BOUNDS FOR COMPACT INVARIANT SETS OF ONE SYSTEM ARISEN IN STUDIES OF PLASMA DYNAMICS MODELS

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Abstract

In this paper we show how to compute bounds for a compact domain which contains all compact invariant sets of one sixdimensional system describing plasma dynamics. Using the first order extremum conditions we obtain formulas for the localization bounds by using several quadratic and rational localizing functions. In addition, by exploiting some rational functions we demonstrate how to refine this localization with help of a removal of some pieces from the localization domain. Conditions of global stability are presented. Results of numerical simulation illustrating the localization domain for the chaotic attractor are provided.

Key words

Modeling, Simulation,Localization

1 Introduction

In this work we compute a localization domain containing all compact invariant sets for one six- dimensional system of polynomial differential equations which describes dynamics of low frequency, long wave-length electromagnetic fields in a non- uniform magnetised plasma:

$$\begin{aligned}
\dot{x}_1 &= -\sigma_1 x_1 + \sigma_1 x_2 + \delta x_2 x_3 \qquad (1)\\
\dot{x}_2 &= -x_1 x_3 + \gamma_1 x_1 - x_2\\
\dot{x}_3 &= x_1 x_2 - \beta_1 x_3\\
\dot{x}_4 &= -\sigma_2 x_4 + \sigma_2 x_5 + \delta_1 x_5 x_6 + \delta_2 x_5 x_3\\
\dot{x}_5 &= -x_4 x_6 + \gamma_2 x_4 - \gamma_3 x_5 + \delta_4 x_3 x_4\\
\dot{x}_6 &= x_4 x_5 - \beta_2 x_6.
\end{aligned}$$

This system was derived in the paper [Banerjee,Saha and Chowdhury, 2001] from some couple of partial differential equations presented in [Shukla,Birk,Dreher and Stenflo, 1996].

The problem of finding a localization set is of interest both in theoretical studies and for practical needs. Reasons for this are as follows. Equations of plasma dynamics may exhibit chaotical behaviour for some values of parameters, see [Banerjee,Saha and Chowdhury, 2001], which leads to a necessity of studies its long-time dynamics. Further, finding a localization set K is important for narrowing the simulation experiment of dynamics of (1) onto K in case of systems with a complex behaviour and/or of high dimensions. Both of these features we meet in case of the system (1).

The physical significance of compact invariant sets of any differentiable right-side system is related to the fact that they may carry information about a long- time behavior of the system, both in the negative and the positive time. Any globally bounded motion of the system is contained in one of compact invariant sets. Further, the existence or the nonexistence of periodic orbits expresses the fact of the presence or the lack correspondingly of repeatable behavior.

The localization problem of compact invariant sets containing in a given domain has been studied intensively for nonlinear multidimensional continuous-time systems during last years, see papers [Krishchenko and Starkov, 2006], [Krishchenko and Starkov, 2007], [Starkov, 2009], [Starkov, 2009] and others. The key idea of the approach described in these papers consists in using the first order extremum conditions.

The physical sense of parameters of this system is given in paper [Banerjee,Saha and Chowdhury, 2001]; according with this all parameters of the system (1) are assumed to be positive throughout this paper. We point out that studies of a location of all compact invariant sets of some systems of plasma dynamics have been already realized in case of the Pikovsky- Rabinovich-Trakhtengerz system describing dynamics of plasma instability, see [Krishchenko and Starkov, 2007].

Based on the fact that our system may be expressed as a cascade connection of two three-dimensional systems of specific types we derive a compact domain

$K(h_1;\rho) \cap K(h_2)$ being an intersection of two elliptic cylinders containing all compact invariant sets of our system. This compact domain is described in terms of parameters of the system. The key idea of our approach is presented in [Krishchenko and Starkov, 2006]. Further, we define the localization domain $K(h_1;\rho) \cap K(h_2)$ by applying some other localizing functions. One global asymptotic stability condition is given. This paper is the reworked and enlarged version of the paper [Starkov and Gamboa, 2011] In addition, in the complete version of the manuscript we illustrate localization results of a chaotic attractor by numerical simulation.

