ON DYNAMICS OF DISCONTINUOUS SYSTEMS WITH TRADITIONAL AND NON-TRADITIONAL IMPACT PAIRS

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Abstract

In this review in general the dynamic effects are considered and described, caused by different discrete and distributed systems with impacts. We will study traditional and modern models of impact pairs and we will show the results of some interesting experiments. And we will show some new dynamic effects.

1. Systems with Serial Impact Pairs

Systems with serial impact pairs include vibro-impact systems with large numbers of degrees of freedom in which all elements except for outer ones are involved in two or more impact pairs.

Some of the typical models of such systems are shown in fig.1.

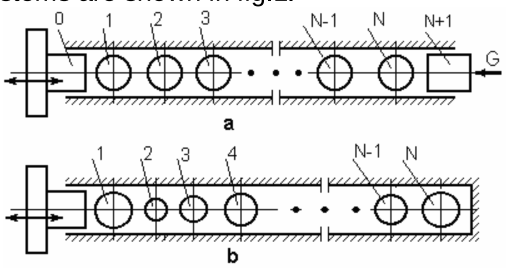

Fig.1

A number of systems with serial impact pairs were examined in the works, where periodic motions in one-dimensional basic systems of the type as shown in fig.1,a,b, are analyzed. Motion of the j-th element during the time interval between collisions is given by the equation

$$m_j\ddot{u}_j + b_{ju}\dot{u}_j = 0. \qquad (1)$$

$$u(\varphi_k)=\Delta_k,\ \dot{u}(\varphi_k+0)=-R_k\dot{u}(\varphi_k-0),$$

$$J_k= m_k(R+1)\dot{u}_t(\varphi_k-0), \qquad (2)$$

where R - coefficient of velocity restoration at impact, which here is supposed to be straight and central; Δ - set-up clearance (interference) value; $m = m_1m_2/(m_1+m_2)$ - reduced mass of colliding bodies; $u_k = x_k - x_{k-1;}$ φ_k and J_k – phases and impulses of impacts.

Evidently, equations for impact elements of the type (1) can be significantly generalized. However in such case motion of a chain with serial impact pairs may become extremely complex.

Systems with serial impact pairs were examined using the fitting technique. In so doing one-dimensional chains of point bodies are considered and it is assumed that energy dissipation occurs only as result of impacts. The analysis is simplified by the obvious condition that in periodic modes impact forces pulse value in all impact pairs J=const. For chains of the type like in fig.1,a with force closure the pulse value J=QT (T - process period), and dynamical length of a chain is set to provide this preliminary determined impulse. For chains with a given length impulse value can be found as a solution of a periodic problem.

It is easy to find out that in a one-dimensional chain of balls with equal masses m and absolutely elastic impacts the pulse J imparted to the first ball propagates along the chain undistorted as a peculiar kind of diffusive wave of the impact force $\Phi(t,x)=J\delta(vt-x_j)$, where $v=J/m$; x_j - coordinates of initial locations of the balls at rest; j - number of a ball in the chain. Evidently, this wave can reflect from a stationary barrier changing the propagation direction.

2. Systems with Parallel Impact Pairs

Systems with parallel impact pairs include complex vibro-impact systems, in which some of the elements of one (basic) subsystem constitute impact pairs with elements of other subsystems, and each element of the basic subsystem can be incorporated in only one impact pair. Examples of such systems are shown in fig.2.

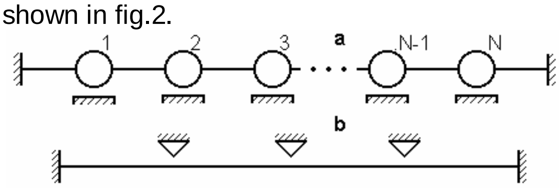

Fig.2

Fig.2,a shows a system with basic subsystem presented by a string with N balls fixed on it. In this case the balls collide with a rigid one-side restrictions, which constitute the second subsystem. Naturally, the restriction may also be double-sided. Besides that, the balls may constitute impact pairs with elements of more complicated form. Fig.2,b displays a transversely oscillating string or beam interacting with point-wise restrictions. Here the subsystem with distributed parameters acts as a basic.

