SELF-TUNING CONTROL BASED ON GENERALIZED MINIMUM VARIANCE CRITERION.

Anna Patete * **Katsuhisa Furuta** ** **Masayoshi Tomizuka** ***

* *Department of Advanced Multidisciplinary Engineering.*
E-mail: apatete@furutalab.k.dendai.ac.jp
** *Department of Computer and Systems Engineering.*
E-mail: furuta@k.dendai.ac.jp
Tokyo Denki University, Hatoyama-cho, Hiki-gun,
Saitama, 350-0394, Japan. Fax: +81-49-296-3286.
*** *Department of Mechanical Engineering.*
University of California, Berkeley, CA 94720, USA.
Fax:+1-510-643-5599. E-mail: tomizuka@me.berkeley.edu

Abstract: The stability of adaptive control systems has been studied extensively for minimum phase systems, mainly for model reference adaptive systems, but complete stability proof for non-minimum phase systems have not been given. In this paper, the stability of two types of self-tuning controllers for discrete time minimum and non-minimum phase plants is studied, namely: recursive estimation of the implicit self-tuning controller parameters based on generalized minimum variance criterion (REGMVC), and another based on generalized minimum variance criterion - β equivalent control approach (REGMVC-β). Stability of the algorithms are proved by the Lyapunov theory.

Keywords: Self-tuning control, discrete-time systems, generalized minimum variance control, and sliding-mode control, linear control systems.

1. INTRODUCTION

Åström and Wittenmark (Astrom and Wittenmark, 1973) developed and studied the convergence of the implicit self-tuning controller in a stochastic setting. The stability of self-tuning algorithms for Model Reference Adaptive Systems (MRAS) have been studied for the strictly positive real model by Landau (Landau, 1980)(Landau, 1982). Latter on, Johansson (Johansson, 1986) studied the stability of MRAS for a minimum phase system, using Lyapunov theory. Global convergence for a class of adaptive control algorithms applied to discrete-time single-input single-output (SISO) and multi-input multi-output (MIMO) linear systems were studied for the minimum variance criterion in a seminal paper by Goodwin, et al. (Goodwin *et al.*, 1980). However in these approaches the considered system should be minimum phase and the extension to consider the measurement noise may be difficult.

Extending the results of Åström (Astrom and Wittenmark, 1973), Clarke and Gawthrop (Clarke *et al.*, 1975) proposed the Generalized Minimum Variance Control (GMVC) for non-minimum phase systems, using a cost function which incorporates system input and set-point variation. For the case of the unknown system parameters, the unknown parameters are estimated using a recursive least-squares algorithm. Latter, in (Clarke *et al.*, 1979) the convergence of the closed-loop system is analyzed using the positive real condition. In (Gawthrop, 1980) some stability analysis are given based on the notion of dissipative systems and conicity properties. However a complete global stability proof of self-tuning control for non-minimum phase systems have not been yet studied.

Sliding mode control (SMC) based on the variable structure systems (VSS), in the continuous-time case, is not robust when uncertainty excess the bound assumed in the design. Slotine (Slotine and Li, 1964) combined variable structure and adaptive control to solve this problem. Furuta (Furuta, 1993*a*) presents a discrete-time VSS type method for the case where systems parameters are unknown. The VSS is designed based on minimum variance control (MVC) or generalized minimum variance control (GMVC) using recursive parameter estimation. Extending (Furuta, 1993*a*), in (Furuta, 1993*b*) a designed parameter is introduced in the control law while maintaining the use of VSS. The stability of self-tuning control based on the certainty equivalence principle has been studied in (Morse, 1992). This approach is said implicit self-tuning control. However parameters are not identified accurately in the closed loop, and the stability is not assured based on the certainty equivalence principle.