2 Preliminaries

Let us consider a quadratic system

$$\dot{x} = F(x) = Ax + f(x), \tag{2}$$

i.e. a system in which f is a homogeneous quadratic vector field; A is a constant $(n \times n)$- matrix; $x \in \mathbf{R}^n$. Let $h(x)$ be a differentiable function such that h is not the first integral of (2). By $h|_B$ we denote the restriction of h on a set $B \subset \mathbf{R}^n$. By $S(h)$ we denote the set $\{x \in \mathbf{R}^n \mid L_F h(x) = 0\}$. Let Q be an open set in $\mathbf{R}^n$. Next, we introduce $h_{\inf}(Q) := \inf\{h(x) \mid x \in S(h) \cap Q\}$; $h_{\sup}(Q) := \sup\{h(x) \mid x \in S(h) \cap Q\}$. If $Q = \mathbf{R}^n$ then we simply write $h_{\inf} = h_{\inf}(\mathbf{R}^n); h_{\sup} = h_{\sup}(\mathbf{R}^n)$. We shall use

Theorem 1 : Each compact invariant set Γ of (2) is contained in the localization set $K(h) = \{h_{\inf} \leq h(x) \leq h_{\sup}\}$, see e.g. [Krishchenko and Starkov, 2006].

The function h applied here is called localizing. It is evident that if all compact invariant sets are located in sets N_1 and N_2, with $N_1; N_2 \subset \mathbf{R}^n$, then they are located in the set $N_1 \cap N_2$ as well.

3 Ellipsoidal localization of the first three plasma equations

As it follows from the structure of our system, the first three equations

$$\begin{aligned} \dot{x}_1 &= -\sigma_1 x_1 + \sigma_1 x_2 + \delta x_2 x_3 \\ \dot{x}_2 &= -x_1 x_3 + \gamma_1 x_1 - x_2 \\ \dot{x}_3 &= x_1 x_2 - \beta_1 x_3 \end{aligned} \tag{3}$$

does not depend on the last three equations. However, the last three equations may be considered as a system with a single input which is the variable $x_3(t)$. Therefore we shall construct a localization domain in two steps. In this Section we find an ellipsoid containing all compact invariant sets of (3). Then in next Sections we improve this localization domain. Further, using the refined localizing bounds for the function $x_3(t)$ we derive the localization for all compact invariant sets for the complete system (1).

By applying the localizing function

$$h_1(x) = \frac{x_1^2}{2} + \frac{(\delta+\mathbf{1})x_2^2}{2} + \frac{1}{2}\left(x_3 - [\gamma_1(\delta+\mathbf{1})+\sigma_1]\right)^2 \tag{4}$$

and using the Lagrange multiplier method we can derive the following result:

Proposition 2 : For the generic system (3) with positive coefficients we have:
1) the localization bound is given by the formula

$$h_{1\,\sup} := \frac{[\gamma_1(\delta+\mathbf{1})+\sigma_1]^2}{2};$$

2) all compact invariant sets are contained in the solid ellipsoid

$$K(h_1;r) = \left\{ \begin{array}{c} \frac{x_1^2}{2} + \frac{(\delta+\mathbf{1})x_2^2}{2} + \\ \frac{1}{2}\left(x_3 - [\gamma_1(\delta+\mathbf{1})+\sigma_1]\right)^2 \leq r := h_{1\,\sup} \end{array} \right\}$$

in $\mathbf{R}^3 = \{(x_1,x_2,x_3)^T\}$;

3) further, for any positive ε the solid ellipsoid $K(h_1;r+\varepsilon)$ is a positively invariant domain.

