A MINIMAX TERMINAL CONTROL PROBLEM IN THE HIERARCHICAL DISCRETE-TIME DYNAMICAL SYSTEM WITH INCOMPLETE INFORMATION

Department of Optimal Control,
Institute of Mathematics & Mechanics UB RAS,
The Center of Economic Safety,
Institute of Economics UB RAS
Russia
shorikov@usue.ru

Abstract

In this paper we consider the discrete-time dynamical system consisting of several controlled objects with incomplete information which has two levels of control. Under investigation for this dynamical system we propose the mathematical formalization in the form of realization of two-level hierarchical minimax program terminal control problem with incomplete information and propose the general scheme for its solving.

Key words

Hierarchical discrete-time dynamical system, incomplete information.

1 Introduction

In this paper we consider the discrete-time dynamical system consisting of several controlled objects with incomplete information which has two levels of control. One level (or first level) is dominating and other level (or second level) is subordinate and both have different criteria of functioning and united a priori by determined information and control relations. We formulate a minimax program terminal control problem with incomplete information for processes in this two-level hierarchical discrete-time dynamical system and propose a general scheme for its solving. The results obtained in this paper are based on [Krasovskii, 1968]–[Bazaraa and Shetty, 1979] and can be used for computer simulation and for designing of optimal digital controlling systems for actual technical, robotics, economic, and other multilevel control processes. Mathematical models of such systems had considered, for example, in [Chernousko, 1994]–[Tarbouriech and Garcia, 1997].

2 Description of the problem

On a given integer time interval $\overline{0,\mathrm{T}} = \{0, 1, \cdots, \mathrm{T}\}$ $(\mathrm{T} > 0)$ we consider a controlled multistep dynamical system consisting of $(n+1)$ objects ($n \in \mathbf{N}$, where $\mathbf{N}$ is the set of all natural numbers). The motion of object I which is a general object and controlled by the dominating player P is described by the following linear discrete-time recurrent vector equation

$$y(t+1) = A(t)y(t) + B(t)u(t)+$$

$$+C(t)v(t) + D(t)\xi(t),\ y(0) = y_0, \qquad (1)$$

and the motion of object II_i $(i \in \overline{1,n})$ which is a subsidiary object corresponding to index i and controlled by the subordinate player E_i is described by the following linear discrete-time recurrent vector equation

$$z^{(i)}(t+1) = A^{(i)}(t)z^{(i)}(t) + B^{(i)}(t)u(t)+$$

$$+C^{(i)}(t)v^{(i)}(t) + D^{(i)}(t)\xi^{(i)}(t),\ z^{(i)}(0) = z_0^{(i)}. \qquad (2)$$

Here, $t \in \overline{0,\mathrm{T}-1}$; $y \in \mathbf{R}^r$ and $z^{(i)} \in \mathbf{R}^{s_i}$ are the phase vectors of the objects I and II_i, respectively ($r, s_i \in \mathbf{N}$; for $k \in \mathbf{N}$, $\mathbf{R}^k$ is the k-dimensional Euclidean space of column vectors); $u(t) \in \mathbf{R}^p$ and $v^{(i)}(t) \in \mathbf{R}^{q_i}$ are the control vectors (the controls) of the players P and E_i, respectively, restricted by the given constraints

$$u(t) \in \mathrm{U}_1,\ v^{(i)}(t) \in \mathrm{V}_1^{(i)};$$

$$\mathrm{U}_1 \subset \mathbf{R}^p,\ \mathrm{V}_1^{(i)} \subset \mathbf{R}^{q_i}\ (p,\ q_i \in \mathbf{N}); \qquad (3)$$

where the sets U_1 and $\mathrm{V}_1^{(i)}$ are finite sets in the spaces $\mathbf{R}^p$ and $\mathbf{R}^{q_i}$, respectively; the control vector $v(t)$ has form $v(t) = (v^{(1)}(t), v^{(2)}(t), \cdots, v^{(n)}(t))' \in \mathbf{R}^q$ $(q = \sum_{i=1}^{n} q_i)$; $\xi(t) \in \mathbf{R}^l$ and $\xi^{(i)}(t) \in \mathbf{R}^{l_i}$ are the vectors of non-controlling parameters (the noises or the simulation errors) of the objects I and II_i, respectively, restricted by the following given constraints

$$\xi(t) \in \Xi_1,\ \xi^{(i)}(t) \in \Xi_1^{(i)};$$

$$\Xi_1 \in \mathrm{comp}(\mathbf{R}^l),\ \Xi_1^{(i)} \in \mathrm{comp}(\mathbf{R}^{l_i})\ (l,\ l_i \in \mathbf{N}), \quad (4)$$

where for any $k \in \mathbf{N}$, $\mathrm{comp}(\mathbf{R}^{\mathbf{k}})$ is the set of all compact subsets of the space $\mathbf{R}^k$; for all $i \in \overline{1,n}$ and $t \in \overline{0,\mathrm{T}-1}$, $A(t), A^{(i)}(t), B(t), B^{(i)}(t), C(t), C^{(i)}(t), D(t)$ and $D^{(i)}(t)$ are real matrices of dimensions $(r \times r), (s_i \times s_i), (r \times p), (s_i \times p_i), (r \times q), (s_i \times q_i), (r \times l)$ and $(s_i \times l_i)$, respectively, and each from matrices $A(t)$ and $A^{(i)}(t)$ have inverse matrices $[A(t)]^{-1}$ and $[A^{(i)}(t)]^{-1}$, respectively; the rank of matrix $D(t)$ is equal to l, the dimension of vector $\xi(t)$, and the rank of matrix $D^{(i)}(t)$ is equal to l_i, the dimension of vector $\xi^{(i)}(t)$, and Ξ_1 and $\Xi_1^{(i)}$ are convex polyhedrons in the spaces $\mathbf{R}^l$ and $\mathbf{R}^{l_i}$, respectively (here and below, a convex polyhedron is convex cover of finite set of vectors in the corresponding Euclidean vector space).

We also assume that for all instants $t \in \overline{0,\mathrm{T}}$ the phase vectors $y(t)$ and $z^{(i)}(t)$ of the objects I and II_i $(i \in \overline{1,n})$, respectively, combined with the initial conditions in equations (1), (2) are restricted by the given following constraints

$$y(t) \in \mathrm{Y}_1,\ z^{(i)}(t) \in \mathrm{Z}_1^{(i)};$$

$$\mathrm{Y}_1 \in \mathrm{comp}(\mathbf{R}^r),\ \mathrm{Z}_1^{(i)} \in \mathrm{comp}(\mathbf{R}^{s_i}), \qquad (5)$$

where Y_1 and $\mathrm{Z}_1^{(i)}$ are convex polyhedrons in the spaces $\mathbf{R}^r$ and $\mathbf{R}^{s_i}$, respectively.