In this paper, the stability of two types of implicit self-tuning controllers for discrete time minimum and non-minimum phase plants, when the system parameters are unknown, is proved. One is the combination of the generalized minimum variance control and identification of control parameter recursively (REGMVC), which has been used in many self-tuning controllers. The stability of the overall adaptive system is proved in this paper, although the parameters are not assured to converge to the true values. The other one (REGMVC-β) considers delay in control input. Stability of the algorithm is proved by the Lyapunov theory. It is not necessary to use VSS nor any additional condition to ensure closed loop system stability for the algorithms studied in this paper. The stability of the closed-loop system is proved in straight forward way in comparison with Goodwin, et al. (Goodwin *et al.*, 1980), and may be extended to the case including the measurement noise (Patete *et al.*, n.d.). This paper consider the non-minimum phase systems contrary to (Goodwin *et al.*, 1980) paper.

The paper is organized as follows; in section 2, the generalized minimum variance criterion is presented. Section 3 deals with parametric uncertainties using self-tuning control based on generalized minimum variance criterion. A simulate examples is given in section 4. Concluding remarks are in 5.

2. GENERALIZED MINIMUM VARIANCE CRITERION

This paper considers a single-input single-output (SISO) time-invariant system. The representation of the plant with input u_k and output y_k is

$$A(z^{-1})y_k = z^{-d}B(z^{-1})u_k \tag{1}$$

where $A(z^{-1})$ and $B(z^{-1})$ have no common factor and z denotes the time shift operator $z^{-t}y_k = y_{k-t}$. In the Laplace transformation, $z = e^{sT_0}$ where T_0 is the sample period (for simplicity, and without loss of generality, we may assume $T_0 = 1$).

The polynomials $A(z^{-1})$ and $B(z^{-1})$ are assumed to be known, and represented as:

$$A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + ... + a_nz^{-n}$$
$$B(z^{-1}) = b_0 + b_1z^{-1} + b_2z^{-2} + ... + b_mz^{-m}$$

where $b_0 \neq 0$ and delay step d, is also assumed to be known.

The objective of the control is to minimize the variance of the controlled variables s_{k+d}, that is defined in the deterministic case as

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k \tag{2}$$

The polynomials

$$C(z^{-1}) = 1 + c_1z^{-1} + c_2z^{-2} + ... + c_nz^{-n}$$
$$Q(z^{-1}) = q_0(1 - z^{-1})$$

are to be designed, r_k is the reference signal, and the error signal e_k is defined as $e_k = y_k - r_k$.

The idea is similar to the discrete time sliding mode control, see (Xinghuo and Jian-Xin, 1992) and (Zinober, 1994). In the case of Goodwin, et al. (Goodwin *et al.*, 1980), $Q(z^{-1})$ is not considered and $C(z^{-1})$ is chosen as $C(z^{-1}) = 1$.

The polynomial $C(z^{-1})$ is Schur, hence the error signal will vanish if (2) is kept to zero. The polynomial $C(z^{-1})$ may be determined by assigning all characteristic roots inside the unit disk of z-plane.

Equation (2) is rewritten as:

$$s_{k+d} = G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+d} \tag{3}$$

where the polynomial $G(z^{-1})$ is defined as $G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1})$, and polynomials $E(z^{-1})$ and $F(z^{-1})$ satisfy the equality,

$$C(z^{-1}) \doteq A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}) \tag{4}$$

Then the minimum generalized variance control input needed to vanish s_{k+d} in (2) is given by:

$$u_k = -G(z^{-1})^{-1}[F(z^{-1})y_k - C(z^{-1})r_{k+d}] \quad (5)$$

where, the polynomials $C(z^{-1})$ and $Q(z^{-1})$ are chosen to make the close-loop characteristic equation to have all zeros inside the unit disc. If the process is minimum phase, $Q(z^{-1}) = 0$ may be chosen, which is the case of minimum variance control.

3. SELF-TUNING CONTROL BASED ON GENERALIZED MINIMUM VARIANCE CRITERION

In the previous section, the generalized minimum variance criterion was described for the case where all plant parameters are known. Yet, in general, plant parameters are not accurately known and parameter identification should be performed. The assumption here is that while model parameters are unknown, the structure of the model (model order) is known a-priori.