4 Refining the localization $K(h_1;r)$ by using rational functions

In papers [Krishchenko and Starkov, 2007], [Starkov, 2009]. It was proposed to use specially constructed rational functions in order to remove some pieces of a localization domain. Here this idea is applied to the first three equations of (1).

Let us take the function

$$h_2 = \frac{x_3}{x_1^2 + \delta x_2^2}$$

Then we get that the set $S(h_2)$ is defined by

$$h_2 \,|_{S(h_2)}\, \frac{\rho_1(x_1,x_2)}{x_1^2+\delta x_2^2} = \frac{x_1 x_2}{x_1^2+\delta x_2^2} \leq \frac{1}{2\sqrt{\delta}},$$

with the quadratic form $\rho_1(x_1,x_2) = (\beta_1 - 2\sigma_1)x_1^2 + (\beta_1\delta - 2\delta)x_2^2 + 2(\sigma_1+\delta\gamma_1)x_1x_2$.

Applying the condition

$$\begin{gathered} \beta_1 > 2\sigma_1 \\ \delta(\beta_1 - 2\sigma_1)(\beta_1 - 2) > (\sigma_1+\delta\gamma_1)^2 \end{gathered}$$

we have that $\rho_1(x_1,x_2)$ is positive definite and

$$\left|h_3 \mid_{S(h_3)}\right| \leq \frac{1}{2\sqrt{\delta}\lambda_{\min}(\rho_1)},$$

where $\lambda_{\min}(\rho_1)$ is the minimal eigenvalue of ρ_1 :

$$\begin{aligned}
\lambda_{\min}(\rho) &= \frac{1}{2}(\lambda_{1,\min}(\rho) + \lambda_{2,\min}(\rho)); \\
\lambda_{1,\min}(\rho) &= \beta_1 - 2\sigma_1 + \beta_1\delta - 2\delta; \\
\lambda_{2,\min}(\rho) &= \sqrt{[\lambda_{1,\min}(\rho)]^2 - \lambda_{3,\min}(\rho)}; \\
&\text{and} \\
\lambda_{3,\min}(\rho) &= 4[(\beta_1\delta - 2\delta)(\beta_1 - 2\sigma_1) - (\sigma_1 + \delta\gamma_1)^2].
\end{aligned}$$

Thus we come to the localization set

$$K(h_2) := \{\mid \frac{x_3}{x_1^2 + \delta x_2^2} \mid \leq \frac{1}{2\sqrt{\delta}\lambda_{\min}(\rho_1)}\}$$

which gives

$$K(h_2) \cap K(h_1; r) \subset \{\frac{x_3}{x_1^2 + \delta x_2^2} \leq \frac{1}{2\sqrt{\delta}\lambda_{\min}(\rho_1)}\}.$$

Below we propose another rational localizing function $h_3(x)$:

$$h_3 = \frac{x_3}{x_1^2 + (\delta+1)x_2^2 + x_3^2}$$

We compute that the set $S(h_3)$ is defined by

$$\begin{aligned}
h_3 \mid_{S(h_3)} \frac{\rho_2(x_1, x_2, x_3)}{x_1^2 + (\delta+1)x_2^2 + x_3^2} &= \\
\frac{x_3}{x_1^2 + (\delta+1)x_2^2 + x_3^2} &\leq \frac{1}{2\sqrt{(\delta+1)}},
\end{aligned}$$

with $\rho_2(x_1, x_2, x_3) = (\beta_1 - 2\sigma_1)x_1^2 + (\delta+1)(\beta_1 - 2)x_2^2 - \beta_1 x_3^2 + 2[\sigma_1 + (\delta+1)\gamma_1]x_1x_2$. Applying the condition

$$\begin{aligned}
\beta_1 &> 2\sigma_1 \\
[\sigma_1 + (\delta+1)\gamma_1]^2 &> (\beta_1 - 2\sigma_1)(\delta+1)(\beta_1 - 2)
\end{aligned}$$

we have that ρ_2 is positive definite and

$$\left|h_3 \mid_{S(h_3)} \frac{\rho_2(x_1, x_2, x_3)}{x_1^2 + (\delta+1)x_2^2 + x_3^2}\right| \leq \frac{1}{2\sqrt{(\delta+1)}}.$$

