Abstract
The paper deals with the active and reactive power control of voltage source converters (VSC) for distributed generation. The control method explained is based on the instantaneous power theory. The power converter regulates the DC-Bus voltage and also controls the reactive power injected to the grid. The presented control strategy is evaluated by simulation in Matlab-Simulink.

Key words
Distributed Generation, Converter control, PLL, Current loop, qd0 Transform

1 Introduction
The rapid increment of renewable power generation is changing the power system shape, and creating new power generation concepts as distributed generation. Distributed generation can be defined, according to [Ackermann, Andersson and Söder, 2001], as an electric power source connected directly to the distribution network or on the consumer side of the meter. On the other hand, CIGRE working group 37-23 defines it as low rating generation that is neither planned nor dispatched centrally and is usually connected to the distribution network [CIGRE working group 37-23, 1999]. Therefore, from the main ideas of both definitions, distributed generation can be understood as low power generation that is connected close to the distribution network. The power generation technologies involved in distributed generation include, for example, wind turbines, small hydro turbines, combined heat and power (CHP) units, also known as cogeneration, fuel cells and photovoltaics (PV) cells.

In order to enhance the quality of the power injected to the grid by the units of distributed generation, a power converter is used between the grid and the generator enabling the system to be regulated. Different control systems have been studied in order to provide different grid supports or ride through faults capability, e.g., frequency support [Hirodontis, Anaya-Lara, Burt and McDonald, 2009][Karlsson, Björnstedt and Ström, 2005], to guarantee power quality [Hornik and Zhong, 2009], voltage dips [Wang, Duarte and Hendrix, 2009], etc. The present paper studies and presents an active and reactive power control of a converter to enable the system to improve the quality of the power delivered.

This paper is organized as follows. In Section II, the system under study is described. The Clarke and Park Transforms are presented in Section III. In section IV, the instantaneous power theory is briefly introduced. The control scheme is developed in section V. In section VI, the proposed control is validated by means of simulation and discussed. Finally, the conclusions are summarized in Section VII.
to the sinusoidal components and its variability in the time-frame.
Therefore, in order to facilitate the control design of three-phase inverters the qd0 reference frame is used. In the case of balanced three-phase circuits, the application of the Park Transform to three AC variables reduces to two DC variables, due to the 0-component in balanced conditions is zero. Thus, simplified calculations can be carried out on these transformed DC variables. Three-phase AC components are recovered with the inverse transform.

3.1 Clarke Transformation
The Clarke Transformation allows to change the three dimension vector into two dimension vector (abc-frame to \(\alpha\beta\)-frame).

\[
\begin{pmatrix}
    x_\alpha \\
    x_\beta
\end{pmatrix} = \frac{2}{3} \begin{pmatrix}
    1 & -\frac{1}{2} & -\frac{1}{2} \\
    0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix} \begin{pmatrix}
    x_a \\
    x_b \\
    x_c
\end{pmatrix}
\] (1)

This transformation can be understood as the projection of the three axis onto a stationary two-axis reference frame, as it is shown in the figure 2. It is important to remark that it is possible to expand the variable change matrix 1 to a 3x3 matrix 2, including a 0 component (called homopolar component), which is orthogonal to \((\alpha, \beta)\) axis and usually a constant value (zero if the system is balanced). In this way the transformation is bijective.

\[
\begin{pmatrix}
    x_q \\
    x_d \\
    x_0
\end{pmatrix} = \frac{2}{3} \begin{pmatrix}
    1 & -\frac{1}{2} & -\frac{1}{2} \\
    0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix} \begin{pmatrix}
    x_a \\
    x_b \\
    x_c
\end{pmatrix}
\] (2)

3.2 Park Transformation
The Park Transformation describes a rotation of an orthogonal system \((\alpha, \beta)\) to \((q, d)\). The rotation is represented in the figure 3.

\[
\begin{pmatrix}
    x_q \\
    x_d \\
    x_0
\end{pmatrix} = \begin{pmatrix}
    -\sin(\theta) & \cos(\theta) & 0 \\
    \cos(\theta) & \sin(\theta) & 0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    x_\alpha \\
    x_\beta \\
    x_0
\end{pmatrix}
\] (3)

Again, the rotation can be expressed as a 3x3 matrix including the rotation orthogonal axis, which is the same axis in \((\alpha, \beta, 0)\) and \((q, d, 0)\) references, although a vector rotation on a constant plain is expressed as a 2x2 matrix.
3.3 qd0 Transformation
The qd0 Transformation includes the Clark and Park Transformations, combining both transformations in one matrix. Hence, the Park Transformation with an angle \( \theta \) of a vector \( x^{abc} \in \mathbb{R}^3 \) in the abc frame is given by

\[
x^{qd0} = T(\theta)x^{abc}
\]

with

\[
T(\theta) = \frac{2}{3} \begin{pmatrix}
cos(\theta) & cos(\theta - \frac{2\pi}{3}) & cos(\theta + \frac{2\pi}{3}) \\
sin(\theta) & sin(\theta - \frac{2\pi}{3}) & sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\]

Then it can be said that the vector \( x^{qd0} \) is the vector \( x^{abc} \) in the dq0 frame on the angle \( \theta \) reference.