Using the dynamic characteristic of an impact pair obtained earlier, we write motion equations for a system with N parallel impact pairs (fig.2,a)

$$m_1\ddot{u}_1+b_1\dot{u}_1+(c_1+c_2)u_1-c_2u_2+\Phi_1(u_1,u_{t1})= =P_1(t),$$
$$m_j\ddot{u}_j+b_j\dot{u}_j+c_j(u_j-u_{j-1})+c_{j+1}(u_j-u_{j+1})+$$
$$+\Phi_j(u_j,u_{tj})=P_j(t), \quad j=2,\ N-1 \qquad (3)$$
$$m_N\ddot{u}_N+b_N\dot{u}_N+(c_N+c_{N+1})u_N-$$
$$-c_Nu_{N-1}+\Phi(u_N,u_{tN})=P_N(t),$$

where u_j - coordinate of the j-th body; m_j - it's mass; c_j – stiffness of the j-th string portion; b_j - coefficient of resistance to the motion of the j-th body; $P_j(t)$ - driving force affecting the j-th body (j=1,...,N); $\Phi_j(u_j,u_j)$ - dynamic characteristic of the j-th impact pair.

Independent of the system structure the equations, etc. may be written uniformely in the operator form. For the required movements field u(x,t):

$$u(x,t)=\sum_{k=1}^{N} L(x,x_k;p)[P_k(t)-\Phi(u_k,pu_k)], \quad (4)$$

where the dynamic compliance operators L(x,y;p) are determined by the structures of the initial interacting subsystems.

For description of the system with concentrated parameters in the operator equation we should set $x=x_j$.

In case of a periodic outside excitation in order to find the T-periodic modes instead of (4) we can obtain:

$$u(x,t) = u_0(x,t) + \sum_{k=1}^{N}\int_0^T \chi(x,x_k;t-)\Phi(u_k,u_k)ds, \quad (5)$$

where $u_0(x,t)$ - steady-state movement field under inducing forces in the absence of impact interactions; $\chi(x,x_k;t\text{-}s)$ - periodic Green function, compliant to the operator $L(x,x_k;p)$.

If assumed that like above the impact is momentary and acts one time during the motion period, so that the function $\Phi(u,u_t)$ can be presented as a combination of singular distributions, the equations (5) can be reduced to the following representation of the vibro-impact process:

$$u(x,t) = u_0(x,t) + \sum_{k=1}^{N} J_k\,\chi(x,x_k;t-t_k) \quad (6)$$

where J_k - impact forces impulse in the k-th impact pair; t_k - moment of the impact in this pair. Representation (6) is called "2N-parametric". The unknown motion parameters can be sought from the impact conditions of impucts.

The solutions obtained should be analysed regarding the stability and feasibility of geometric conditions of the type $u_k(x,t_k)\le\Delta_k$.The final solution of the problem in a visible analytical general form can be obtained for a limited number of models of such kind, however, for particular parameters it is almost always possible to find a corresponding numerical-analytical solution. Besides that, basing on the representation (6) it is possible to build up some approximate solutions.

Further we will discuss some effects revealed as a result of analysis the model (5) with a periodic structure: for each j all m_j, b_j and c_j values are the same and equal to m, b, and c correspondingly. The outside excitation was chosen sinusoidal.

The main result is the discovery of existence of periodic modes with synchronous impacts in remote impact pairs. Such modes were calls “claps” or “puffs”. At excitation of these modes the string with the balls fixed on it generates one-, two- or multitrapezoid forms quite similar to the corresponding inherent oscillations forms of a linear system in terms of alternation of nodes and crests of waves, and feasible in frequency areas situated at the right from the inherent frequencies of the corresponding linear system. Fig.3 shows an example of claps for two lowest forms in the system with six symmetrical impact pairs. The trapezoid form appears even with two impact pairs.

