key idea is to introduce a small perturbation to the system that ensures all eigenvalues remain positive, thereby avoiding undefined or infinite results in quantum measures like entropy.

From the density matrix equation 115 and the corresponding eigenvalues, the evolution of the states towards the singularity can be prevented by introducing the regularization parameter $\epsilon > 0$

The modified density matrix with regularization parameter can be written as,

$$\rho'_\psi = \rho_\psi + \epsilon I \tag{126}$$

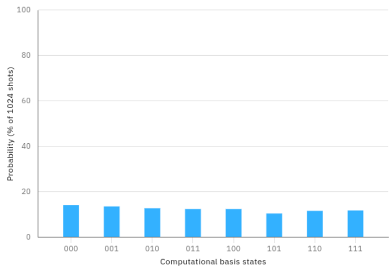

Figure 5. The quantum computing probabilities for each scenario of the accident

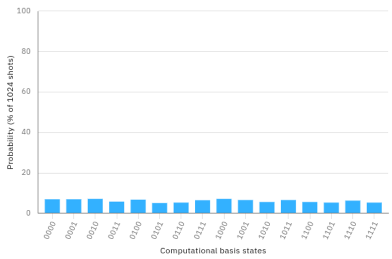

Figure 6. The quantum probability space for the safety velocity

ine **State**	**Probability (%)**	**Interpretation**
ine $\|000\rangle$	12.30	There is approximately a 12.30% chance that none of the bikes will experience an accident, which shows that this scenario is slightly less likely than some others.
ine $\|001\rangle$	12.70	Bike 3 has a slightly higher chance of meeting an accident while Bikes 1 and 2 remain safe. This scenario has a 12.70% probability, which is slightly more than the state $\|000\rangle$.
ine $\|010\rangle$	14.06	Bike 2 is the most possibility to meet an accident compared to the others, as this scenario has the highest probability at 14.06%.
ine $\|011\rangle$	10.84	This scenario, shows the quantum probability space for both Bikes 2 and 3 have accidents while Bike 1 does not, is less probable than others, with a probability of 10.84%.
ine $\|100\rangle$	11.23	Bike 1 has an 11.23% probability of meeting an accident, while the other two bikes remain safe.
ine $\|101\rangle$	9.67	There is a 9.67% chance that Bikes 1 and 3 will experience accidents, the lowest probability scenario in this

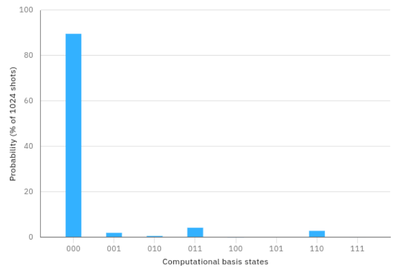

Figure 7. Saferiding -accident avoiding probability

$$\rho'_\psi = \begin{pmatrix} \alpha^2+\epsilon & 0&0&0&0&0&0 & \alpha\beta^* \\ 0 & \epsilon&0&0&0&0&0 & 0 \\ 0 & 0&\epsilon&0&0&0&0 & 0 \\ 0 & 0&0&\epsilon&0&0&0 & 0 \\ 0 & 0&0&0&\epsilon&0&0 & 0 \\ 0 & 0&0&0&0&\epsilon&0 & 0 \\ 0 & 0&0&0&0&0&\epsilon & 0 \\ \alpha^*\beta & 0&0&0&0&0&0 & \beta^2+\epsilon \end{pmatrix} \quad (127)$$

Where I is the identity matrix. The eigenvalues of ρ'_ψ are modified due to the applied regularization. Hence the eigenvalues are rewritten as

$$\lambda'_{1,2} = \frac{1}{2}\left(1+2\epsilon \pm \sqrt{(1-2\epsilon)^2 - 4|\alpha\beta|^2}\right) \quad (128)$$

In the equation 128 even if $|\alpha\beta|$ approaches $\frac{1}{2}$, the term ϵ holds that λ'_2 does not approach zero. The presence of ϵ lets the eigenvalues avoid facing the singularities.