When the parameters of the plant are not accurately known, the controller polynomials $G(z^{-1})$ and $F(z^{-1})$ are estimated, i.e.

$$\hat{F}(z^{-1}) = \hat{f}_0 + \hat{f}_1 z^{-1} + ... + \hat{f}_{n-1} z^{-n+1}$$

$$\hat{G}(z^{-1}) = \hat{g}_0 + \hat{g}_1 z^{-1} + ... + \hat{g}_{m+d-1} z^{-(m+d-1)}$$

Let,

$$\phi^T = [y_k, y_{k-1}, ..., y_{k-n+1}, u_k, ..., u_{k-m-d+1}]$$

be a vector containing measured and control signal data,

$$\theta^T = [f_0, f_1, ..., f_{n-1}, g_0, ..., g_{m+d-1}]$$

the vector containing the controller parameters, and

$$\hat{\theta}_k^T = [\hat{f}_0, \hat{f}_1, ..., \hat{f}_{n-1}, \hat{g}_0, ..., \hat{g}_{m+d-1}]$$

the estimation of θ.

Åström and Wittenmark (Astrom and Wittenmark, 1989) have studied self-tuning estimation by minimizing the least-squares criterion function

$$J = \frac{1}{2}\sum_{j=0}^{k}[\varepsilon_j]^2 \quad (6)$$

by the recursive least-squares algorithm

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{P_{k-1}\phi_{k-d}}{1+\phi_{k-d}^T P_{k-1}\phi_{k-d}}\varepsilon_k \quad (7)$$

$$P_k = P_{k-1} - \frac{P_{k-1}\phi_{k-d}\phi_{k-d}^T P_{k-1}}{1+\phi_{k-d}^T P_{k-1}\phi_{k-d}} \quad (8)$$

where ε_k is the prediction error.

In this paper the control law is:

$$u_k = -\hat{G}(z^{-1})^{-1}[\hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d}] \quad (9)$$

where the estimates of control parameters are given by (Furuta, 1993*a*),

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\Gamma_{k-1}\phi_{k-d}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}} \quad (10)$$

$$\cdot[s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-1}]$$

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\Gamma_{k-1}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}} \quad (11)$$

The stability of the self-tuning control algorithm, recursive estimates of controller parameters based on generalized minimum variance criterion (REG-MVC), is analyzed by using Lyapunov function as follows:

Theorem 1. . Recursive estimates of controller parameters based on generalized minimum variance criterion: Given Γ_0 and $\hat{\theta}_0$, the estimate $\hat{\theta}_k$ of the controller (9) satisfies the recursive equations (10) and (11), then the overall self-tuning controller combining (9), (10) and (11) for system (1) is stable.

Proof. s_{k+d} is written as

$$s_{k+d} = \quad (12)$$

$$\hat{G}(z^{-1})u_k + \hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d} + \phi_k^T\tilde{\theta}_{k+d}$$

where $\tilde{\theta}_k = \theta - \hat{\theta}_k$. Using the control law (9), (12) is rewritten as

$$s_{k+d} = \phi_k^T\tilde{\theta}_{k+d} \quad (13)$$

Consider the candidate Lyapunov function:

$$V_k = \frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k, (\Gamma_0 > 0) \quad (14)$$