Since the matrix \( T(\theta) \) is invertible; the inverse transformation can be obtained as

\[
x^{abc} = T^{-1}(\theta)x^{qd0}
\]

with

\[
T^{-1}(\theta) = \begin{pmatrix}
cos(\theta) & sin(\theta) & 1 \\
cos(\theta - \frac{2\pi}{3}) & sin(\theta - \frac{2\pi}{3}) & 1 \\
cos(\theta + \frac{2\pi}{3}) & sin(\theta + \frac{2\pi}{3}) & 1
\end{pmatrix}
\]

In conclusion, the qd0 transform can be interpreted as the projection of the abc-frame variables onto a rotating two-axis reference frame.

4 Instantaneous power theory
Throughout the paper, the instantaneous power theory for balanced three-phase systems under sinusoidal conditions developed by Akagi [Akagi, Watanabe and Areedes, 2007] is considered. A balanced three-phase system under sinusoidal conditions is defined as a three-phase system with equal amplitudes and \(2\pi/3\) rad \((120^\circ)\) the angle between all the phases.

Therefore, the instantaneous voltages and currents in abc frame can be written as:

\[
x_a = \sqrt{2}Xcos(\omega t + \phi) \\
x_b = \sqrt{2}Xcos(\omega t + \phi - 2\pi/3) \\
x_c = \sqrt{2}Xcos(\omega t + \phi + 2\pi/3)
\]

Electrical theory defines 3 different types of electrical power: Apparent (S), Active (P) and Reactive (Q). The Apparent power in abc frame is expressed as: \( S = V \cdot I^* = P + jQ \)

Where \( V \) and \( I \) are the phasor (phase vector) representation of the voltage and the current.

Applying the Clarke and Park transformations described previously, the active and reactive power can be expressed as:

\[
P = \frac{3}{2}(v_a i_a + v_b i_b)
\]

\[
Q = \frac{3}{2}(v_a i_b - v_b i_a)
\]

in the \((\alpha,\beta)\) frame and

\[
P = \frac{3}{2}(v_q i_q + v_d i_d)
\]

\[
Q = \frac{3}{2}(v_q i_d - v_d i_q)
\]

in the \((q,d)\) frame.

5 Converter Control
The purpose of the control of the power converter is to regulate the reactive power and DC bus voltage which regulates active power. The DC bus voltage and the reactive power references determine the current references which determine the voltages to be applied on the grid side.

Figure 4 depicts the block diagram of grid side control. The reference currents are calculated by means of independent variables as active and reactive power set-points, that is explained in the current loops subsection. Furthermore, by means of the qd0 Transformation, explained above, the control change abc-frame currents to qd-currents, to compare them with the reference currents. The angle necessary, to calculate the Park Transformation and to synchronize the system with the grid, is obtained by means the PLL, that is explained later, from the grid voltage.
to obtain the desired reference. In this case, it is interesting to set the reference where voltage d-component is zero.

To find this angle it is usually used a phase locked loop (PLL). A basic scheme of a three-phase PLL, as it is shown in 5, consists in a comparison of the voltage d-component with a constant zero, the obtained error is regulated by a PI controller. The output of the controller is the grid frequency. The grid angle is obtained by integration of the frequency. The grid angle is introduced in the Park Transform to calculate the qd voltage components.

The design of the controller parameters is detailed by S.K. Chung [Chung, 2000]

\[ i_{DCP} = \frac{2}{3} \cdot \frac{P_{DCp}}{v_{zq}} \]  
\[ i_{DCI} = \frac{2}{3} \cdot \frac{P_{DCI}}{v_{zq}} \]  

(15a)  
(15b)

The control loop scheme is shown in Figure 6; The design of both proportional and integral parameters of the PI controller is explained in [Junyent-Ferrè, 2007].