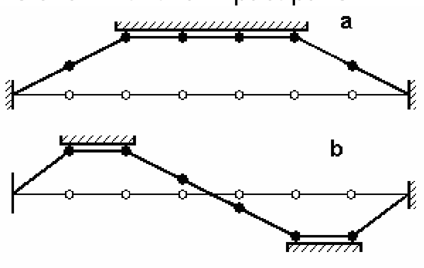

Fig.3

3. Systems with distributed impact pairs

In this chapter we will briefly examine the problems concerned with the models of systems with distributed impact pairs.

References considered models of distributed linear media of complicated structure. The most specific feature of mentioned structures is the presence of two main "medium parts", namely: "carrier" and so called "attached" parts. Dynamic description of such systems, in general, consists of two groups of motion equations - reflecting "carrier" and "attached" subjects constrained behavior respectively. Likewise all the models in the multipolar mechanics, the concept of a point is a subject of significant revision: its state can be defined by unspecified number of kinematic parameters.

Such models utilization arise to be productive for solution of some practical problems, such as dynamical analysis of vibro-states of complicated mechanical structures, consisting of, said, distributed single dimensioned "carrier" and of gross number flexural "attached" solid devices. Monograph, particularly, considered models of media with non-linear damping.

At the same time, paper considered the problem of random oscillations of a distributed carrier rod with gross number of separated impact pairs being attached flexural along it.

It seems that mentioned considerations could be useful by dynamic models creation for the systems consisting of "carrier" and "attached" parts with multiply breaks in it. Due to the system nature the possibility of different type collisions arises in the "attached" subsystems.

Then, unlike the approach of where it was assumed that the impacting elements concentration is low, we consider completely distributed model. Thus, assuming the impact pairs to be "spreader" within a certain space, we can use the concept of distributed impact elements.

Now, let's overview briefly some papers concerning the distributed impact elements. We can obtain the model of distributed impact element by at least two ways.

Firstly, in some cases it appears impossible to disregard wave process arising in the impact pairs itself. Impacting bodies can't be considered as the solid bodies since the lengths of the waves generated by collisions are comparable with the impacting surfaces dimensions.

Secondly, considering the dynamic system with amount of convenient impact pairs to be large enough we can perform long-wave approximation and transit to the distributed model with distributed impact element.

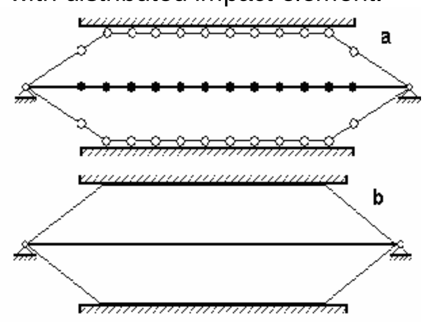

Fig.4

Let's consider vibrating string or supported flexible beam, colliding with obstacles of various kinds. For example, using the Timoshenko beam model and denoting by $u(x,t)$ and $y(x,t)$ - $u_x(x,t)$ the beam instantaneous linear and angular deflection shape, $\Phi(u)$ impact force distribution, the motion equations will be written as following: $\rho U_{2t} - k'FGu_{2x} + k'FGy_{2x} + \Phi(u) = P(x,t)$; $E\Gamma y_{2x} - k'FGu_x - \Gamma\rho F^{-1} y_{2t} = 0$ with the boundary conditions $u(0,t) = u(l,t) = 0$, etc. $0<x< l$. $-\infty<t<\infty$. Singular function $\Phi(u)$ is defined by type of the obstacle and properties of model. The standard set of parameters and modules enters into equation Timoshenko.

In spite of the complexity of the motion equations, in some cases it appears possible to perform not only the corresponding numerical analysis of mentioned models, but to obtain an approximate analytical representation of the required motion distributions as well. These representations can be obtained by means of the modified frequency-time analysis of vibro-impact processes and other methods of the modern non-linear mechanics. In many cases the equations of motion can be reduced to the following "two-functions" representation of the vibro-impact process $[u(x,t)]$:

$$u(x,t)=u0(x,t)-\int_{X} J(z)\chi[x,z;t-f(z)]dz,$$

field under inducing forces in the absence of impact interactions; **X** – some integration area ; $\chi(x,z;t)$ - PGF compliant to some operator $L(x,z;p)$ (built up similarly the motion equation). Unknown two functions $J(x)$ (the impact force impulse distribution) and $f(z)$ (the impact force phase distribution) can be sought from the impact conditions.

