SYNCHRONIZATION AND PARAMETER IDENTIFICATION OF UNCERTAIN COMPLEX NETWORK VIA ADAPTIVE-IMPULSIVE CONTROL

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Abstract

Based on adaptive-impulsive control method, we investigate the synchronization and parameters identification of uncertain general complex dynamical networks, with non-delayed and delayed coupling. Several criteria are proposed for the uncertain general complex networks with nonidentical nodes, which guarantee that the uncertain node's parameters can be identified when the synchronization occurs. Finally, some numerical simulation results are given to validate the feasibility of the proposed methods.

Key words

synchronization, parameter identification, uncertain complex network, adaptive-impulsive control, delay

1 Introduction

Synchronization, as a universal phenomenon exhibited in many natural systems and complex networks, has been widely studied in recent years. The researchers studied the synchronization for general complex networks, random directed networks, time-varying complex dynamical networks, and most of their results are about the "inner synchronization" presented in [Li, Sun and Kurths, 2007]. Later, some researchers began to study the "outer synchronization" between two complex networks [Tang, Chen, Lu and Tse, 2008; Chen, Jiao, Wu and Wang, 2009]. All those results mainly studied the synchronization of the complex networks with known system parameters and known topological structure.

However, in real-world complex networks, there exists various uncertain information, such as unknown or uncertain topological structure and node dynamics. Therefore, synchronization as well as parameters identification are important issues in the research of complex networks.

The impulsive control, which exerts the control action on systems only at some discrete instants, is more efficient and practical than many other control methods. Therefore, it is popular to synchronize chaotic systems or complex networks via impulsive control. Zhou *et.al* reported some results of impulsive synchronization in general complex delayed dynamical networks [Zhou, Wu, Xiang, Cai and Liu, 2011]. Further, by combining the adaptive control and impulsive control, Wan and Sun studied the adaptive impulsive synchronization of chaotic systems [Wan and Sun, 2011] and Chen *et.al* discussed the adaptive impulsive synchronization of uncertain chaotic systems [Chen, Hwang and Chang, 2010]. Later, Sun *et.al* investigated the adaptive-impulsive synchronization in drive-response networks [Sun, Zeng, Tao and Tian, 2009], K. Li and C.H. Lai analyzed the adaptive-impulsive synchronization of uncertain complex dynamical networks [Li and Lai, 2008], Jiang investigated the hybrid adaptive and impulsive synchronization of uncertain complex dynamical networks by the generalized Barbalat's lemma [Jiang, 2009]. However, all those works only realize the synchronization but do not consider the evolution of the uncertain parameters. To our best knowledge, few work has been done for the adaptive-impulsive synchronization and parameter identification of uncertain complex networks. Very recently, our group obtained some theoretical results about the parameter identification and synchronization of uncertain general complex networks via adaptive-impulsive control, in which the adaptive controllers are used in each node of the response network [Zhang, Luo and Wan, 2013]. In the present paper, we further investigate the adaptive-impulsive synchronization and system parameter tracking of general uncertain complex dynamical networks, with non-delayed and time-varying delayed coupling. Based on some impulsive controllers and adaptive laws of unknown parameters, several novel criteria have been obtained to realize the synchronization and the tracking of unknown parameters, for the impulsively controlled general complex networks consisting of nonidentical nodes.

2 Model and assumption

Consider a class of n-dimensional dynamical system, which is described by the following differential equation

$$\dot{x}_i(t) = F_i(t, x_i(t), \Theta_i), \tag{1}$$

in which $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^{\mathrm{T}} \in \mathbf{R}^n$, $\Theta_i \in \mathbf{R}^p$ is the parameter vector and assume that the vector-valued function $F_i(t, x_i(t), \Theta_i)$ satisfies the uniform Lipschitz condition, that is

$$\|F_i(t, y_i(t), \Theta_i) - F_i(t, x_i(t), \Theta_i)\| \le L_i \|y_i(t) - x_i(t)\|.$$

Further, it can be rewritten as

$$\dot{x}_i(t) = f_i(t, x_i(t)) + g_i(t, x_i(t)) \cdot \Theta_i, \tag{2}$$

where $f_i(t, x_i(t)) : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^n$ is the continuous nonlinear function vector without unknown parameters and $g_i(t, x_i(t)) : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^{n\times p}$ is a continuous function matrix, $\Theta_i \in \mathbf{R}^p$ is the unknown parameter vector.