The control process in the discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

In the field of interests of the player P are both the admissible phase vectors of the object I and of each object II_i $(i \in \overline{1,n})$, and for any instant $\tau \in \overline{1,\mathrm{T}}$ and any time interval (simply interval) $\overline{0,\tau} \subseteq \overline{0,\mathrm{T}}$ $(0 < \tau)$ he knows the collection: $y(\tau)$ is the phase vector of object I at instant τ; $u_\tau(\cdot) = \{u(t)\}_{t\in\overline{0,\tau-1}}$ is the past realizations of his control on the interval $\overline{0,\tau}$ $(\forall\ t \in \overline{0,\tau-1} : u(t) \in \mathrm{U}_1)$; $v_\tau^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t\in\overline{0,\tau-1}}$ is the past realizations of the control of player E_i $(i \in \overline{1,n})$ on the interval $\overline{0,\tau}$ $(\forall\ t \in \overline{0,\tau-1} : v^{(i)}(t) \in \mathrm{V}_1^{(i)})$; $\omega_\tau^{(i)}(\cdot) = \{\omega^{(i)}(t)\}_{t\in\overline{0,\tau}}$ $(\omega^{(i)}(t) \in \mathbf{R}^{m_i};\ m_i \in \mathbf{N}, m_i \le s_i)$ is the past realizations of the information signal S_i on the interval $\overline{0,\tau}$, and its values $\omega^{(i)}(t)$ $(\omega^{(i)}(0) = \omega_0^{(i)}$ is fixed) are generated for each index $i \in \overline{1,n}$ and for each instant $t \in \overline{0,\tau}$ by the discrete-time vector equation

$$\omega^{(i)}(t) = G^{(i)}(y(t))z^{(i)}(t) + F^{(i)}(t)\hat{\xi}^{(i)}(t), \qquad (6)$$

where $\hat{\xi}^{(i)}(t)$ is a measurement error satisfying the following constraint

$$\hat{\xi}^{(i)}(t) \in \hat{\Xi}_1^{(i)},\ \hat{\Xi}_1^{(i)} \in \mathrm{comp}(\mathbf{R}^{\hat{l}_i})\ (\hat{l}_i \in \mathbf{N}). \qquad (7)$$

For all $i \in \overline{1,n}$, $t \in \overline{0,\mathrm{T}}$, and vectors $y(t) \in \mathbf{R}^r$ we assume that $G^{(i)}(y(t))$ and $F^{(i)}(t)$ are real matrices of dimensions $(m_i \times s_i)$ and $(m_i \times \hat{l}_i)$, respectively, and for all vectors $y(t) \in \mathbf{R}^r$ each matrix $G^{(i)}(y(t))$ has the dimension m_i, and it is equal to the dimension of vector $\omega^{(i)}$; $\hat{\Xi}_1^{(i)}$ is convex polyhedron in the space $\mathbf{R}^{\hat{l}_i}$.

During this control process the player P knows the set $\mathrm{Z}^{(i)}(0) = \mathrm{Z}_0^{(i)} \subseteq \mathrm{Z}_1^{(i)}$ $(i \in \overline{1,n})$ of all admissible initial phase vectors $z^{(i)}(0) = z_0^{(i)}$ of the object II_i which are consistent [Krasovskii, 1968] with the initial signal $\omega_0^{(i)}$ and it is a convex polyhedron in the space $\mathbf{R}^{s_i}$.

Suppose that the player P also knows a formation principle of the controls $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ $(\forall\ t \in \overline{\tau,\mathrm{T}-1} : v^{(i)}(t) \in \mathrm{V}_1^{(i)})$ by each player E_i $(i \in \overline{1,n})$ on the interval $\overline{\tau,\mathrm{T}}$ which will be described below.

We also assumed that the player P knows a choice of realization of the control $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ $(\forall\ t \in \overline{\tau,\mathrm{T}-1} : v^{(i)}(t) \in \mathrm{V}_1^{(i)})$ by each player E_i $(i \in \overline{1,n})$ on any interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$, and he can use it for construct his control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ on this interval $(\forall\ t \in \overline{\tau,\mathrm{T}-1} : u(t) \in \mathrm{U}_1)$.

It is assumed that a result of the control process for the player P is estimated on considered interval $\overline{\tau,\mathrm{T}}$ by convex functional α, which defined on admissible terminal (final) phase vectors of the object I and on n information sets [Krasovskii and Subbotin, 1988]–[Shorikov, 1997] corresponding this control process, and both defined at final instant T, and functional α meets the corresponding Lipschitz condition. Note that each information set contains admissible final phase vectors of the corresponding object II_i, $i \in \overline{1,n}$.

The player P using his information and control possibilities has interest in such result for realization of the control process in the dynamical system (1)–(7) on the interval $\overline{\tau,\mathrm{T}}$ when functional α has minimal admissible value by means way using to choice of his admissible program control $u(\cdot) = \{u(t)\}_{t\in\overline{0,\mathrm{T}-1}}$ $(\forall\ t \in$

$\overline{\tau, \mathrm{T}-1}: \ u(t) \in \mathrm{U}_1)$, and each player E_i $(i \in \overline{1,n})$ to help him in it. Note that for this control process the player P do not exclude situation when the parameters $\xi(\cdot) = \{\xi(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}, \ \xi^{(i)}(\cdot) = \{\xi^{(i)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ and $\omega^{(i)}(\cdot) = \{\omega^{(i)}(t)\}_{t\in\overline{\tau+1,\mathrm{T}}}$ may be realized on the interval $\overline{\tau,\mathrm{T}}$ by worst form for him, namely, when they determines maximal admissible value of the functional α.

Then considering these conditions we will say that such possibilities of behavior of the player P combined with the objects I and II_i $(i \in \overline{1,n})$ define the I level or the dominating level of the control process for considered system.

It is assumed that in a field of interests of each player E_i $(i \in \overline{1,n})$ are only admissible phase states of the object II_i, and for any instant $\tau \in \overline{1,\mathrm{T}}$ and the interval $\overline{0,\tau} \subseteq \overline{0,\mathrm{T}}$ $(0 < \tau)$ he knows the collection: $z^{(i)}(\tau)$ is the phase vector of object II_i at instant τ; $u_\tau(\cdot) = \{u(t)\}_{t\in\overline{0,\tau-1}}$ is the past realizations of the control of player P on the interval $\overline{0,\tau}$ $(\forall\, t \in \overline{0,\tau-1} : u(t) \in \mathrm{U}_1)$; $v_\tau^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t\in\overline{0,\tau-1}}$ is the past realizations of his control on the interval $\overline{0,\tau}$ $(\forall\, t \in \overline{0,\tau-1}$: $v^{(i)}(t) \in \mathrm{V}_1^{(i)})$. We also assume that on this interval $\overline{\tau,\mathrm{T}}$ each player E_i knows a choice of realization of the control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ $(\forall\, t \in \overline{\tau,\mathrm{T}-1}$: $u(t) \in \mathrm{U}_1)$ of player P on this interval, which he can use for construct his control $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ on the interval $\overline{\tau,\mathrm{T}}$ $(\forall\, t \in \overline{\tau,\mathrm{T}-1} : v^{(i)}(t) \in \mathrm{V}_1^{(i)})$.

We also assume that at every instant $t \in \overline{0,\mathrm{T}-1}$ of main interval $\overline{0,\mathrm{T}}$ a choice of the control $v^{(i)}(t)$ by the player E_i depend not only from the restriction (3) but it also depend from a choice of the control $u(t) \in \mathrm{U}_1$ by the player P on the base of a priori mapping $\Psi_1^{(i)}$.