The time difference of (14) is:

$$\triangle V_k = V_k - V_{k-1} \quad (15)$$

$$= -\frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1} \quad (16)$$

$$+\frac{1}{2}s_k^2$$

$$= -\frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^T\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1}) + \frac{1}{2}s_k^2 \quad (17)$$

$$-\frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} + \Gamma_{k-1}^{-1})\tilde{\theta}_k - \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}$$

$$= -\frac{1}{2}s_{k-1}^2 - \frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^T \Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1}) \quad (18)$$

$$+\frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-1}^{-1})\tilde{\theta}_k + \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_k - \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}$$

$$-\frac{1}{2}s_k^2 + s_k^2$$

From (13), s_k^2 is

$$s_k^2 = \tilde{\theta}_k^T \phi_{k-d}\phi_{k-d}^T \tilde{\theta}_k \quad (19)$$

substituting (19) into (18), the following relation is derived:

$$\triangle V_k = -\frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^T \Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1}) \quad (20)$$

$$-\frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_k$$

$$+\tilde{\theta}_k^T\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k)$$

If

$$\frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_k \quad (21)$$

$$+\tilde{\theta}_k^T\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k) = 0$$

then, $\triangle V_k$ is proven to be negative semidefinite, i.e. $\triangle V_k \leq 0$.

From the first term on the left of (21) the following is derived:

$$\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T = 0 \quad (22)$$

$$\Gamma_k = (\Gamma_{k-1}^{-1} + \phi_{k-d}\phi_{k-d}^T)^{-1} \quad (23)$$

$$\Gamma_k = \Gamma_{k-1} - \quad (24)$$

$$\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T(\Gamma_{k-1}^{-1} + \phi_{k-d}\phi_{k-d}^T)^{-1}$$

this yields (11) by the matrix inversion lemma.

From the second term of (21) the following is derived

$$\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k = 0 \quad (25)$$

$$\tilde{\theta}_k + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k = \tilde{\theta}_{k-1} \quad (26)$$

$$(I + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_k = \quad (27)$$

$$(I + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_{k-1} - \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_{k-1}$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \quad (28)$$

$$\Gamma_{k-1}\phi_{k-d}(1 + \phi_{k-d}^T\Gamma_{k-1}\phi_{k-d})^{-1}\phi_{k-d}^T(\theta - \hat{\theta}_{k-1})$$

from (3),

$$s_k = \phi_{k-d}^T\theta - C(z^{-1})r_k \quad (29)$$

thus (10) is derived.

Thus $\triangle V_k \leq 0$, which implies that as k goes to infinity s_k and $\hat{\theta}_k - \hat{\theta}_{k-1}$ approach to zero then the stability of the overall self-tuning is proved. The stability is proved in straight forward way. The results are just the extension of Goodwin, et al. (Goodwin *et al.*, 1980); but the proof is so straight forward and simple.

(20) gives, s_k shall vanish when $\triangle V_k$ is negative semidefinite for all k, and the generalized minimum variance is minimized.

Remark 1. Note that we do not prove, or claim, that $\hat{\theta}_k$ converges to θ. Instead each element of $\hat{\theta}_k$ approaches to constant values.

Now, instead of (9), consider the following control law,

$$u_k = \hat{G}(z^{-1})^{-1}[C(z^{-1})r_{k+d} - \hat{F}(z^{-1})y_k + \beta s_k] \quad (30)$$

where $0 < \beta < 1$. Substituting (30) into (3), if the controller parameters are known exactly, the following relation is derived

$$s_{k+d} = \beta s_k \quad (31)$$

The idea is similar to the introduction of an additional control parameter to give system robustness. Some results on this topic are found in (Furuta, 1993*b*). When (30) is used instead of (9), the method will be referred as generalized minimum variance criterion - β equivalent control approach (GMVC-β). The following theorem, recursive estimates of controller parameters based on generalized minimum variance criterion - β equivalent control approach (REGMVC-β), establishes overall system stability when (30) is used.

Theorem 2. . Recursive estimates of controller parameters based on generalized minimum variance criterion - β equivalent control approach: Given Γ_i and $\hat{\theta}_i$, $i = 0, -1, -2, ..., -d$, the estimate $\hat{\theta}_k$ of the controller (30) satisfies the recursive equations,

$$\hat{\theta}_k = \hat{\theta}_{k-d} + \frac{\Gamma_{k-d}\phi_{k-d}}{1 + \phi_{k-d}^T\Gamma_{k-d}\phi_{k-d}} \quad (32)$$

$$\cdot[s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-d} + \beta s_{k-d}]$$

$$\Gamma_k = \Gamma_{k-d} - \frac{\Gamma_{k-d}\phi_{k-d}\phi_{k-d}^T\Gamma_{k-d}}{1 + \phi_{k-d}^T\Gamma_{k-d}\phi_{k-d}} \quad (33)$$

then the overall self-tuning controller combining (30), (32) and (33) for system (1) is stable.