3 Synchronization and parameter identification of uncertain complex networks without delay

A drive network with unknown system parameters is described by

$$\dot{x}_i(t) = F_i(t, x_i(t), \Theta_i) + \sum_{j=1}^{N} c_{ij} A x_j(t), \quad i = 1..N, \tag{3}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^{\mathrm{T}} \in \mathbf{R}^n$ are the state variables of node i, and $C = (c_{ij})_{N\times N}$ is the weighted configuration matrix. If there is a directed coupling from node i to j ($j \ne i$), then $c_{ij} \ne 0$ and c_{ij} is the weight; otherwise, $c_{ij} = 0$. The matrix $A = (a_{ij})_{n\times n} \in \mathbf{R}^{n\times n}$ is the inner connecting matrix of each node.

Another impulsively controlled slave network is designed by

$$\begin{cases} \dot{y}_i(t) = F_i(t, y_i(t), \hat{\Theta}_i) + \sum\limits_{j=1}^{N} c_{ij} A y_j(t), \ t \ne t_k \\ \Delta y_i(t^+) = d_{ik}(y_i(t) - x_i(t)), \quad t = t_k, \ \ k = 1, 2, \ldots \\ y_i(t_0) = y_{i0}, \end{cases} \tag{4}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \cdots, y_{in}(t))^{\mathrm{T}} \in \mathbf{R}^n$ are the response state variables of node i, $\hat{\Theta}_i$ is the estimation of the unknown system parameters Θ_i, and d_{ik} are the adaptive impulsive feedback gain received by the ith node at t_k impulsive moment. Moreover, $\Delta y_i(t_k^+) = y_i(t_k^+) - y_i(t_k^-)$, $y_i(t_k^+) = \lim\limits_{t\to t_k^+} y_i(t)$ and any solution of (4) is left continuous at each t_k, i.e. $y_i(t_k^-) = y_i(t_k)$. The moments of impulse satisfy $t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ and $\lim\limits_{k\to\infty} t_k = \infty$, $\tau_k = t_k - t_{k-1} < \infty$.

Theorem 1. *Let λ be the largest eigenvalue of $(C \otimes A) + (C \otimes A)^{\mathrm{T}}$, where $\otimes$ is Kronecker product. If there exists a constant $\gamma > 0$ such that*

$$2\alpha_k \tau_k + \ln \bar{d}_k + \gamma < 0, \quad k = 1, 2, \ldots \tag{5}$$

where $\alpha_k = \frac{\lambda}{2} + \max\limits_i \{\hat{L}_i(t_k)\}$, $\bar{d}_k = \max\limits_i \{(1 + d_{ik})^2\}$. Under the following updating laws

$$\begin{aligned} \dot{\hat{\Theta}}_i &= -g_i^{\mathrm{T}}(t, y_i(t)) e_i(t), \\ \dot{\hat{L}}_i &= e_i^{\mathrm{T}}(t) e_i(t), \end{aligned}$$

where $e_i(t) = y_i(t) - x_i(t)$, then the network (3) and impulsively controlled network (4) is adaptive-impulsive synchronous. Moreover, for any $1 \le i \le N$, $\hat{\Theta}_i \to \Theta_i$.

4 Synchronization and parameter identification of uncertain complex networks with time-varying coupling delay

Suppose that time-varying delay $\tau(t)$ is differentiable and satisfies $\dot{\tau}(t) < \mu < 1$, where μ is a constant. Obviously, this assumption holds for any constant delay $\tau(t) \equiv \tau$.