Let mapping $\Psi_1^{(i)}$ for all index $i \in \overline{1,n}$ is defined by the following description

$$\Psi_1^{(i)} : \ \mathrm{U}_1 \ \rightarrow \ \mathrm{comp}(\mathrm{V}_1^{(i)});$$

$$\forall\, t \in \overline{0,\mathrm{T}-1}, \ \forall\, u(t) \in \mathrm{U}_1,$$

$$v^{(i)}(t) \in \Psi_1^{(i)}(u(t)) \in \mathrm{comp}(\mathrm{V}_1^{(i)}), \qquad (8)$$

where $\Psi_1^{(i)}(u(t))$ is convex polyhedron in the space $\mathbf{R}^{q_i}$ for all $u(t) \in \mathrm{U}_1$. Therefore, it mean that a choice of admissible realization of the control $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ by the player E_i on the interval $\overline{\tau,\mathrm{T}}$ at every instant $t \in \overline{\tau,\mathrm{T}-1}$ constrained not only by condition (3), but also constrained by admissible realization of the control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ of player P, which communicate to the player E_i $(i \in \overline{1,n})$, and values of the control $u(t)$ at every instant $t \in \overline{\tau,\mathrm{T}-1}$ define the corresponding condition (8). It mean that the constraint (8) is a condition that define a behavior of each player E_i $(i \in \overline{1,n})$ for achievement of his aim in the control process (it will formulate below) and obviously depend from a behavior of the player P.

It is assumed that a result of the control process for each player E_i $(i \in \overline{1,n})$ is estimated on considered interval $\overline{\tau,\mathrm{T}}$ by convex functional β_i, which defined on admissible terminal (final) phase vectors of the object II_i at final instant T and this functional meets the corresponding Lipschitz condition.

We assume that each player E_i $(i \in \overline{1,n})$ using his information and control possibilities has interest in such result for realization of the control process in the dynamical system (1)–(8) on the interval $\overline{\tau,\mathrm{T}}$ when the functional β_i has minimal admissible value by means way using to a choice of his admissible control $v^{(i)}(\cdot) = \{v^{(t)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ $(\forall\, t \in \overline{\tau,\mathrm{T}-1}$: $v^{(i)}(t) \in \mathrm{V}_1)$ and using realization of the control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ $(\forall\, t \in \overline{\tau,\mathrm{T}-1} : \ u(t) \in \mathrm{U}_1)$ of player P on this interval. Note, that for this control process the player E_i do not exclude situation when the parameter $\xi^{(i)}(\cdot) = \{\xi^{(i)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ may be realized on the interval $\overline{\tau,\mathrm{T}}$ by worst form for him, namely, when they determines maximal admissible value of the functional β_i.

Then considering these circumstances we will say that such possibilities of a behavior of all players $E_i, \ i \in \overline{1,n}$, combined with all objects $II_i, \ i \in \overline{1,n}$, define the II level or the subordinate level of the control process for considered system (which is subordinate to the I level or the dominating level of the control process). Also, the collection n of the players $E_i, \ i \in \overline{1,n}$ we will call by the player E.

It is also assumed that in this control process for all instant $t \in \overline{0,\mathrm{T}}$ the player P knows all equations and constraints (1)–(8) and each player E_i knows (2)–(8) $(i \in \overline{1,n})$ for fixed value of the index i.

3 Formulation of problems and general scheme for solution of main problem

For a strict mathematical formulation of the minimax program terminal control problem with incomplete information for the control process in this two-level hierarchical discrete-time dynamical system (1)–(8) we introduce some definitions.

For any fixed number $k \in \mathbf{N}$ and interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau \leq \vartheta)$, we denote by $\mathbf{S}_k(\overline{\tau,\vartheta})$ the metric space of the functions $\varphi : \ \overline{\tau,\vartheta} \ \longrightarrow \ \mathbf{R}^k$ of an integer argument with the metric ρ_k defined by the relation

$$\rho_k(\varphi_1(\cdot),\varphi_2(\cdot)) = \max_{t\in\overline{\tau,\vartheta}} \parallel \varphi_1(t) - \varphi_2(t) \parallel_k$$

$$((\varphi_1(\cdot),\varphi_2(\cdot)) \in \mathbf{S}_k(\overline{\tau,\vartheta}) \times \mathbf{S}_k(\overline{\tau,\vartheta})),$$

and by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau,\vartheta}))$ we denote the set of all nonempty and compact (in this metric) subsets in the space $\mathbf{S}_k(\overline{\tau,\vartheta})$. Hear $\|\cdot\|_k$ is the Euclidean norm in $\mathbf{R}^k$.

Using the constraint (3) we define the set $\mathbf{U}(\overline{\tau,\vartheta}) \in \mathrm{comp}(\mathbf{S}_p(\overline{\tau,\vartheta-1}))$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\vartheta-1}}$ of the player P on the interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \vartheta)$ by the following relation

$$\mathbf{U}(\overline{\tau,\vartheta}) = \{u(\cdot):\ u(\cdot) \in \mathbf{S}_p(\overline{\tau,\vartheta-1}),$$

$$\forall\, t \in \overline{\tau,\vartheta-1},\ u(t) \in \mathrm{U}_1\}.$$

Similarly, using the constraints (3), (4) and (7) we define the following sets: $\Xi(\overline{\tau,\vartheta})$ is the set of all admissible program errors of simulating a dynamic of object I; $\mathbf{V}^{(i)}(\overline{\tau,\vartheta})$ is the set of all admissible program controls of player E_i $(i \in \overline{1,n})$; $\Xi^{(i)}(\overline{\tau,\vartheta})$ is the set of all admissible program errors of simulating a dynamic of object II_i $(i \in \overline{1,n})$; $\hat{\Xi}^{(i)}(\overline{\tau,\vartheta})$ is the set of all admissible program errors of simulating a realization of information signal S_i $(i \in \overline{1,n})$ which is described by the relations (6), (7) and all of these sets defined on the interval $\overline{\tau,\vartheta}$.

We also introduce the following sets

$$\mathbf{V}(\overline{\tau,\vartheta}) = \prod_{i=1}^{n} \mathbf{V}^{(i)}(\overline{\tau,\vartheta}),\ \bar{\Xi}(\overline{\tau,\vartheta}) = \prod_{i=1}^{n} \Xi^{(i)}(\overline{\tau,\vartheta}),$$

$$\hat{\Xi}(\overline{\tau,\vartheta}) = \prod_{i=1}^{n} \hat{\Xi}^{(i)}(\overline{\tau,\vartheta}),$$

which are the sets of all admissible collections $v(\cdot) = (v^{(1)}(\cdot), v^{(2)}(\cdot), \cdots, v^{(n)}(\cdot)) \in \mathbf{V}(\overline{\tau,\vartheta})$ of the program controls of company players E_i, $i \in \overline{1,n}$, or all admissible program controls $v(\cdot)$ of player E, and all admissible collections $\bar{\xi}(\cdot) = (\xi^{(1)}(\cdot), \xi^{(2)}(\cdot), \cdots, \xi^{(n)}(\cdot)) \in \bar{\Xi}(\overline{\tau,\vartheta})$ of program errors of simulating a dynamics of all objects II_i, $i \in \overline{1,n}$, and all admissible collections $\hat{\xi}(\cdot) = (\hat{\xi}^{(1)}(\cdot), \hat{\xi}^{(2)}(\cdot), \cdots, \hat{\xi}^{(n)}(\cdot)) \in \hat{\Xi}(\overline{\tau,\vartheta})$ of program errors of simulating realizations of all information signals S_i, $i \in \overline{1,n}$, and each of their defined on the interval $\overline{\tau,\vartheta}$.