Proof. Using (12) and control law (30), s_{k+d} is

$$s_{k+d} = \beta s_k + \phi_k^T\tilde{\theta}_{k+d} \quad (34)$$

The time difference of the candidate Lyapunov function is given by:

$$\triangle V_k = V_k - V_{k-d} \quad (35)$$

where the candidate Lyapunov function is the same as in (14). Then,

$$\triangle V_k = \quad (36)$$

$$\frac{1}{2}s_k^2 - \frac{1}{2}s_{k-d}^2 + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-d}^T\Gamma_{k-d}^{-1}\tilde{\theta}_{k-d}$$

$$= -\frac{1}{2}s_{k-d}^2 - \frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-d})^T\Gamma_{k-d}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-d}) \quad (37)$$

$$+\frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} + \Gamma_{k-d}^{-1})\tilde{\theta}_k - \tilde{\theta}_k^T\Gamma_{k-d}^{-1}\tilde{\theta}_{k-d}$$

$$= -\frac{1}{2}s_{k-d}^2 - \frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-d})^T\Gamma_{k-d}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-d}) \quad (38)$$

$$+\frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-d}^{-1})\tilde{\theta}_k + \tilde{\theta}_k^T\Gamma_{k-d}^{-1}\tilde{\theta}_k - \tilde{\theta}_k^T\Gamma_{k-d}^{-1}\tilde{\theta}_{k-d}$$

$$-\frac{1}{2}s_k^2 + s_k^2$$

from (34), s_k^2 is

$$s_k^2 = \quad (39)$$

$$\beta^2 s_{k-d}^2 + 2\beta s_{k-d}\phi_{k-d}^T\tilde{\theta}_k + \tilde{\theta}_k^T\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k$$

Substituting (39) into (38), the following relation is derived

$$\triangle V_k = \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-d}^{-1} - \phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_k \quad (40)$$

$$-\frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-d})^T\Gamma_{k-d}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-d}) + \tilde{\theta}_k^T\Gamma_{k-d}^{-1}$$

$$\cdot(\tilde{\theta}_k - \tilde{\theta}_{k-d} + \Gamma_{k-d}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k + \Gamma_{k-d}\phi_{k-d}\beta s_{k-d})$$

$$-\frac{1}{2}s_{k-d}^2(1-\beta^2)$$

If

$$\frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-d}^{-1} - \phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_k + \tilde{\theta}_k^T\Gamma_{k-d}^{-1} \quad (41)$$

$$\cdot(\tilde{\theta}_k - \tilde{\theta}_{k-d} + \Gamma_{k-d}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k + \Gamma_{k-d}\phi_{k-d}\beta s_{k-d})$$

$$= 0$$

then the difference of the candidate Lyapunov function (35) is proven to be negative semidefinite, $\triangle V_k \leq 0$.

From the first term on the far left of (41) the following is derived:

$$\Gamma_k^{-1} - \Gamma_{k-d}^{-1} - \phi_{k-d}\phi_{k-d}^T = 0 \quad (42)$$

$$\Gamma_k = (\Gamma_{k-d}^{-1} + \phi_{k-d}\phi_{k-d}^T)^{-1} \quad (43)$$

$$\Gamma_k = \Gamma_{k-d} - \quad (44)$$

$$\Gamma_{k-d}\phi_{k-d}\phi_{k-d}^T(\Gamma_{k-d}^{-1} + \phi_{k-d}\phi_{k-d}^T)^{-1}$$