If the drive network is described by

$$\dot{x}_i(t) = F_i(t, x_i(t), \Theta_i) + \sum_{j=1}^{N} c_{ij} A x_j(t - \tau(t)), \ i = 1..N, \tag{6}$$

and the impulsively controlled slave network is given by

$$\begin{cases} \dot{y}_i(t) = F_i(t, y_i(t), \hat{\Theta}_i) + \sum\limits_{j=1}^{N} c_{ij} A y_j(t - \tau(t)), t \ne t_k \\ \Delta y_i(t^+) = d_{ik}(y_i(t) - x_i(t)), \quad t = t_k, \ \ k = 1, 2, \ldots \\ y_i(t_0) = y_{i0}, \end{cases} \tag{7}$$

Theorem 2. *Let ρ be the largest eigenvalue of $(C \otimes A)(C \otimes A)^{\mathrm{T}}$, if there exists a constant $\xi > 0$ such that*

$$2\beta_k \tau_k + \ln \bar{d}_k + \xi < 0, \quad k = 1, 2, \ldots \tag{8}$$

where $\beta_k = \frac{\rho}{2} + \frac{1}{2(1-\mu)} + \max\limits_i \{\hat{L}_i(t_k)\}$ and $\bar{d}_k = \max\limits_i \{(1 + d_{ik})^2\}$. Under the following updating laws

$$\begin{aligned} \dot{\hat{\Theta}}_i &= -g_i^{\mathrm{T}}(t, y_i(t)) e_i(t), \\ \dot{\hat{L}}_i &= e_i^{\mathrm{T}}(t) e_i(t), \end{aligned}$$

then the network (6) and the impulsively controlled delayed network (7) is adaptive-impulsive synchronous. Moreover, for any $1 \le i \le N$, $\hat{\Theta}_i \to \Theta_i$.

5 Numerical simulation examples

In the following, we assume the network inner-coupling matrix A is the identity matrix, i.e. $A = I_{n\times n}$.

Example 1 Consider an uncertain complex network composed with identical nodes without delay. Here, the chaotic Lorenz system is taken as the node's dynamical function, which is given by

$$F(t, x_i(t), \Theta) = \begin{pmatrix} a(x_{i2} - x_{i1}) \\ cx_{i1} - x_{i1}x_{i3} - x_{i2} \\ x_{i1}x_{i2} - bx_{i3} \end{pmatrix}, \quad i = 1, 2, \ldots, 200 \tag{9}$$

where the parameters are given by $a = 10, b = 8/3, c = 28, \Theta = (a, b, c)^{\mathrm{T}}$. The network model in this example is a B-A scale-free network with 200 nodes, which is generated as follows:

(1) Growth: starting with a small number $(m_0 = 2)$ of nodes, at every time step, add a new node with $m = 2$ edges, that link this new node to m different existing nodes in the network.

(2) Preferential attachment: the probability p_i of a new node being connected to i-th node is $\frac{k_i}{\sum\limits_{j\neq i} k_j}$, where k_i is the degree of i-th node.

Select the impulsive feedback gain constant d, impulsive interval τ_k and exponent constant γ respectively as

$$\begin{cases} d = -0.99, \\ \tau_k = 0.02, \\ \gamma = 2. \end{cases} \tag{10}$$

one can easily verify that inequality (5) in Theorem 1 holds with the parameters given by (10). Figure 1 shows the adaptive-impulsive synchronization errors of $e_{i1}(t), e_{i2}(t), e_{i3}(t)$ $(i = 1, 2, \ldots, 200)$ respectively. Clearly, all synchronization errors are rapidly converging to zero. At the same time, Figure 2 shows the identification process of unknown system parameters a, b, c.

Example 2 For the complex networks (6) and (7) with delayed coupling, set $\tau(t) \equiv 0.04$. Each isolate node is the periodically driven double-well Duffing oscillator

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = -px_{i1} - x_{i1}^3 - qx_{i2} + r\cos(wt) \end{cases} \tag{11}$$

which is a classic and popular model of nonlinear phenomena and the solution of Eq.(11) approaches to a chaotic attractor with parameters $p = -1.1, q = 0.4, r = 2.1, w = 1.8$. Here we assume that part of the parameters, such as p, q are unknown.