Using the constraints (3), (6) for fixed admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ of player P, and for each index $i \in \overline{1,n}$ we define the set $\mathbf{\Psi}^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot)) \in \mathrm{comp}(\mathbf{S}_{q_i}(\overline{\tau,\vartheta-1}))$ of all admissible program controls $v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,\vartheta})$ of player E_i on the interval $\overline{\tau,\vartheta}$, corresponding to admissible program control $u(\cdot)$ of player P by the following relation

$$\mathbf{\Psi}^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot)) = \{v^{(i)}(\cdot):\ v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,\vartheta}),$$

$$\forall\, t \in \overline{\tau,\vartheta-1},\ v^{(i)}(t) \in \Psi^{(i)}_1(u(t))\}.$$

We also introduce the set

$$\mathbf{\Psi}_{\overline{\tau,\vartheta}}(u(\cdot)) = \prod_{i=1}^{n} \mathbf{\Psi}^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot)).$$

Now, by virtue of (1)–(7), we define the set $\hat{\mathbf{\Omega}}^{(i)}(\overline{\tau,\vartheta}) \subset \mathbf{S}_{m_i}(\overline{\tau+1,\vartheta})$ of all admissible program realizations of information signal $\omega^{(i)}(\cdot) = \{\omega(t)\}_{t\in\overline{\tau+1,\vartheta}}$ on the interval $\overline{\tau,\vartheta}$ and define the set $\hat{\mathbf{\Omega}}(\overline{\tau,\vartheta}) = \prod_{i=1}^{m} \hat{\mathbf{\Omega}}^{(i)}(\overline{\tau,\vartheta}) \subset \mathbf{S}_m(\overline{\tau+1,\vartheta})$.

Then for any instant $\tau \in \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$ let $\hat{\mathbf{W}}(\tau) = \{\tau\} \times \mathbf{R}^r \times \mathrm{comp}(\mathbf{R}^s)$ is the set of all admissible τ-positions $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \{\tau\} \times \mathbf{R}^r \times \mathrm{comp}(\mathbf{R}^s)$ of player P (where $Z(\tau) = \prod_{i=1}^{n} Z^{(i)}(\tau)$ and for any index $i \in \overline{1,n}$ let $Z^{(i)}(\tau)$ is the set of admissible phase vectors $z^{(i)}(\tau) \in \mathbf{R}^{s_i}$ of object II_i at instant τ; $s = \sum_{i=1}^{n} s_i$; $w(0) = w_0 = \{0, y_0, Z_0\}$, $Z(0) = Z_0 = \prod_{i=1}^{n} Z^{(i)}(0)$; $w^*(0) = w_0^* = \{0, y_0, Z_0^*\}$, $Z_0^* = \prod_{i=1}^{n} Z_0^{(i,*)}$; $\hat{\mathbf{W}}(0) = \hat{\mathbf{W}}_0 = \{w(0) = w_0 :\ w_0 = \{0, y_0, Z_0\} \in \{0\} \times \mathbf{R}^r \times \mathrm{comp}(\mathbf{R}^s)\}$), where for any index $i \in \overline{1,n}$ the nonempty set $Z_0^{(i,*)}$ defined, by virtue (6), (7), by the following relation

$$Z_0^{(i,*)} = \{z_0^{(i)}:\ z_0^{(i)} \in Z_0^{(i)},\ \exists\, \hat{\xi}_0^{(i)} \in \hat{\Xi}_1^{(i)},$$

$$\omega_0^{(i)} = G^{(i)}(y_0) z_0^{(i)} + F^{(i)}(0)\hat{\xi}_0^{(i)}\}.$$

For any instant $\tau \in \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$ we denote by $\hat{\mathbf{G}}^{(i)}(\tau) = \overline{0,\mathrm{T}} \times \mathbf{R}^{s_i}$ the set of all admissible τ-positions $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \overline{0,\mathrm{T}} \times \mathbf{R}^{s_i}$ of player E_i $(i \in \overline{1,n}$; $\hat{\mathbf{G}}^{(i)}(0) = \{g^{(i)}(0)\} = \hat{\mathbf{G}}_0^{(i)} = \{g_0^{(i)}\}$, $g^{(i)}(0) = g_0^{(i)} = \{0, z_0^{(i)}\})$ and denote by $\hat{\mathbf{G}}(\tau) = \overline{0,\mathrm{T}} \times \prod_{i=1}^{n} \mathbf{R}^{s_i}$ the set of all admissible τ-positions $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \overline{0,\mathrm{T}} \times \prod_{i=1}^{n} \mathbf{R}^{s_i}$ of all company of players E_i, $i \in \overline{1,n}$, or of player E, for the II level of this control process ($\hat{\mathbf{G}}(0) = \{g(0)\} = \hat{\mathbf{G}}_0 = \{g_0\}$, $g(0) = g_0 = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\}$).

Let for any interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \vartheta)$, for any index $i \in \overline{1,n}$, and for fixed admissible realizations, by virtue of (1)–(8), the τ-position $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ (where $Z(\tau) = Z^{(1)}(\tau) \times Z^{(2)}(\tau) \times \cdots \times Z^{(n)}(\tau)$), and the controls $u(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau,\vartheta})$ of players P and E, respectively, and the information signal $\omega^{(i)}(\cdot) \in \hat{\mathbf{\Omega}}^{(i)}(\overline{\tau,\vartheta})$ we denote by

$\mathbf{R}^{(i)}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot))$ the set of all collections $(\tilde{z}^{(i)}(\tau), \tilde{\xi}^{(i)}(\cdot)) \in Z^{(i)}(\tau) \times \mathbf{\Xi}^{(i)}(\overline{\tau,\vartheta})$ consistent (see [Krasovskii, 1968], [Kurzhanskii, 1977], [Shorikov, 1997]) with this information on the interval $\overline{\tau,\vartheta}$, by the following relation

$$\mathbf{R}^{(i)}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot)) =$$

$$= \{(\tilde{z}^{(i)}(\tau), \tilde{\xi}^{(i)}(\cdot)) :$$

$$(\tilde{z}^{(i)}(\tau), \tilde{\xi}^{(i)}(\cdot)) \in Z^{(i)}(\tau) \times \mathbf{\Xi}^{(i)}(\overline{\tau,\vartheta}),$$

$$\exists\, \xi_*(\cdot) \in \mathbf{\Xi}(\overline{\tau,\vartheta}),\ \exists\, \hat{\xi}_*^{(i)}(\cdot) \in \hat{\mathbf{\Xi}}^{(i)}(\overline{\tau,\vartheta}),$$

$$\forall\, t \in \overline{\tau+1,\vartheta} :$$

$$\omega^{(i)}(t) = G^{(i)}(y_*(t))\tilde{z}^{(i)}(t) + F^{(i)}(t)\hat{\xi}_*^{(i)}(t)$$

$$(y_*(t) = y_t(\overline{\tau,\vartheta}, y(\tau), u(\cdot), v(\cdot), \xi_*(\cdot)),$$