which yields (33). From the second term of (41),

$$\tilde{\theta}_k - \tilde{\theta}_{k-d} + \Gamma_{k-d}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k \quad (45)$$

$$+\Gamma_{k-d}\phi_{k-d}\beta s_{k-d} = 0$$

$$\hat{\theta}_k = \hat{\theta}_{k-d} - \Gamma_{k-d}\phi_{k-d}(1 + \phi_{k-d}^T\Gamma_{k-d}\phi_{k-d})^{-1} \quad (46)$$

$$\cdot(\phi_{k-d}^T(\theta - \hat{\theta}_{k-d}) + \beta s_{k-d})$$

Noting from (3),

$$s_k = \phi_{k-d}^T\theta - C(z^{-1})r_k \quad (47)$$

(32) is obtained from (46).

Thus $\triangle V_k \leq 0$, and s_{k-d} and $\hat{\theta}_k - \hat{\theta}_{k-d-1}$ approach to zero as k goes to infinity, and the stability of the overall self-tuning has been proved.

Remark 2. Note that for the case $d = 1$ when $\beta = 0$, REGMVC-β algorithm is same as REGMVC.

4. SIMULATION RESULTS

As an hypothetical example consider the following true non-minimum phase plant with $d = 2$,

$$(1 - 0.7z^{-1})y_k = z^{-d}(1 + 2.8z^{-1})u_k \quad (48)$$

The nominal plant is

$$(1 - 0.5z^{-1})y_k = z^{-d}(1 + 2.5z^{-1})u_k \quad (49)$$

For the nominal controller design using the generalized minimum variance criterion, the following polynomials are chosen:

$$C(z^{-1}) = 1 + z^{-1} + 0.3z^{-2} \quad (50)$$

$$Q(z^{-1}) = 40(1 - z^{-1}) \quad (51)$$

which lead to the following polynomials for the controller law,

$\hat{F}(z^{-1}) = 1.05$, $\hat{G}(z^{-1}) = 41 - 36z^{-1} + 3.75z^{-2}$

$\hat{F}(z^{-1})$ and $\hat{G}(z^{-1})$ give initial estimates of the controller parameters.

When polynomial $C(z^{-1})$ and $Q(z^{-1})$ in (50) and (51) respectively, are used to computed the optimal controller polynomials $F(z^{-1})$ and $G(z^{-1})$ for the real plant (48), the following polynomials are obtained,

$F(z^{-1}) = 1.49$, $G(z^{-1}) = 41 - 35.5z^{-1} + 4.76z^{-2}$

In Fig. 1 the output responses of the system (48), using the fixed controller GMVC, and the self-tuning controllers REGMVC and REGMVC-β (for selected values of β: $\beta = 0.4$ and $\beta = 0.8$), are shown. The initial condition for Γ is set to the identity matrix, i.e. $\Gamma_i = I$, $(i = 0, -1)$. The reference signal is chosen as a sequence of steps with length of 50 samples. Note that the generalized minimum variance controller (GMVC) is very sensitive to the presence of parametric