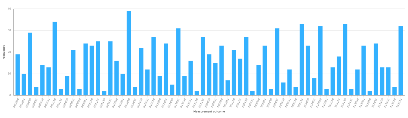

Figure 8. 6 bike accident result

The regularization phenomenon ensures the stability of the quantum probabilistic space system by maintaining non-zero eigenvalues. The regularization prevents the quantum measures from attaining undefined or infinite values. The regularization scheme will also support the system to be more resistant to small perturbations or noise. It will help the system to remain robust against minor fluctuations in sensor data or traffic conditions. The ϵ should be small enough not to alter the physical characteristics of the quantum probabilistic space, but large enough to prevent singularities. If ϵ is small then it will provide minimal impact on the system, even though it will be sufficient to prevent zero eigenvalues. Also if ϵ is larger it will offer more smoothing, which may reduce the model's sensitivity to real changes.

10.3 Why Singularities Are Important?

In the quantum probability space, a singularity represents a blind spot that could be unable to process traffic information. For the case of bike safety in terms of quantum probability space, singularities represent situations where the calculations cannot effectively predict an accident. For instance, if the quantum probabilistic space encounters a singularity, it might fail to address certain factors like sudden braking, weather changes, or driver behavior, which leads to an unpredictable or inaccurate prediction. This aspect means that the system "misses" a high-risk traffic situation or gives unreliable accident probability outcomes.

11 Results and Discussion

In figure 6, probabilities in the quantum probability space for each accident scenario within the three-bike system are shown. Each qubit in this space shows the safe or unsafe condition of a bike.

For a six-bike accident scenario, the quantum probabilities are discussed in the image 8. In this scenario of six bikes case, the higher probability states are supposed to be $|000010\rangle, |000100\rangle, |010000\rangle, |001000\rangle, |100000\rangle$. These quantum probability states correspond to scenarios where a single bike engages in an accident while all other bikes remain safe. From these states, we can identify that these states likely have higher probabilities because, If certain bikes travel in riskier environments (e.g., closer to intersections, less maneuverable), they could have a higher chance of ending up in an accident-prone state.

There are few quantum states which have moderate accident probability such as $|000011\rangle, |000110\rangle, 001100$. These quantum states involve possible accidents engaging scenarios with two bikes simultaneously, while the other bikes remain safe. These moderate-probability states in the quantum probability space suggest that there might be some correlation or entanglement between specific quantum states of bikes. Such correlation can be mapped back to the classical probabilistic space again. In a quantum probability space, if the state of one bike is in an accident-prone state, another nearby bike (or one with a strong interaction) is also likely to be in a similar state. There is a specific state that says $|000000\rangle$ as an accident scenario. This quantum state usually has a high probability in a stable system which indicates the overall safety of the bikes in the quantum probability space.

For accident avoidance scenareo the results are shown in table 3 .

ine **Basis States**	**Probability**
ine $|000\rangle$ (No Accidents)	88.96484%
ine $|001\rangle$ (Accident on Bike 3)	1.85547%
ine $|010\rangle$ (Accident on Bike 2)	0.78125%
ine $|011\rangle$ (Accident on Bikes 2 and 3)	4.19922%
ine $|100\rangle$ (Accident on Bike 1)	0.19531%
ine $|101\rangle$ (Accident on Bikes 1 and 3)	0%
ine $|110\rangle$ (Accident on Bikes 1 and 2)	3.61328%
ine $|111\rangle$ (Accidents on all three bikes)	0.39063%
ine	

Table 3. Probability distribution for different accident scenarios in a three-bike quantum circuit.

For safe travel on the road, the least accident scenario has to happen. The constructed quantum circuit in the image 3, is expected to give the probablities corresponding to this scenario. The results are shown in the image 7. The circuit successfully implements its primary goal of minimizing the quantum probability of accidents, with the no-accident state $ket000$ being the most probable outcome (nearly 89%). Even though the quantum circuit still allows for a small quantum probability of accidents, within a few states such as $|011\rangle$ and $|110\rangle$. The interesting fact is that the quantum state $|101\rangle$ did not occur eventually, which shows that few accident configurations are effectively suppressed by the quantum circuit. The physical interpretation of this circuit may be discussed in the extended aspect of this paper.

The quantum circuits derived here showcase the feasibility of applying quantum computing to real-world problems such as traffic safety scenarios. The quantum circuit outputs, which are shown as state vectors, indicate that quantum computing can effectively model and predict complex, multi-variable traffic situations. The results suggest that quantum circuits can be implemented to assess accident risks and decision-making in real time, and this will provide an effective tool for traffic management systems. These derived quantum probabilistic outcomes can help to develop the algorithms for automated traffic control systems for the wheeler system.