$$\tilde{z}^{(i)}(t) = z_t^{(i)}(\overline{\tau,\vartheta}, \tilde{z}^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \tilde{\xi}^{(i)}(\cdot)),$$

$$v(\cdot) = (v^{(1)}(\cdot), v^{(2)}(\cdot), \cdots, v^{(n)}(\cdot)))\}, \qquad (9)$$

where by $y_*(t) = y_t(\overline{\tau,\vartheta}, y(\tau), u(\cdot), v(\cdot), \xi_*(\cdot))$ and $\tilde{z}^{(i)}(t) = z_t^{(i)}(\overline{\tau,\vartheta}, \tilde{z}^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \tilde{\xi}^{(i)}(\cdot))$ we denoted the sections at instant $t \in \overline{\tau+1,\vartheta}$ of the motions of objects I and II_i, respectively, on the interval $\overline{\tau,\vartheta}$. By virtue of (1) and (2), the motions of objects I and II_i are generated by the collections $(y(\tau), u(\cdot), v(\cdot), \xi_*(\cdot))$ and $(\tilde{z}^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \tilde{\xi}^{(i)}(\cdot))$, respectively.

Let

$$\mathbf{Z}_\vartheta^{(i,e)}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot)) =$$

$$= \{z^{(i,e)}(\vartheta) :\ z^{(i,e)}(\vartheta) \in \mathbf{R}^{s_i},$$

$$z^{(i,e)}(\vartheta) = z_\vartheta^{(i)}(\overline{\tau,\vartheta}, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \xi^{(i)}(\cdot)),$$

$$(z^{(i)}(\tau), \xi^{(i)}(\cdot)) \in$$

$$\in \mathbf{R}^{(i)}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot))\} \qquad (10)$$

be the information set of player P by relatively of the player E_i and of object II_i (see [Krasovskii and Subbotin, 1988], [Kurzhanskii, 1977], [Shorikov, 1997]) for a posteriori minimax filtering process in the discrete-time dynamical system (1)–(8) on the interval $\overline{\tau,\vartheta}$, corresponding to the instant ϑ and admissible collection $(w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot)) \in \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau,\vartheta}) \times \mathbf{V}(\overline{\tau,\vartheta}) \times \hat{\mathbf{\Omega}}^{(i)}(\overline{\tau,\vartheta})$. Note that it is the set of all admissible realizations of the phase vectors of object II_i at the instant ϑ which are consistent with all information about this system known to the player P on the interval $\overline{\tau,\vartheta}$ about behavior of the player II_i.

For any fixed interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \vartheta)$, τ-position $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ of player P and controls $u(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ and $v(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot))$ of players P and E, respectively, we define the following sets

$$\mathbf{\Omega}^{(i)}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), v(\cdot)) = \{\omega^{(i)}(\cdot) :$$

$$\omega^{(i)}(\cdot) \in \hat{\mathbf{\Omega}}(\overline{\tau,\vartheta}),\ \forall\, t \in \overline{\tau+1,\vartheta},$$

$$\omega^{(i)}(t) = G^{(i)}(y(t))z^{(i)}(t) + F^{(i)}(t)\hat{\xi}^{(i)}(t),$$

$$y(t) = y_t(\overline{\tau,\vartheta}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot)),$$

$$z^{(i)}(t) = z_t(\overline{\tau,\vartheta}, z^{(i)}(\tau), v^{(i)}(\cdot), \xi^{(i)}(\cdot)),$$

$$\hat{\xi}^{(i)}(t) \in \hat{\Xi}_1^{(i)},\ (\xi(\cdot), z^{(i)}(\tau), \xi^{(i)}(\cdot)) \in$$

$$\in \mathbf{\Xi}(\overline{\tau,\vartheta}) \times Z^{(i)}(\tau) \times \mathbf{\Xi}^{(i)}(\overline{\tau,\vartheta})$$

$$(Z(\tau) = Z^{(1)}(\tau) \times Z^{(2)}(\tau) \times \cdots \times Z^{(n)}(\tau)),$$

$$v(\cdot) = (v^{(1)}(\cdot), v^{(2)}(\cdot), \cdots, v^{(n)}(\cdot)))\}; \qquad (11)$$

$$\mathbf{W}(\tau, w(\tau), \vartheta, u(\cdot), v(\cdot)) = \{w(\vartheta) :\ w(\vartheta) \in \hat{\mathbf{W}}(\vartheta),$$

$$w(\vartheta) = \{\vartheta, y(\vartheta), Z(\vartheta)\},$$

$$y(\vartheta) = y_\vartheta(\overline{\tau, \vartheta}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot)),$$

$$\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \vartheta}),$$

$$Z(\vartheta) = Z^{(1)}(\vartheta) \times Z^{(2)}(\vartheta) \times \cdots \times Z^{(n)}(\vartheta),$$

$$\forall i \in \overline{1, n} :$$

$$Z^{(i)}(\vartheta) = \mathbf{Z}_\vartheta^{(i,e)}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot)),$$

$$\omega^{(i)}(\cdot) \in \mathbf{\Omega}^{(i)}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), v(\cdot))\}, \qquad (12)$$

which will be called the sets of all admissible information signals S_i on the interval $\overline{\tau, \vartheta}$ and all admissible ϑ-positions of player P, respectively, corresponding to the τ-position $w(\tau)$, and the controls $u(\cdot)$ and $v(\cdot)$ of players P and E_i, respectively.

It is known [Shorikov, 1997], that the information set $\mathbf{Z}_\vartheta^{(i,e)}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot)) = Z^{(e)}(\vartheta) \neq \emptyset$ ($Z^{(e)}(0) = Z_0^*$ of a posteriori minimax filtering process for discrete-time dynamical system (1)–(8) is convex and may be approximate by convex polyhedron in the space $\mathbf{R}^s$ and may be construct by way to realization of finite sequence of one-step operations only.

Note that the information set $\mathbf{Z}_\vartheta^{(i,e)}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \omega^{(i)}(\cdot))$ is main element for solving of a posteriori minimax filtering problem for discrete-time dynamical system (1)–(8) (see [Shorikov, 1997], [Shorikov, 1997]) and will be use for formalization and solving main problem of this paper.