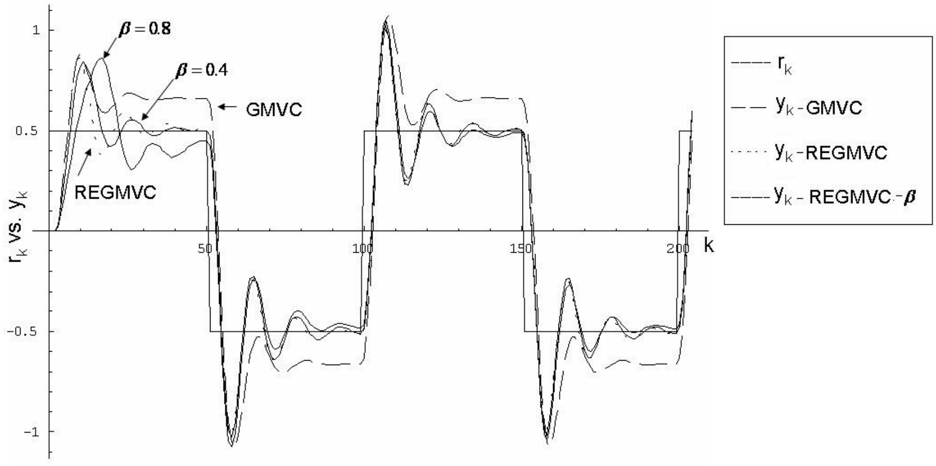

Fig. 1. y_k vs. r_k using GMVC, REGMVC, and REGMVC-β, for the true non-minimum phase system (48), $d = 2$. $\Gamma_{(i)i=0,-1} = I$ and $\beta = 0.4$ and 0.8.

uncertainties, and that the self-tuning controller can improve the responses as the parameters are identified. In this particular example, the case REGMVC-β for small values of β, e.g. $\beta \leq 0.2$ the response behave very similar to the case REGMVC.

5. CONCLUSIONS

Two design methods for the self-tuning stabilizing controller based on generalized minimum variance for discrete-time systems were studied: REGMVC and REGMVC-β. Parameter identification has been used in the presence of parametric uncertainties. The overall stability for the two algorithms was proved by the Lyapunov theory. The robustness and performance of the self-tuning algorithms has been shown through a simulated example.

REFERENCES

Astrom, K.J. and B. Wittenmark (1973). On self-tuning regulators. *Automatica* **9**, 185–199.

Astrom, K.J. and B. Wittenmark (1989). *Adaptive control.* Addison-Wesley. USA.

Clarke, D.W., D. Phil and P.J. Gawthrop (1975). Self-tuning controller. *IEE Proceedings* **122(9)**, 929–934.

Clarke, D.W., D. Phil and P.J. Gawthrop (1979). Self-tuning control. *IEE Proceedings* **126(6)**, 633–640.

Furuta, K. (1993*a*). Vss type self-tuning control. *IEEE Transaction on Industrial Electronics* **40(1)**, 37–44.

Furuta, K. (1993*b*). Vss-type self-tuning control-beta equivalent control approach. *Proceedings of the 1993 American control conference (ACC), San Francisco, California* pp. 980–984.

Gawthrop, P.J. (1980). On the stability and convergence of a self-tuning controller. *International Journal of Control* **31(5)**, 973–998.

Goodwin, G.C., P.J. Ramadge and P.E. Caines (1980). Discrete-time multivariable adaptive control. *IEEE Transaction on Automatic Control* **25(3)**, 449–456.

Johansson, R. (1986). Direct adaptive control. global lyapunov stability and exponential convergence. *Proceedings of the 25th Conference on Decision and Control, Athens, Greece* pp. 813–818.

Landau, I.D. (1980). An extention of a stability theorem applicable to adaptive control. *IEEE Transactions on Automatic Control* **25(4)**, 814–817.

Landau, I.D. (1982). Near supermartingales for convergence analysis of recursive identification and adaptive control schemes. *International Journal of Control* **35(2)**, 197–226.

Morse, A. (1992). Towards a unified theory of parameter adaptive control - part ii: certainty equivalence and implicit tuning. *IEEE Transactions on Automatic Control* **37(1)**, 15–29.

Patete, A., K. Furuta and M. Tomizuka (n.d.). Self-tuning control based on generalized minimum variance criterion for auto regresive models. *Submitted to Automatica.*

Slotine, J.E. and W. Li (1964). *VSS controller design for discrete-time system.* Prentice Hall.

Xinghuo, Y. and X. Jian-Xin (1992). Advances in variable structure systems. *IEEE Proceedings of thr 6th International Workshop on Variable Structure Systems.*

Zinober (1994). Variable structure and lyapunov control. In: *An introduction to sliding mode variable structure control* (A.S.I Zinober, Ed.). pp. 1–22. Springer-Verlag. London.