On Optimizing Control in Aerospace Engineering at Perturbations, Restrictions and Stream of Faults

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Abstract: This paper presents the study of necessary optimum control conditions for stochastic systems of random abrupt nature to describe the operation of a flying vehicle being bounded by various system objectives and requirements, and being subjected to disturbances conditioned by random parameters spread, additive and multiplicative noise, and random Poisson stream of subsystem failures. The inter-orbit flight problem being subject to a failure of one of two sections of simultaneously working engines is analyzed.

Keywords: flying vehicle, control, optimization, random disturbance, failure

1. INTRODUCTION

Despite known optimization outcomes of stochastic systems with random structure (Kazakov and Artem'ev, 1980; Bodner et al., 1987; Solodov, 1988), the problem of statistical optimization of the flying vehicle and its control systems being the subject to choice of main design objectives and characteristics to be closely connected with the accuracy and reliability of flight missions or statistically defined by sets of conditions and application goals turned out to be the least researched. Considering reliability factors while optimizing the flying vehicle movement leads to a change of known programs of control as well as optimum parameters. The factors mentioned point out to the relevance of stochastic system optimization of flying vehicles, the failures being taken into account.

2. THE PROBLEM STATEMENT

We consider nonlinear stochastic system

$$dX_i^k = \sum_{q=1}^{n} \left(C_{iq}^j(t, B^j) \right) dt + \ dW_{iq}) \ \varphi_{iq}^j \left(t, X^k, u, a, B^j \right)$$

$$+ \sum_{q=1}^{n} \sigma_{iq}^j \ \left(t, X^k, B^j \right) \ d\eta_q(t), \ X_i^k \left(t_0 \right) \ = \ X_{i0}, \qquad (1)$$

$$(i \ = \ 1, \ldots, \ n; \ j \ = \ 0, \ldots, k), \quad t \ \in \ [T_j, T_{j+1}],$$

$$T_0 \ = \ t_0, \ T_{k+1} \ = \ t_f,$$

which describes an operation of a flying vehicle, its switching structure forming a stationary Poisson stream of random events with probabilities

$$P_k = \frac{\lambda^k \left(t_f - t_0 \right)^k}{k!} \exp\left(-\lambda \left(t_f - t_0 \right) \right),$$

at the time-segment $[t_0, t_f]$ or non-stationary one

$$P_k \ = \ \frac{1}{k!} \left(\int_{t_0}^{t_f} \lambda(t) \, dt \right)^k \ \exp \ \left(- \int_{t_0}^{t_f} \lambda(t) \, dt \right), \qquad (2)$$

$$(k \ = \ 0, \ 1, ..., \ k_0)$$

with k break-points at the segment $[t_0, t_f]$, with stream density $\lambda = \lambda(t)$ as a known function of time.

Control objectives $\nu \ = \ (u, a)$ and different system requirements are described by mixed limitations of equalities of system parameters, control functions, and phase coordinates:

$$I_s \left(\nu \right) = M[F_s(X_f^k) \,|\, k \ \leq \ k_0] = c_s, \ s \in J_1 = \{ \, 1, \ldots, q \, \} \, . (3)$$

Efficiency of control $\nu \ = \ (u, a)$ of the system (1) is described by the minimum of the functional

$$I_0 \left(v \right) = M[F_0(X_f^k, a) \,|\, k \ \leq \ k_0]. \qquad (4)$$

In (1)–(3) there is t as time; t_0, t_f being initial and final points of the considered time-interval $[t_0, t_f]$. $T_1, ..., T_k$ – the system of random values over $[t_0, t_f]$ with k break-points having distribution density $\psi_k(t_1, ..., \ t_k| \, k)$ at $T_1 < T_2 \ < \cdots < \ T_k$ for stationary and non-stationary Poisson stream respectively being defined by the following formulas (Kozhevnikov, 1966):

$$\psi_k(t_1, ..., \ t_k| \, k) = \ k! \, (t_f - t_0)^{-k},$$

$$\psi_k(t_1, ..., \ t_k| \, k) = \ k! \, \prod_{j=1}^{k} \lambda(t_j) \left(\int_{t_0}^{t_f} \lambda(t) \, dt \right)^{-k}, \qquad (5)$$

where

$t_1, ..., \ t_k$ are realizations of random values $T_1, ..., T_k$;