Then, for the estimating a quality of this control process by player P on the I level of control we define the following terminal functional

$$\alpha : \ \hat{\mathbf{W}}(\tau) \times \mathbf{V}(\overline{\tau, \mathrm{T}}) \times \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) \times \hat{\mathbf{\Omega}}(\overline{\tau, \mathrm{T}}) =$$

$$= \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \qquad (13)$$

and it such that for admissible on the interval $\overline{\tau, \mathrm{T}}$ realizations $w(\tau) \in \hat{\mathbf{W}}(\tau)$, $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$, $v(\cdot) = (v^{(1)}(\cdot), v^{(2)}(\cdot), \cdots, v^{(n)}(\cdot)) \in \mathbf{V}(\overline{\tau, \mathrm{T}})$, $\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \mathrm{T}})$, $\omega(\cdot) = (\omega^{(1)}(\cdot), \omega^{(2)}(\cdot), \cdots, \omega^{(m)}(\cdot)) \in \hat{\mathbf{\Omega}}(\overline{\tau, \mathrm{T}})$, and $\mathbf{Z}_\mathrm{T}^{(i,e)}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot))$ its values are defined by the following concrete relation

$$\alpha(w(\tau), g(\tau), u(\cdot), v(\cdot), \xi(\cdot), \omega(\cdot)) =$$

$$= \kappa^{(0)} \cdot \hat{\gamma}^{(0)}(y(\mathrm{T})) +$$

$$+ \sum_{i=1}^{n} \kappa^{(i)} \cdot \hat{\gamma}^{(i)}(\mathbf{Z}_\mathrm{T}^{(i,e)}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot))). \qquad (14)$$

Where $y(\mathrm{T}) = y_\mathrm{T}(\overline{\tau, \mathrm{T}}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$ and $\mathbf{Z}_\mathrm{T}^{(i,e)}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), v(\cdot),\ \omega^{(i)}(\cdot))$ are the sections of motion of object I and of the information set of player P relatively the player E_i ($i \in \overline{1, n}$), respectively, on the interval $\overline{\tau, \mathrm{T}}$ at final (terminal) instant T; $\kappa^{(0)} \in \mathbf{R}^1$ and $\kappa^{(i)} \in \mathbf{R}^1$, $i \in \overline{1, n}$, are any fixed numerical parameters which satisfying the following conditions

$$\kappa^{(0)} \geq 0; \ \forall\, i \in \overline{1, n} : \ \kappa^{(i)} \geq 0;$$

$$\sum_{i=1}^{n} \kappa^{(i)} = 1 - \kappa^{(0)}. \qquad (15)$$

We assume that the functional $\hat{\gamma}^{(0)}$ and all functionals $\hat{\gamma}^{(i)}$, for each $i \in \overline{1, n}$, are convex and each of it meets the corresponding Lipschitz condition.

Let we consider the following functionals

$$\gamma^{(0)} : \ \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{V}(\overline{\tau, \mathrm{T}}) \times \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) =$$

$$= \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \gamma^{(0)}) \longrightarrow \mathbf{E}; \qquad (16)$$

$$\gamma^{(i)} : \ \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{V}(\overline{\tau, \mathrm{T}}) \times \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) \times$$

$$\times \hat{\mathbf{\Omega}}^{(i)}(\overline{\tau, \mathrm{T}}) = \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \gamma^{(i)}) \longrightarrow \mathbf{E}, \qquad (17)$$

and its values are defined by the relations

$$\gamma^{(0)}(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) = \hat{\gamma}^{(0)}(y(\mathrm{T})); \qquad (18)$$

$$\gamma^{(i)}(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \omega^{(i)}(\cdot)) =$$

$$= \hat{\gamma}^{(i)}(\mathbf{Z}^{(i,e)}_{\mathrm{T}}(\overline{\tau,\mathrm{T}}, w(\tau), u(\cdot), v(\cdot), \omega^{(i)}(\cdot))), \quad (19)$$

where the player P estimate by the functional $\gamma^{(0)}$ and by each functional $\gamma^{(i)}$ $(i \in \overline{1,n})$ a quality of realization of final phase vectors of objects I and II_i, respectively, on the I level of this control process in the dynamical system (1)–(8) on interval $\overline{\tau,\mathrm{T}}$. And let we consider vector-functional $\delta = (\gamma^{(0)}, \gamma^{(1)}, \gamma^{(2)}, \cdots, \gamma^{(n)})$ such that it defined by the relation

$$\delta: \, \mathbf{\Gamma}(\overline{\tau,\mathrm{T}}, \gamma^{(0)}) \times \prod_{i=1}^{n} \mathbf{\Gamma}(\overline{\tau,\mathrm{T}}, \gamma^{(i)}) \longrightarrow \mathbf{E}^{n+1}, \quad (20)$$

and its $(n+1)$ values for admissible on the interval $\overline{\tau,\mathrm{T}}$ realizations of all arguments are defined according to the relations (13)–(15). Then we can assert that functional α which is defined by (13)–(15) is its convolution after using the scalar's method for this vector-functional.

Now for estimating a quality of the control process by each player E_i $(i \in \overline{1,n})$ on the II level of control we define the corresponding following terminal functional, namely

$$\beta^{(i)}: \, \hat{\mathbf{G}}^{(i)}(\tau) \times \mathbf{U}(\overline{\tau,\mathrm{T}}) \times \mathbf{V}^{(i)}(\overline{\tau,\mathrm{T}}) \times \mathbf{\Xi}^{(i)}(\overline{\tau,\mathrm{T}}) =$$

$$= \mathbf{\Gamma}^{(i)}(\overline{\tau,\mathrm{T}}, \beta^{(i)}) \longrightarrow \mathbf{E}, \quad (21)$$

and it such that for admissible on the interval $\overline{\tau,\mathrm{T}}$ realizations $g^{(i)}(\tau) \in \hat{\mathbf{G}}^{(i)}(\tau)$, $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$, $v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,\mathrm{T}})$ and $\xi^{(i)}(\cdot) \in \mathbf{\Xi}^{(i)}(\overline{\tau,\mathrm{T}})$ its values are defined by the following concrete relation

$$\beta^{(i)}(g^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \xi^{(i)}(\cdot)) =$$

$$= \hat{\beta}^{(i)}(z^{(i)}(\mathrm{T})), \quad (22)$$

where $z^{(i)}(\mathrm{T}) = z^{(i)}_{\mathrm{T}}(\overline{\tau,\mathrm{T}}, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \xi^{(i)}(\cdot))$ is the section at final (terminal) instant T of motion of object II_i on the interval $\overline{\tau,\mathrm{T}}$. We assume that each functional $\hat{\beta}^{(i)}$ $(i \in \overline{1,n})$ is convex and meets the corresponding Lipschitz condition.

The aim of each player E_i $(i \in \overline{1,n})$ in this program control process on any fixed interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$ may be formulate by the following way. The player E_i $(i \in \overline{1,n})$ using his information and control possibilities has interest in such result on the II level of control in the dynamical system (1)–(8) on the interval $\overline{\tau,\mathrm{T}}$ when the functional $\beta^{(i)}$ which determined by the relations (21), (22) for each admissible realizations of his τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \hat{\mathbf{G}}^{(i)}(\tau)$ $(g^{(i)}(0) = g^{(i)}_0 \in \hat{\mathbf{G}}^{(i)}_0)$ and the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of player P on it time interval has minimal admissible value by means way of using to choice his admissible program control $v^{(i)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$.

Note that on the II level of this control process we not exclude situation when the parameter $\xi^{(i)}(\cdot)$ may be realized on the interval $\overline{\tau,\mathrm{T}}$ by worst form for the player E_i, $(i \in \overline{1,n})$, namely, when it determine maximal admissible value of the functional $\beta^{(i)}$.

Then for realization of the aim of player E_i $(i \in \overline{1,n})$ we can formulate the following multistep program minimax terminal control problem by object II_i on the II level of two-level hierarchical dynamical system described by the relations (1)–(8).