Now if we denote by $\lambda_{\min}(\rho_2)$ the minimal eigenvalue of ρ_2 then we have

$$\left|h_3 \mid_{S(h_3)}\right| \leq \frac{1}{2\lambda_{\min}(\rho_2)\sqrt{\delta+1}},$$

where

$$\begin{aligned}
\lambda_{\min}(\rho_2) &= \frac{1}{2}(\lambda_{1,\min}(\rho_2) + \lambda_{2,\min}(\rho_2)); \quad (5) \\
\lambda_{1,\min}(\rho_2) &= \beta_1 - 2\sigma_1 + (\delta+1)(\beta_1 - 2) \\
\lambda_{2,\min}(\rho_2) &= \sqrt{[\lambda_{1,\min}(\rho_2)]^2 - \lambda_{3,\min}(\rho_2)}
\end{aligned}$$

where

$$\lambda_{3,\min}(\rho_2) = 4[(\beta_1 - 2)(\beta_1 - 2\sigma_1)(\delta+1) - [\sigma_1 + (\delta+1)\gamma_1]^2]; \quad (6)$$

Thus we come to the localization set $K(h_3)$ defined by the inequality

$$\mid \frac{x_3}{x_1^2 + (\delta+1)x_2^2 + x_3^2} \mid \leq \frac{1}{2\sqrt{\delta+1}\lambda_{\min}(\rho_2)}$$

which gives

$$K(h_3) \cap K(h_1; r) \subset \{\frac{x_3}{x_1^2 + \delta x_2^2} \leq \frac{1}{2\sqrt{\delta+1}\lambda_{\min}(\rho_2)}\}.$$

5 Localization by additional localizing functions

Here we describe how to get closer to compact invariant sets of the system (3) by finding additional localizing functions.

1. By using a localizing function

$$h_4 = \frac{1}{2}x_1^2 + \frac{\delta}{2}x_2^2 - (\sigma_1 + \gamma_1\delta)x_3$$

we obtain that the set $S(h_4)$ is defined by the equation

$$x_3 = \frac{\sigma_1}{(\sigma_1 + \gamma_1\delta)\beta_1}x_1^2 + \frac{\delta}{(\sigma_1 + \gamma_1\delta)\beta_1}x_2^2.$$

As a result, we come to the following conclusion: $h_{4\,\inf} = 0$, with $\frac{1}{2} > \frac{\sigma_1}{\beta_1}$ and $\frac{\delta}{2} > \frac{\delta}{\beta_1}$; $h_{4\,\sup} = 0$, with $\frac{\sigma_1}{\beta_1} > \frac{1}{2}$ and $\frac{\delta}{\beta_1} > \frac{\delta}{2}$; $h_4 \mid_{S(h_4)} = 0$, with $\beta_1 = 2$; $\sigma_1 = 1$.

Hence, if $(\beta_1 - 2\sigma_1)^2 + \delta^2(\beta_1 - 2)^2 > 0$ then we obtain the localization set $K(h_4)$ in the following equations

$$\left\{ \begin{array}{c} \frac{1}{2}x_1^2+\frac{\delta}{2}x_2^2-(\sigma_1+\gamma_1\delta)x_3\geq 0; \\ \frac{1}{2}\geq\frac{\sigma_1}{\beta_1} \quad \text{and} \quad \frac{\delta}{2}\geq\frac{\delta}{\beta_1} \end{array} \right\};$$
$$\left\{ \begin{array}{c} \frac{1}{2}x_1^2+\frac{\delta}{2}x_2^2-(\sigma_1+\gamma_1\delta)x_3\leq 0; \\ \frac{1}{2}\leq\frac{\sigma_1}{\beta_1} \quad \text{and} \quad \frac{\delta}{2}\leq\frac{\delta}{\beta_1} \end{array} \right\}.$$