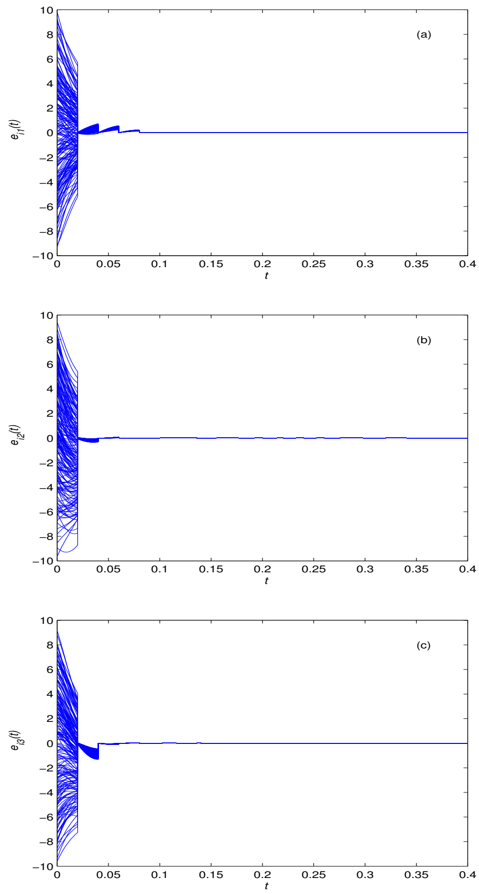

Figure 1. The adaptive-impulsive synchronization errors $e_{i1}(t), e_{i2}(t), e_{i3}(t)$ $(i = 1, 2, \ldots, 200)$ for the network in Example 1.

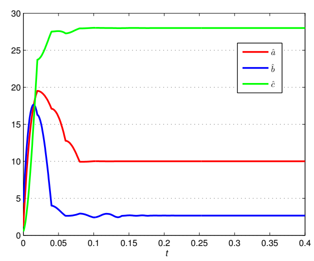

Figure 2. The parameters tracking of $\hat{a}$, $\hat{b}$ and $\hat{c}$ is successful and those parameters are identified by $\hat{a} = 10$, $\hat{b} = 8/3$, $\hat{c} = 28$.

The outer-coupling matrix is

$$C = \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{pmatrix},$$

after simple calculating by Matlab, one gets $\rho = 21.3262$. Set other parameters as

$$\begin{cases} \mu = 0.1, \\ d = -0.99, \\ \tau_k = 0.1, \\ \xi = 2, \end{cases} \tag{12}$$

then one can get that the Eq.(8) in the Theorem 2 is satisfied. Figure 3 shows the variations of the synchronization errors, and the parameters estimation are displayed in Figure 4.

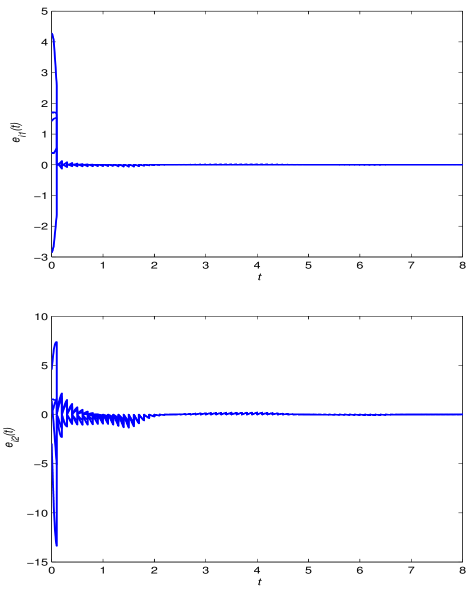

Figure 3. The adaptive-impulsive synchronization errors $e_{i1}(t), e_{i2}(t)$ $(i = 1, 2, \ldots, 5)$ for the network in Example 2.

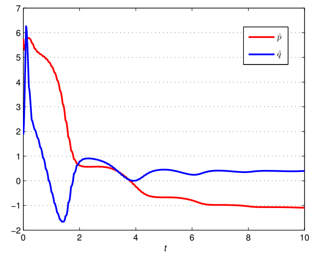

Figure 4. The parameters tracking of $\hat{p}, \hat{q}$ is successful and those parameters are identified by $\hat{p} = -1.1, \hat{q} = 0.4$.

6 Conclusion

The adaptive-impulsive synchronization and parameters identification of unknown general complex dynamical networks, with non-delayed and delayed coupling are studied in this paper. Specially, some uncertain factors, such as some unknown system parameters, are taken into account in this network model. By constructing another suitable impulsively controlled slave network, several novel adaptive laws and control criteria are derived. These criteria are efficient to achieve the adaptive-impulsive synchronization and identify the unknown system parameters of general uncertain complex networks. Finally, numerical simulation results have been presented to demonstrate the effectiveness of the proposed criteria about synchronization and parameters tracking.

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