Problem 1. For any fixed index $i \in \overline{1,n}$ and time interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$, and admissible on the II level of two-level hierarchical dynamical system described by the relations (1)–(8) realizations of the τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \hat{\mathbf{G}}^{(i)}(\tau)$ $(g^{(i)}(0) = g^{(i)}_0 \in \hat{\mathbf{G}}^{(i)}_0)$ of player E_i and any program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of player P on the I level of this control process it is required to find the set $\mathbf{V}^{(i,e)}(\overline{\tau,\mathrm{T}}, g^{(i)}(\tau), u(\cdot)) \subseteq \mathbf{\Psi}^{(i)}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ of program minimax controls $v^{(i,e)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ of player E_i, corresponding the control $u(\cdot)$ of player P which is determined by the following relation

$$\mathbf{V}^{(i,e)}(\overline{\tau,\mathrm{T}}, g^{(i)}(\tau), u(\cdot)) = \{v^{(i,e)}(\cdot):$$

$$v^{(i,e)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau,\mathrm{T}}}(u(\cdot)), \; c^{(e)}_{\beta^{(i)}}(\overline{\tau,\mathrm{T}}, g^{(i)}(\tau), u(\cdot)) =$$

$$= \max_{\xi^{(i)}(\cdot) \in \mathbf{\Xi}^{(i)}(\overline{\tau,\mathrm{T}})} \beta^{(i)}(g^{(i)}(\tau), v^{(i,e)}(\cdot), u(\cdot), \xi^{(i)}(\cdot)) =$$

$$= \min_{v^{(i)}(\cdot) \in \, \mathbf{\Psi}^{(i)}_{\overline{\tau,\mathrm{T}}}(u(\cdot))} \{$$

$$\max_{\xi^{(i)}(\cdot) \in \mathbf{\Xi}^{(i)}(\overline{\tau,\mathrm{T}})} \beta^{(i)}(g^{(i)}(\tau), v^{(i)}(\cdot), u(\cdot), \xi^{(i)}(\cdot))\}, \quad (23)$$

where the functional $\beta^{(i)}$ is defined by the relations (21), (22).

The set $\mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u(\cdot)) = \prod_{i=1}^{n} \mathbf{V}^{(i,e)}(\overline{\tau,\mathrm{T}}, g^{(i)}(\tau), u(\cdot))$ which constructed from solving of n problems 1 for $i \in \overline{1,n}$ (where $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ is the τ-position of player E) we shall call the set of program minimax controls of player E

on II level of this control process in the dynamical system (1)–(8) and corresponding to it the value of vector $c^{(e)}_{\beta}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot)) = (c^{(e)}_{\beta(1)}(\overline{\tau, \mathrm{T}}, g^{(1)}(\tau), u(\cdot)), c^{(e)}_{\beta(2)}(\overline{\tau, \mathrm{T}}, g^{(2)}(\tau), u(\cdot)), \cdot\cdot, c^{(e)}_{\beta(n)}(\overline{\tau, \mathrm{T}}, g^{(n)}(\tau), u(\cdot)))' \in \mathbf{E}^n$ we shall call the value of result of programm minimax control of player E on II level of control in this control process. It should be noted that the number $c^{(e)}_{\beta}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot))$ is the concrete value of the vector functional $\beta = (\beta^{(1)}, \beta^{(2)}, \cdots, \beta^{(n)})'$ which defined by the relation (12) and such that may be determine by the following mapping

$$\beta: \prod_{i=1}^{n} \mathbf{\Gamma}^{(i)}(\overline{\tau, \mathrm{T}}, \beta^{(i)}) \longrightarrow \mathbf{E}^n,$$

where for any index $i \in \overline{1, n}$ the value of the functional $\beta^{(i)}$ is defined by the relations (21), (22). Note that we can use the vector functional β as quality test for behavior of the player E (or company of all players $E_i,\ i \in \overline{1, n}$) on the II level of this control process in situation when all players $E_i,\ i \in \overline{1, n}$ have common aim and they organize common coalition.

Note that the solution of problem 1 on the interval $\overline{\tau, \mathrm{T}}$ determine principle of forming the set $\mathbf{V}^{(i,e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot)) \subseteq \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of program minimax controls by each player $E_i,\ (i \in \overline{1, n})$ on II level of the control process in dynamical system (1)–(8), corresponding to the realizations of his τ-position $g^{(i)}(\tau)$ and subordinating to a choice of the admissible programm control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ by player P on I level of this control process.

According to the definitions and assumptions made above about all parameters and information relations for the dynamical systems (1)–(8), the aim of player P in this program control process on any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ may be formulate by the following way. The player P using his information and controls possibilities has interest in such result on the I level of control in the dynamical system (1)–(8) on the interval $\overline{\tau, \mathrm{T}}$ when the functional α determined by the relations (13)–(15) for each admissible realizations of his τ-position $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ has minimal admissible value by means way of using to choice of his admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ for any program minimax control $v^{(e)}(\cdot) = \{v^{(1,e)}(\cdot), v^{(2,e)}(\cdot), \cdots, v^{(n,e)}(\cdot)\} \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot))$ of player E (its forming by players $E_i,\ i \in \overline{1, n}$ from solving of n problems 1 for all indexes $i \in \overline{1, n}$) which subordinate of the player P, where the τ-position $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(g(0) = \{0, z^{(1)}_0, z^{(2)}_0, \cdots, z^{(n)}_0\} = g_0 \in \hat{\mathbf{G}}_0)$ and determine the phase vectors of all objects $II_i,\ i \in \overline{1, n}$ on II level of this control process at instant τ. Note that for all index $i \in \overline{1, n}$: $z^{(i)}(\tau) \in Z^{(i)}(\tau)$, where $Z^{(1)}(\tau) \times Z^{(2)}(\tau) \times \cdots \times Z^{(n)}(\tau) = Z(\tau)$.

Also note that on the I level of this control process we not exclude situation when the parameters $\xi(\cdot)$ and $\omega^{(i)}(\cdot),\ i \in \overline{1, n}$ may be realized on the interval $\overline{\tau, \mathrm{T}}$ by worst form for the player P, namely, when it determine maximal admissible value of the functional α.

Below for realization of the aim of player P we can formulate the following multistep programm minimax terminal control problem by objects I and $II_i,\ i \in \overline{1, n}$ with incomplete information on the I level of two-level hierarchical dynamical system described by the relations (1)–(8).

Problem 2. For any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ and admissible on the I level of two-level hierarchical dynamical system described by the relations (1)–(8) realization of the τ-position $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, Z0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P it is required to find the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ of program minimax controls of player P which is determined by the following relation

$$\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{u^{(e)}(\cdot) :\ u^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}}),$$

$$c^{(e)}_{\alpha}(\overline{\tau, \mathrm{T}}, w(\tau)) = \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot))} \{$$

$$\max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) \\ \omega(\cdot) \in \mathbf{\Omega}^{(1)}(u^{(e)}(\cdot))}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \omega(\cdot))\} =$$