Besides, we have the localization set in a form of a quadratic surface

$$K(h_4):=\left\{ \begin{array}{c} \frac{1}{2}x_1^2+\frac{\delta}{2}x_2^2-(\sigma_1+\gamma_1\delta)x_3=0; \\ \beta_1=2;\sigma_1=1 \end{array} \right\}.$$

2. Now by applying the quadratic localizing function

$$h_5=-\frac{1}{2\sigma_1}x_1^2+\frac{1}{2\gamma_1}x_2^2+\frac{1}{2}\left(\frac{\sigma_1+\gamma_1\delta}{\sigma_1\gamma_1}\right)x_3^2$$

we get that the set $S(h_5)$ is given by

$$\xi=x_1^2=\frac{1}{\gamma_1}x_2^2+\left(\frac{\sigma_1+\gamma_1\delta}{\sigma_1\gamma_1}\right)\beta_1x_3^2.$$

Hence, if $(\sigma_1-1)^2+(\sigma_1-\beta_1)^2>0$ then we obtain the localization set

$$K(h_5):=\left\{ \begin{array}{c} -\frac{1}{2\sigma_1}x_1^2+\frac{1}{2\gamma_1}x_2^2+\frac{1}{2}\left(\frac{\sigma_1+\gamma_1\delta}{\sigma_1\gamma_1}\right)x_3^2\leq 0, \\ \left(1-\frac{1}{\sigma_1}\right)\leq 0 \quad \text{and} \quad \left(1-\frac{\beta_1}{\sigma_1}\right)\leq 0 \end{array} \right\};$$

$$K(h_5):=\left\{ \begin{array}{c} -\frac{1}{2\sigma_1}x_1^2+\frac{1}{2\gamma_1}x_2^2+\frac{1}{2}\left(\frac{\sigma_1+\gamma_1\delta}{\sigma_1\gamma_1}\right)x_3^2\geq 0, \\ \left(1-\frac{1}{\sigma_1}\right)\geq 0 \quad \text{and} \quad \left(1-\frac{\beta_1}{\sigma_1}\right)\geq 0\} \end{array} \right\}.$$

Besides, we have a localization set in a form of a quadratic surface

$$K(h_5):=\left\{ \begin{array}{c} -\frac{1}{2\sigma_1}x_1^2+\frac{1}{2\gamma_1}x_2^2+\frac{1}{2}\left(\frac{\sigma_1+\gamma_1\delta}{\sigma_1\gamma_1}\right)x_3^2=0, \\ \text{with} \quad \sigma_1=\beta_1=1. \end{array} \right\};$$

3. Here we apply yet another quadratic localizing function

$$h_6=-\frac{(\gamma_1+1)}{2\sigma_1}x_1^2+\frac{1}{2}x_2^2+\frac{\psi}{2}x_3^2+x_3,$$

with $\psi=\frac{\sigma_1+\delta(\gamma_1+1)}{\sigma_1}$. Then the set $S(h_6)$ is defined by

$$x_1^2=\frac{1}{(\gamma_1+1)}\left[x_2^2+\psi\beta_1x_3^2+\beta_1x_3\right].$$

As a result, we come to the localization set $K(h_6)$ given by

$$\left\{ \begin{array}{c} \frac{(\gamma_1+1)}{2\sigma_1}x_1^2-\frac{1}{2}x_2^2-\frac{\psi}{2}x_3^2-x_3\leq \\ \frac{(2\sigma_1-\beta_1)^2}{8\psi\sigma_1(\sigma_1-\beta_1)}, \quad \text{with} \quad 1>\frac{1}{\sigma_1},\sigma_1>\beta_1 \end{array} \right\};$$
$$\left\{ \begin{array}{c} \frac{(\gamma_1+1)}{2\sigma_1}x_1^2-\frac{1}{2}x_2^2-\frac{\psi}{2}x_3^2-x_3\geq \\ \frac{(2\sigma_1-\beta_1)^2}{8\psi\sigma_1(\sigma_1-\beta_1)}, \quad \text{with} \quad 1<\frac{1}{\sigma_1},\sigma_1<\beta_1 \end{array} \right\}.$$