Calculations of a number of concrete systems enable to find out and systematize various dynamical effects arising there. Let's note the most specific and impressive ones:

4. Some principal dynamic effects.

One-dimensional extended objects (strings, beams, etc.) vibrating near straight obstacles:

- arising of trapezoid standing waves ("claps"), characterized by synchronous coming of distant points of distributed systems to the restrictions (fig.4);

- emergence of higher forms of claps (multitrapezoid standing waves);

- arising, as the claps occur, of the effects, characteristic to "impact oscillators": "delaying", multivaluedness of "amplitude"-and-frequency characteristic, feasibility of "hard excitation" and others;

- arising of near-periodic standing waves;

-retention of trapezoid profiles of standing waves at different kinds of outside and self-running excitation;

- arising of standing waves characterized by specific profiles of complex nature (quasi-adhesion, emergence of "inside out" configurations etc.)

5.Some results of the experiments (waves picture)

The experiments with a distributed impact element were carried out at the stand schematized in fig.5. Here a rubber tourniquet, one end of which is connected to the force sensor FS fixed on the carriage K1, and the other is linked with the rod of electro-dynamic vibrator B, was used as a distributed impact element РЭ. The tourniquet tension can be adjusted by moving the carriage C1 using the screw S1. It's oscillations are restricted by the extended plate П, fixed on the carriage C2, and moving it with the micrometric screw S2 enables to change the clearance between the tourniquet and the restriction. The vibrator V is energized by the control generator of sinusoid oscillations CG.

The signal from the force sensor FS, proportional to the angle of cord rotation was registered by the cathode-ray oscillograph O. The tourniquet's standing waves were observed in stroboscope light, generated by movement analyzer MA, which lamp's L bursts are synchronized with the control generator. The phase-rotator built in the analyzer enables to stop and photograph (with camera C) the tourniquet's form at any movement phase, and setting of a small detuning between the frequencies of bursts and excitation enables to observe a slowed-down display of evolution of standing waves.

The stand allows setting the second restriction), which enables to examine systems with both one-side and two-side restrictions of the string oscillations.

We cite some results of the examination of periodic standing waves, observed at this unit.

At excitation frequencies $\omega<\omega_1<\Omega_1$, where Ω_1 - the first inherent frequency of the linear system, w_1 - the frequency at which pre-resonance branch of amplitude-and-frequency characteristic passes the restriction level, sinusoid standing waves arise without contacts with the restriction and with amplitudes within the clearance.

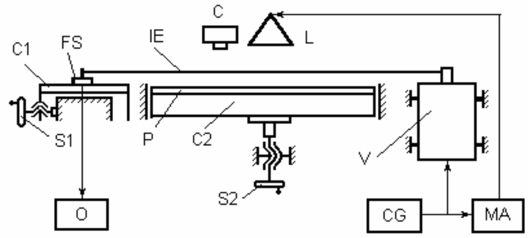

Fig.5

In the range $\omega_1<\omega<\Omega_1$ the waves were found called "on-running without rebound". These waves are different in that the string points reaching the restriction stop immediately and rest at it for some time. As this happens, the wave running on the restriction "spreads" along it until the string takes a certain "final" configuration. Then the string points leave the obstacle and the process recurs.

At passing of the linear resonance frequency $\omega=\Omega_1$ on-running waves disappear and resonance modes of the "claps" type arise. At puffs points of a certain tourniquet part concurrently reach the restriction and rebound immediately. The characteristic standing wave evolution in a system with one-side restriction is shown in fig.6,a where it is evident that intermediate trapezoid wave configurations at the end position degenerate into an isosceles triangle, as predicted by the theory.

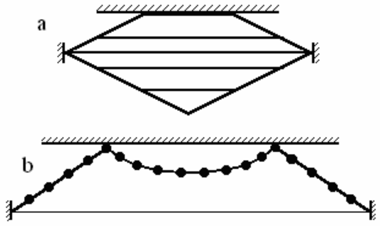

Fig.6

Claps are excited in the frequency range $\Omega_1<\omega<\omega_*$, where $\omega_*>\omega_2$, ω_* - the ultimate puffs excitation frequency, ω_2 - frequency, at which the over-resonance branch of the system's resonance curve crosses the restriction level. In the area $\omega_2<\omega<\omega_*$ in par.