$X^k(t)$ is $n-$dimensional random system state vector-function with k break-points being $t-$continuous over each time-interval $[T_j, T_{j+1}]$, (j =0,. . . , k);

$u(t)$ is a sectionally-continuous deterministic $r-$dimensional vector-function of control over time-segments $[T_j, T_{j+1}]$;

a is a deterministic $m-$dimensional vector of control that defines the rated parameters of design and energy for the system in question;

B^j is a random $l-$dimensional vector, being constant over $[T_j, T_{j+1}]$, its components relating to random continuous values to characterize, specifically, deviations of the system control parameters from their rated values a over time-segments $[T_j, T_{j+1}]$;

$dW_{iq}(t)$, $d\eta_q(t)$ are Stratonovich stochastic differentials of Wiener processes $W_{iq}(t)$, $\eta_q(t)$.

The right-hand members of (1) meet the known requirements for a solution to exist over continuity segments $[T_j, T_{j+1}]$ (Gihman and Skorohod, 1977). The upper index j on the right sides of (1) characterizes the system structure over the time-segment $[T_j, T_{j+1}]$, $I_s(\nu)$ $s \in \{0\} \cup J_1$ are the limited functionals being differentiable over the set of variables to represent the conditional mathematical expectation values. The averaging operation of (6) is carried out over the partial area $[0, \infty]$ of realizations of a random argument k, $k_0-$is a nonnegative integer.

As it is known (Gihman and Skorohod, 1968), random parameters being present on the right sides of (1), the process described by (1) is not necessarily Markovian. So, in order for (1) to describe the Markovian process, a vector-function of extended phase coordinates is introduced $Z^k = (X^k, B^j)$. Then (1) being relative to Z^k comes to the equivalent system of diffusion stochastic differential equations

$$dZ_i^k = \sum_{q=1}^{n} \Big(C_{iq}^j \left(t, Z^k\right) dt + dW_{iq}(t) \Big) \varphi_{iq}^j \left(t, Z^k, u, a\right)$$

$$+ \sum_{q=1}^{n} \sigma_{iq}^j \left(t, Z^k\right) d\eta_q(t),\ Z_i^k(t_0) = X_{i0},\ (i = 1, \ldots, n); \quad (6)$$

$$dZ_i^k = 0, \quad Z_i^k(t_0) = B_i^1, \qquad (i = n+1, \ldots n+l), \quad (7)$$

$$(j = 0, ..., k;\ k = 0, ..., k_0), \quad t \in [T_j, T_{j+1}],$$

$$T_0 = t_0,\ T_{k+1} = t_f.$$

The equations (6), (7) describe the diffusion Markovian process with discontinuous coefficients of drift and diffusion over $[t_0, t_f]$, and over the adjoining segments $[T_j, T_{j+1}]$, $(j = 1, ..., k)$ in a successive manner, its probability density of states $p(t, z)$ over the process continuity segments $Z^k(t)$ meeting the equation of Kolmogorov-Fokker-Plank (KFP) (9) and coupling conditions (10), (11) at break-points t_j. So, with respect to the expanded vector of states $Z^k = (X^k, B^j)$, the initial problem described by (1)–(4) is reduced to the equivalent one – the problem with distributed parameters (8) – (12) being relative to the probability density $p(t, z)$:

$$I_0(\nu) = M[F_0(X_f^k, a) \,|\, k \le k_0] \quad \to \min \quad (8)$$

$$\frac{\partial p(t,z)}{\partial t} = -\sum_{i=1}^{n} \frac{\partial}{\partial z_i} \left[A_i^j(t, v, z)\, p(t,z) \right]$$