4. Now we apply the localizing function $h_7=x_3$. Then the set $S(h_7)$ is given by $\beta_1x_3=x_1x_2$. Therefore

$$h_7\mid_{S(h_7)\cap K(h_1;r)}\leq\frac{2r}{\beta_1\sqrt{\delta+1}}$$

and we have the localization set

$$K(h_7)=\{\mid x_3\mid\leq\frac{2r}{\beta_1\sqrt{\delta+1}}\}$$

which may be improved with respect to x_3 for the values

$$\beta_1>\frac{r}{\sqrt{\delta+1}\left[\gamma_1(\delta+\mathbf{1})+\sigma_1\right]}$$

as

$$K(h_1;r)\cap K(h_7)\subset\{0\leq x_3\leq x_{3,\max}:=\frac{2r}{\beta_1\sqrt{\delta+1}}$$

6 Compact localization of the (1) plasma equations

In this section we derive some compact localization domain for the system (1) in the form of intersection of two solid ellipsoids. In order to realize this idea we notice that the system (1) can be considered as two cascade- connected systems. The first one is the first three equations of the (1) system and the second one is the last three equations of the (1) system. Below we shall exploit the localization condition

$$0\leq x_3\leq x_{3,\max}.$$

Let us apply the function:

$$h_8(x)=x_4^2+(\delta_1+1)x_5^2+x_6^2+qx_6, \qquad (7)$$

with parameter q to be defined below. Then we derive that

$$-L_f h_8(x) = 2\sigma_2 x_4^2 + 2(\delta_1+1)\gamma_3 x_5^2 + A_1(q)x_4x_5 + A_2x_3x_4x_5 + 2\beta_2 x_6^2 + q\beta_2 x_6,$$

with $A_1(q) := 2\sigma_2 + 2(\delta_1+1)\gamma_2 + q$; $A_2 := 2\delta_2 + 2\delta_1\delta_4 + 2\delta_4$.
Thus the set $S(h_8) \cap \{0 \le x_3 \le x_{3,\max}\}$ is contained in the set M defined by the inequality

$$\frac{q^2}{8\beta_2} \ge 2\sigma_2(\mid x_4 \mid + \frac{A_1(q)+x_{3,\max}A_2}{4\sigma_2} \mid x_5 \mid)^2 + [2(\delta_1+1)\gamma_3 - \frac{(A_1(q)+x_{3,\max}A_2)^2}{8\sigma_2}]x_5^2 + 2\beta_2(x_6 + \frac{q}{4\beta_2})^2$$

provided q is such that $A_1(q) \ge 0$.
As a result, we obtain
Proposition 3: Suppose that q is chosen in a such way that $A_1(q) \ge 0$ and

$$A_3(q) := 2(\delta_1+1)\gamma_3 - \frac{(A_1(q)+x_{3,\max}A_2)^2}{8\sigma_2} > 0.$$

Then M is a solid ellipsoid in $\mathbf{R}^3 = \{(x_4, x_5, x_6)^T\}$.
From this fact we come to the main assertion stated as
Theorem 4 : In conditions of the last Proposition we establish that all compact invariant sets of the system (1) are contained in the set $K(h_1; r) \cap K(h_8)$, with $K(h_8) = \{h_8 \le h_{8\,\sup}\}$, and the value for $h_{8\,\sup}$ is estimated by

$$R_4 := \frac{q}{4\sqrt{\beta_2\sigma_2}} + \frac{A_1(q)+x_{3,\max}A_2}{8\sigma_2}\frac{q}{\sqrt{2\beta_2 A_3(q)}};$$
$$R_5 := \frac{q}{2\sqrt{2\beta_2 A_3(q)}};$$
$$h_{2\,\sup} \le R_4^2 + (\delta_1+1)R_5^2 + \frac{q^2}{4\beta_2^2}.$$