At the same time with claps there exist sinusoid standing waves within the clearance without contacting the restriction. Claps-type modes are of a clearly defined non-linear resonance nature; the essential non-linear effects typical for an ordinary impact oscillator are characteristic for these modes: delaying by frequency and amplitude, quenching of oscillations, feasibility of hard starting of vibro-impact modes. Similar effects were also observed in the neighborhood of higher forms of linear system oscillations when either one-side or two-sides restrictions were installed.

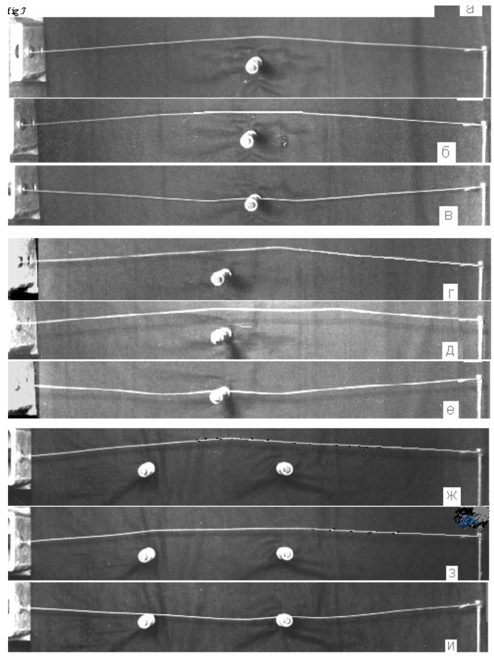

Fig.7

And also similar effects were observed in case of lattices, point-mass obstacles (in fig.7 the string in various phases of movement in systems with one and two obstacles is shown) and at interaction with T-obstacles (the typical structure of a standing wave is shown in fig.8).

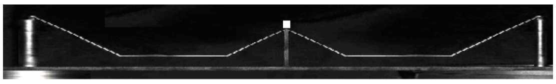

Fig.8

5. The experimental study of systems with parallel impact pairs

We can arrive to the concept of distributed impact element by continualizing of discrete systems. An example of impact element obtained in such manner is so called "concentrated beads", consisting of a string with closely located beads on it. The experiments with such system were carried out at the stand similar to the above described, where a distributed impact element was simulated by a rubber tourniquet with 50 beads of 5 mm diameter located uniformly on it. In this system when passing through frequency range the qualitative character of waves configuration retains: on-running (fig.6,b) without rebounds in pre-resonance area and claps in over-resonance area of the linear system. This system's behavior differs in that standing waves were detected in the neighborhood of the linear resonance frequency, given the title "on-running with rebound". At such modes central beads bumping against the restriction rebound immediately, and then the same happens with neighboring beads as they approach the restriction. As this happens the wave forms a configuration, central part of which is expanding progressively along the restriction and has an "inside-out" form (fig.6, b). Such waves arise if excitation intensity extends a certain ultimate level.

A system, containing small number of beads cannot be reduced to a distributed impact element and represents a typical example of an object with parallel impact pairs. Experiments with a "rarefied" system, containing 3 beads, were performed at the same stand, supplied with impact detectors, which present restrictions. In the frequency range $\omega_1<\omega<\Omega_1$ there were "sluggish" modes on record with collisions of the central bead, sometimes accompanied with rattling. In this area there were also irregular vibro-impact modes observed.

In the band $\Omega_1<\omega<\omega_*$ there were recorded stable periodic modes of the claps type with concurrent collisions of all the three beads, which were clearly corroborated by the oscillograms of the signals from the impact detectors.

At further increasing of frequency after passing the third inherent frequency stable confined oscillations of the soliton (breather) type were observed in a thin frequency range .At this mode, which can be obtained by delaying of one-bead oscillations by amplitude, this bead performs intensive oscillations with collisions, while the other two are almost at rest.

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