$$+\frac{1}{2} \sum_{i,p=1}^{n} \frac{\partial^2}{\partial z_i \partial z_p} \left[B_{ip}^j(t, v, z)\, p(t,z) \right]; \quad p(t,z)|_{t=t_0} = p_0,$$

$$(j = 0, ..., k;\ k = 0, ..., k_0), \quad t \in [t_j, t_{j+1}], \quad (9)$$

$$\left[A_i^j(t_j, \cdot) p(t_j, z) - \frac{1}{2} \sum_{p=1}^{n} \frac{\partial}{\partial z_p} (B_{ip}^j(t_j, \cdot) p(t_j, z)) \right] = 0, \quad (10)$$

$$(i = 1, ..., n; j = 1, ..., k),$$

$$[B_{ip}^j(t_j, \cdot) p(t_j, z)] = 0, \quad (11)$$

$$i = 1, ..., n; p = 1, ..., n; j = 1, ..., k;$$

$$I_s(\nu) = M[F_s(Z_f^k) \,|\, k \le k_0] = c_s, s \in J_1; \quad (12)$$

$A_i^j(t, \nu, z)$ are coefficients of drift of the process described by the stochastic differential equations (6), (7), with discontinuities of the first kind at time points t_j :

$$A_i^j(t, \nu, z) = \sum_{q=1}^{n} C_{iq}^j \varphi_{iq}^j + \frac{1}{2} \sum_{k,p,q=1}^{n} \left[\frac{\partial \varphi_{ik}^j}{\partial z_p} \left(\varphi_{pq}^j G_{ikpq}^W \right.\right.$$

$$\left.\left. + \sigma_{pk}^j G_{ikq}^{W\eta} \right) + \frac{\partial \sigma_{ik}^j}{\partial z_P} \left(\varphi_{pq}^j G_{kpq}^{\eta W} + \sigma_{pq}^j G_{kq}^{\eta} \right) \right],$$

$B_{ip}^j(t, \nu, z)$ — the coefficients of the diffusion process matrix (6), (7) with discontinuities of the first kind at time-points t_j

$$B_{ip}^j(t, \nu, z) = \sum_{k,q=1}^{n} \left(\varphi_{ik}^j \varphi_{pq}^j G_{ikpq}^W + \varphi_{ik}^j \sigma_{pq}^j G_{ikq}^{W\eta} \right.$$

$$\left. + \sigma_{ik}^j \varphi_{pq}^j G_{kpq}^{\eta W} + \sigma_{ik}^j \sigma_{pq}^j G_{kq}^{\eta} \right).$$

G_{ik}^W, G_{pq}^W, G_k^η , G_q^η are intensities of Wiener processes $W_{ik}(t)$, $W_{pq}(t)$, $\eta_k(t), \eta_q(t)$. $G_{ikpq}^W, G_{ikq}^{W\eta}; G_{kpq}^{\eta W}, G_{kq}^{\eta}$ — the mutual intensities of Wiener processes.

The expressions in square brackets (11)–(12) $[F(\cdot)] = F(\cdot)_- - F(\cdot)_+$ designate a difference of the expressions contained in them left and right off break-points t_j.

3. NECESSARY OPTIMUM CONTROL CONDITIONS

Optimum control conditions of the problem described by (6) – (10), similarly to Rodnischev (2001) are defined by

Theorem 1 (the weak principle of the minimum). If (p^*, ν^*) is the optimum solution (6) – (10), then there exist a vector $\gamma = (\gamma_1, ..., \gamma_q)$ and a function $\lambda(t, z) \in C^{1,2}$ to be simultaneously nonzero, the function $\lambda(t, z) \in C^{1,2}$ being defined by the solution to the boundary problem

$$\frac{\partial \lambda}{\partial t} = \sum_{i=1}^{n} \frac{\partial \lambda}{\partial z_i} A_i^j(t, \nu, z) + \frac{1}{2} \sum_{i,p=1}^{n} \frac{\partial^2 \lambda}{\partial z_i \partial z_p} \partial B_{ip}^j(t, \nu, z),$$