7 Conditions of global asymptotic stability

By using Lyapunov functions and the cascade structure of the system (1) one may derive conditions of global stability of (1). To this end let us apply the Lyapunov candidate function

$$V = \frac{x_1^2}{2} + \frac{(\delta+1)}{2}x_2^2 + \frac{x_3^2}{2},$$

$$\dot{V} = -\sigma_1 x_1^2 + x_1x_2[\sigma_1 + \gamma_1(\delta+1)] - (\delta+1)x_2^2 - \beta_1 x_3^2$$

and get that $\dot{V}$ is negative definite if it satisfies the condition

$$\delta + 1 > \frac{[\sigma_1 + \gamma_1(\delta+1)]^2}{4\sigma_1}. \tag{8}$$

Now let us consider another Lyapunov candidate function

$$V_1 = \frac{x_4^2}{2} + \frac{(\delta_1+1)}{2}x_5^2 + \frac{x_6^2}{2}.$$

Then we derive that

$$\dot{V}_1 = -\sigma_2\left[x_4 - \frac{(\rho_1+\rho_2x_3)}{2\sigma_2}x_5\right]^2 - x_5^2\left[(\delta_1+1)\gamma_3 - \frac{(\rho_1+\rho_2x_3)^2}{4\sigma_2}\right] - \beta_2x_6^2,$$

with $\rho_1 = \sigma_2 + \gamma_2(\delta_1+1)$ and $\rho_2 = \delta_2 + \delta_4(\delta_1+1)$.
Suppose, in addition to (8), that if $x \in K(h_1, \rho) \cap K(h_2)$ then its component x_3 is satisfied to the inequality

$$4\sigma_2(\delta_1+1)\gamma_3 > (\rho_1+\rho_2x_3)^2. \tag{9}$$

Now we notice that by construction of $K(h_1, \rho) \cap K(h_2)$ each trajectory enters into the compact invariant domain $K(h_1, \rho) \cap K(h_2)$ and remains there. Hence $\dot{V}_1(x) < 0$ for $x \in K(h_1, \rho) \cap K(h_2)$, $x \ne 0$, and conclude that the system (1) is globally asymptotically stable under (8)-(9).
Now we get that if

$$2\sqrt{\sigma_2(\delta_1+1)\gamma_3} - \rho_1 > 2\rho_2[\gamma_1(\delta+1)+\sigma_1] \tag{10}$$

then (9) holds in $K(h_1, \rho) \cap K(h_2)$. So we have
Proposition 5 : Assume that parameters of (1) and parameter q are such that $A_1(q) > 0$, $A_3(q) > 0$ and (8) and (10) hold. Then (1) is globally asymptotically stable.

8 Conclusions

In this paper we give results concerning a locus of all compact invariant sets of one sixdimensional system derived by Banerjee and others. This system describes dynamics of low- frequency, long wave-length electromagnetic fields propagating in the inhomogeneous magnetised plasma. The domain containing all compact invariant sets is bounded and defined by several quadratic surfaces in the explicit way in terms of parameters of this system. The shape of this localization domain is complex and may contain two holes which are computed explicitly as well. In this study we compute the compact localization domain with help of a representation of the sixdimensional system (1) as a cascade connection of two threedimensional subsystems with a single input $x_3(t)$ going to the second subsystem. One global asymptotic stability condition is given. As an example, we demonstrate that in case of chaotic values of parameters found in [Banerjee,Saha and Chowdhury, 2001] we can efficiently localize a chaotic attractor. The localizing bounds computed in this work are useful not only for numerical experiments aimed for studies of this plasma dynamics model but also may be useful for constructing control laws in chaotic regimes.

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