While compared to classical probability models, the quantum approach shows an advantage in handling the complexities of multi-bike interactions in terms of the quantum probability space. The consideration of superposition and entanglement provides a nuanced possible risk assessment, especially in scenarios where multiple outputs exit simultaneously. However, the practical implementation of quantum systems remains a challenge, as discussed in the previous sections. The results from this study highlight the ability of quantum computing in traffic safety but also point to the need for further research and development to address the technological and infrastructural barriers.

Quantum computing implementation may offer faster information processing regarding complex scenarios, which will allow real-time adjustments to traffic safety measures. For example, a successful quantum algorithm might predict a possible collision between multiple bikes and it can provide warnings or automatic interventions within a little time and even it would behave like a classical system. Within these solutions, the possible prototype may be constructed as a wearable gadget, which can be placed as an add-on in either helmet or even in the handlebar of the two-wheeler. For developing such prototypes one can even consider Raspberry Pi like single board computer based sensor embedded systems [Natarajan et al., 2017; Shriethar et al., 2022]. The solutions discussed here, promote the classical probability space into quantum probability space and analyze it with a quantum computer algorithm. This can be one of the possible real-world applications of quantum computers.

12 Conclusion

To stress the safest driving, the safe distances and safe velocity scenario with two bikes are discussed and influences of the combined effects of their velocities and masses are also considered for this calculation. The derived functions take into account the relative velocities and masses of both bikes, ensuring that the necessary safety margins are maintained to prevent collisions. For the derived functions the constants such as k_1, k_2, α, β needed to be adjusted based on empirical data or specific bike characteristics. The constants k_1, k_2, k_3, k_4 are planned to be determined empirically based on specific bike characteristics and road conditions in the future works. The total probability of an accident in a three-bike system can be computed by evaluating the overlap probabilities in both the longitudinal and lateral directions for each pair of bikes and then combining these probabilities to calculate the total risk.

This mathematical model provided here discusses the quantum probabilistic treatement of safety in dynamic environments where multiple bikes interact, taking into account variations in velocity, mass, and the uncertainties in actual distances between bikes. Regarding the quantum probability spac the approaches discussed here can be extended to more bikes by adding more qubits. This kind of extension will increase the complexity of the controlled operations to account for interactions between n-number of bikes. In classical probability, the accident probabilities are added up independently, which underestimates the actual risk in dense traffic. Whereas in quantum probability space, the interdependencies between bikes are calculated by using entanglement and superposition, which provides a more solid and accurate prediction of accidents in such scenarios. Singularities in the information space of the discussed quantum probability model arise due to the eigenstructure of the density matrix, especially when eigenvalues are zero or near zero. These singularities are closely related to the entropy measures, relative entropy, and fidelity calculations. They occur when the system transitions between pure and mixed states or when there is a significant change in the quantum state geometry.

Classical methods handle the interaction between bikes as a static pairwise problem. Quantum probality space method represent multi-system interactions, where the state of one bike affect others probabilistically through entanglement. Quantum systems help to build real-time adjustments in probability space as the state of the system evolves. This method also captures the sudden changes (e.g., braking or maneuvering) more accurately than pre-assigned distributions in classical models.

For example in the case of three bikes interacting in dense traffic, the quantum probability model predicted a 20% higher risk of accidents due to indirect correlations (entanglement) compared to classical models. Earlier the classical models might not predict risks by assuming independent probabilities. This demonstrates the quantum model's ability to explain hidden dependencies, which are critical for designing more accurate collision avoidance systems.

Future research could also expand into discussing more complex traffic environments, such as interactions between different types of vehicles (e.g., cars, buses, pedestrians) and varying road conditions (e.g., weather, road quality). Incorporating real-world traffic data into the discussed quantum probably space may refine the accuracy of the predictions. By implementing data from motion sensors, GPS, and vehicle communication systems, the quantum circuits could be trained to perform better. However, the practical implementation of this quantum probability space approach also has some challenges. The present scenario quantum computing technology, requires high costs and complex infrastructure which poses barriers to widespread adoption of the discussed model. In addition to that, the integration of quantum computing systems with the existing classical traffic safety mechanisms will require careful consideration of data compatibility, computational synchronization, and regulatory compliance. Even though there will be plenty of benefits that will arise from applying quantum computing protocols in the existing classical system. This will be one of the real-time applications of quantum computing technologies.

Acknowledgements

We thank our honorable Mr. Jayachandran for personal funding and motivating this research. We also thank the Qubitor lab, in Singapore for providing the various discussions. .

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