5.2 Reference calculation

The q-axis may be aligned to the grid voltage, allowing active and reactive decoupled control. To control the reactive power, a \( i_{dq} \) reference is computed as:

\[ i_{dq} = \frac{2}{3} \cdot \frac{Q_{E}}{v_{zq}} \]  

(12)

The active power, which is responsible for the evolution of the dc bus voltage, is controlled by the \( i_{dq} \). A linear controller is usually designed to control the dc bus voltage and to keep it constant.

\[ i_{dq} = \frac{2}{3} \cdot \frac{P_{E}}{v_{zq}} \]  

(13)

The DC link voltage behavior is defined (in Laplace domain) by the following equation

\[ E_{DC} = E_{DC0} + \frac{1}{s \cdot C} (i_{DCI} - i_{DCP}) \]  

(14)

where the current values \( i_{DCP} \) and \( i_{DCI} \) can be calculated as

\[ i_{DCP} = \frac{\frac{v_{abc}}{E} \cdot v_{abc}^{*}}{E} = \frac{P_{DCp}}{E} \]  
\[ i_{DCI} = \frac{\frac{v_{abc}^{*}}{E} \cdot v_{abc}^{*}}{E} \]  

(15a)  
(15b)

5.4 Current Controller Tuning

The controller is designed by means of the so-called internal model control (IMC) theory [Harnefors and Nee, 1998]. The parameters of a PI controller, with the goal of obtain a desired bandwidth \( \alpha \), which appear due to a low-pass filter included by the IMC, are:

\[ K_{p} = \alpha \cdot L_{t} \]  
\[ K_{i} = \alpha \cdot r_{i} \]  

(18)

6 Simulation Results

The system, previously described, is simulated with Matlab-Simulink. The control of the converter is evaluated in two different scenarios: a change in the active power
reference and a change in the reactive power reference. Also both PLL and qd0 Transformation response are evaluated to show how properly they work.

6.1 PLL & qd0 Transform
In the Figure 7, it can be observed that the Phase Locked Loop (PLL) presented above has a good behavior, reaching quickly the desired value. It is also interesting to note that the PLL controller behaves as a first order system. So, it satisfies the plant definition that appears in [Chung, 2000], where is related the PI controller parameters with the period of the system.

Figure 7. Dynamical response of the Phase Locked Loop system

In the Figure 8, can be observed an example of three-phase voltages in abc and what shape they take after the Park Transform. It is important to be noted that the chosen angle, which is given by the PLL controller, is the right one because the voltages in qd frame are constant values.

Figure 8. Application of Qd0 Transform to a sinusoidal balanced three-phase voltage

6.2 Variation of Active Power
The first scenario presented is a change of the active power reference. Ensuring the capability of deliver variable active power, it is required due to the variability of the power demand from the consumers. In this scenario the reactive power reference is set in zero.

In the Figure 9 can be seen the active power reference varying with steps, and how the active power delivered to the grid reaches quickly this new setting.

Figure 9. Plot of the change of the active power

Again, as it is shown in the Figure 10, the dc bus voltage is following the defined value. The peaks appear on the graphic because of the control dynamics, which acts during the reference value variations.

Figure 10. Variation of the DC Bus Voltage driven by the active power change

In Figures 11 and 12, it is plotted both q and d components, either of real and reference currents. The difference between them is the input of the IMC to calculate the voltage values. It can be seen that the q-component of the current presents the same behavior of the active power profile, its calculation is driven by the expression (13). In the same way, the d-component which is guide by the expression (12) has a constant zero value due to the reactive power is set zero.

6.3 Change of Reactive Power
The second scenario presented is the change of the reactive power reference. Regulation of the reactive power delivery allows the system to keep constant the voltage in the connection point. In this scenario, the active power reference is set in 1MW.

In the Figure 13, it can be seen the reactive power reference change, which as in the former case has a step behavior. As it is expected the reactive power delivered
In the Figures 15 and 16, it is plotted the q and d components of the currents. It can be seen that the d-component of the current presents a behavior similar to the reactive power profile. However, in this case, the q-component has not a constant value as it would be expected, since the active power demanded is still the same. Then, as the voltage of the network remains constants, it is needed a change in the injected current.

In the Figure 14, the dc bus voltage is maintained at constant value. Again, as in the first scenario, the voltage presents peaks due to the reactive power changes, that affects the active power sent to the network.

7 Conclusions

This paper has presented an active and reactive power control for converters in distributed generation systems. The control of these converters demands the manipulation of three-phase sinusoidal variables, which makes difficult to compute the control variables. To simplify the design Clark and Park transformations have been used. In this transformed frame of reference, the control design can be cast as the regulation of DC variables in two decoupled control loops based on simple proportional-integral-derivative (PID) controllers. Simulation of the control methodology is presented in order to illustrate the transformation of the variables and also the ability of the system to regulate the active and reactive power.
References
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