$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})} \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot))} \{$$

$$\max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) \\ \omega(\cdot) \in \mathbf{\Omega}^{(1)}(u(\cdot))}} \alpha(w(\tau), u(\cdot), v^{(e)}(\cdot), \xi(\cdot), \omega(\cdot))\}, \quad (24)$$

where the functional α is defined by the relations (13)–(15); the collection of information signals $\omega(\cdot) = \prod_{i=1}^{n} \omega^{(i)}(\cdot) \in \mathbf{\Omega}^{(1)}(u(\cdot)) = \prod_{i=1}^{n} \mathbf{\Omega}^{(i)}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), v(\cdot))$; the τ-position $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(g(0) = \{0, z^{(1)}_0, z^{(2)}_0, \cdots, z^{(n)}_0\} = g_0 \in \hat{\mathbf{G}}_0)$ of player E and determine the realization of phase vectors of all objects $II_i,\ i \in \overline{1, n}$ at instant τ on II level of this control process and for any index $i \in \overline{1, n}$: $z^{(i)}(\tau) \in Z^{(i)}(\tau)$ $(Z(\tau) = \prod_{i=1}^{n} Z^{(i)}(\tau))$; the set $\mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot)) = \{v^{(e)}(\cdot) = \{v^{(1,e)}(\cdot), v^{(2,e)}(\cdot), \cdots, v^{(n,e)}(\cdot)\}\} \subseteq \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of programm minimax controls of player E for II level of the control process in dynamical system (1)–(8) for any realizations τ-position $g(\tau) \in \hat{\mathbf{G}}(\tau)$ $(g(0) = g_0 \in \hat{\mathbf{G}}_0)$ of player E and programm control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P and construct from solving of n problems 1 for all values of index $i \in \overline{1, n}$.

The set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ which constructed from solving of problem 2 we shall call the set of optimal programm minimax controls of player P on I level of this control process in the dynamical system (1)–(6) and corresponding to it the number $c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ we shall call the value of result of programm minimax control of player P on I level of control in this control process.

Note that the solution of problem 2 on the interval $\overline{\tau, \mathrm{T}}$ determine principle of forming the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ of optimal program minimax controls by the player P on I level of the control process in dynamical system (1)–(8), corresponding to the realizations of his τ-position $w(\tau)$.

On base of the solutions formulated problems 1 and 2 we consider the following problem.

Problem 3. For any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ and admissible on the I level of two-level hierarchical dynamical system described by the relations (1)–(8) realization of the τ-position $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P and admissible on the II level of the control process realization τ-position $g(\tau) \in \hat{\mathbf{G}}(\tau)$ $(g(0) = g_0 \in \hat{\mathbf{G}}_0)$ of player E and any admissible realization of optimal program minimax control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on the I level of the control process which formed from solving problem 2 it is required to find the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(u^{(e)}(\cdot))$ of optimal program minimax controls $\hat{v}^{(e)}(\cdot) = \{\hat{v}^{(1,e)}(\cdot), \hat{v}^{(2,e)}(\cdot), \cdots, \hat{v}^{(n,e)}(\cdot)\} \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot))$ of player E on II level of the control process and vector $c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot)) = (c_{\beta(1)}^{(e)}(\overline{\tau, \mathrm{T}}, g^{(1)}(\tau), u^{(e)}(\cdot)), c_{\beta(2)}^{(e)}(\overline{\tau, \mathrm{T}}, g^{(2)}(\tau), u^{(e)}(\cdot)), \cdots, c_{\beta(n)}^{(e)}(\overline{\tau, \mathrm{T}}, g^{(n)}(\tau), u^{(e)}(\cdot)))' \in \mathbf{E}^n$ of optimal value of result of optimal program minimax control of player E on II level of the control process for considered dynamical system and corresponding to the control $u^{(e)}(\cdot)$ of player P which are determined by the following relations

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot)) = \{\hat{v}^{(e)}(\cdot) :$$

$$\hat{v}^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot)),$$

$$c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) =$$

$$= \max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) \\ \omega(\cdot) \in \mathbf{\Omega}^{(1)}(u^{(e)}(\cdot))}} \alpha(w(\tau), u^{(e)}(\cdot), \hat{v}^{(e)}(\cdot), \xi(\cdot), \omega(\cdot)) =$$

$$= \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u^{(e)}(\cdot))} \{$$

$$\max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) \\ \omega(\cdot) \in \mathbf{\Omega}^{(1)}(u^{(e)}(\cdot))}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \omega(\cdot))\}; \tag{25}$$

$$\forall\, i \in \overline{1, n} : \; c_{\beta(i)}^{(e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u^{(e)}(\cdot)) =$$

$$= \max_{\xi^{(i)}(\cdot) \in \mathbf{\Xi}^{(i)}(\overline{\tau, \mathrm{T}})} \beta^{(i)}(g^{(i)}(\tau), \hat{v}^{(i,e)}(\cdot), u^{(e)}(\cdot), \xi^{(i)}(\cdot)) =$$

$$= \min_{v^{(i)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u^{(e)}(\cdot))} \{$$

$$\max_{\xi^{(i)}(\cdot) \in \mathbf{\Xi}^{(i)}(\overline{\tau, \mathrm{T}})} \beta^{(i)}(g^{(i)}(\tau), v^{(i)}(\cdot), u^{(e)}(\cdot), \xi^{(i)}(\cdot))\}, \tag{26}$$

where as an above the set $\mathbf{\Omega}^{(1)}(u^{(e)}(\cdot)) = \prod_{i=1}^{n} \mathbf{\Omega}^{(i)}(\overline{\tau, \vartheta}, w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot))$.

Note that we may consider solutions of formulated problems 1–3 which in union determined the problem of two-level programm minimax terminal control in hierarchical discrete-time dynamical system (1)–(8).

Then the main scheme of realization the process of program minimax terminal control with incomplete information in two-level hierarchical dynamical system (1)–(8) for any fixed and admissible time interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ and realizations τ-position $w(\tau) = \{\tau, y(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(Z(\tau) = Z^{(1)}(\tau) \times Z^{(2)}(\tau) \times \cdots \times Z^{(n)}(\tau))$ $(w(0) = \{0, y_0, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of the player P on the I level of the control process and τ-position $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(g(0) = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = g_0 \in \hat{\mathbf{G}}_0)$ (for all index $i \in \overline{1, n} : z^{(i)}(\tau) \in Z^{(i)}(\tau)$) of the player E on the II level of the control process we can describe in the form of following sequence of actions:

1) for any admissible control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of the player P on the I level of the control process and index $i \in \overline{1, n}$ from solution of the corresponding problem 1 we construct the set $\mathbf{V}^{(i,e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot))$ of the programm minimax controls of the player E_i and the number $c_{\beta(i)}^{(e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot))$ which is the value of result of the program minimax control for this

player on the II level of the control process which corresponding of the control $u(\cdot)$ and satisfying the relation (23); on the base of this elements and from the solution of n problems 1 for all values of index $i \in \overline{1,n}$ we form the set $\mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u(\cdot))$ and vector $c_\beta^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u(\cdot))$;

2) from the solution of problem 2 we construct the set $\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau))$ of the optimal program minimax controls of the player P on the I level of the control process and the number $c_\alpha^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau))$ which is value of the result of the program minimax control of the player P on the I level of the control process which satisfying the relation (24);

3) for any optimal program minimax control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau))$ of the player P on the I level of the control process from the solution of the problem 3 we construct the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot))$ and vector $c_\beta^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot))$ which satisfying the relation (25).

4 Conclusion

In conclusion we note that the concrete algorithm for realization of two-level hierarchical minimax program terminal control process with incomplete information in discrete-time dynamical system (1)–(8) can be described on the base of the algorithms for solving program terminal control problem with incomplete information which are proposed in works [Shorikov, 1997] and [Shorikov, 2005].

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