$$t \in [t_{j+1}, t_j], (j = (0 \ldots, k);\ k = (0, \ldots, k_0),$$

$$\lambda(t_f, z)_- = F_0^{k+1} - \sum_{s=1}^{q} \gamma_s F_s^{k+1},$$

$$\lambda(t_j, z)_- = \lambda(t_j, z)_+ + F_0^j - \sum_{s=1}^{q} \gamma_s F_s^j; \quad (13)$$

$$\left[\lambda(t_j, z)\,{}_i^j(t_j, \cdot) + \frac{1}{2} \sum_{p=1}^{n} \frac{\partial \lambda(t_j, z)}{\partial z_p} B_{ip}^j(t_j, \cdot) \right] = 0, \quad (14)$$

$$(i = 1, \ldots, n; \quad j = 1, \ldots, k),$$

$$[\lambda(t_j, z) B_{ip}^j(t_j, \cdot)] = 0, \quad (15)$$

$$(i = 1, \ldots, n; \quad p = 1, \ldots, n; \quad j = 1, \ldots, k);$$

the vector $\gamma = (\gamma_1, \ldots, \gamma_q)$ and the function $\lambda(t, z) \in C^{1,2}$ are so that:

a) the optimum control $u^* = u^*(t)$ for almost all $t \in [t_j, t_{j+1}]$, $(j = 0, \ldots, k)$ and all $u \in U$ in the uniformly close neighborhood of the point u^* satisfies the inequality

$$\left(P_0 M \left(\frac{\partial R^0}{\partial u} \right) + \sum_{k=1}^{k_0} P_k \frac{\partial G_j^k}{\partial u} \right) (u - u^*) \geq 0; \tag{16}$$

b) the parameters a^* satisfy the following conditions:

$$P_0 M \left(\frac{\partial F_0(Z_f^0, a)}{\partial a} + \int_{t_0}^{t_f} \frac{\partial R^0}{\partial a} dt \right) +$$

$$+ \sum_{k=1}^{k_0} P_k \int_{t_0}^{t_f} \int_{t_1}^{t_f} \cdots \int_{t_{k-1}}^{t_f} M[\partial F_0(Z_f^k, a)/\partial a$$

$$+ \sum_{j=0}^{k} \int_{t_j}^{t_{j+1}} \frac{\partial R^k}{\partial a} dt \mid k, t_1, \ldots, t_k] \, \psi_k dt_k \cdots dt_1; \tag{17}$$

$$G_j^k = \sum_{j=0}^{k} \underbrace{\int_{t_0}^{t} \cdots \int_{t_{j-1}}^{t}}_{j} \underbrace{\int_{t}^{t_f} \int_{t_{j+1}}^{t_f} \cdots \int_{t_{k-1}}^{t_f}}_{k-j} M[R^k(\cdot)|$$

$$k, t_1, \ldots, t_k] \, \psi_k dt_k \cdots dt_1.$$

To define the problem of optimum control (8)–(13), that meets the optimum conditions stipulated by theorem 1, the solutions to the KFP-equation (10) as well as the coupled Bellman parabolic equation are necessary. However, as it is known, solving them for higher order systems, only the solutions to linear stochastic systems can be obtained (Krasovsky, 1974; Kazakov, 1977); only special cases of nonlinear systems of no higher order than third can also be solved (Kolosov, 1984). Therefore, to solve (8) – (12), it is suggested to use a method by Rodnishchev (2001) based on employing statistics – semi-invariants of the process (6), (7), the distribution density $p(t, z)$ being involved.

4. OPTIMUM CONTROL DETERMINATION USING SYSTEM PHASE-STATE STATISTICS

With respect to the statistics, (9) – (13) is narrowed down to the problem of

$$I_0(\nu) = M[F_0(X_f^k, a) \,|\, k \leq k_0] \quad \rightarrow \min \tag{18}$$

$$\dot{\omega}_1^i = M[A_i^j(\cdot)], \ (i = 1, \ldots, n);$$

$$\dot{\omega}_{11}^{ip} = M[\tilde{z}_i A_p^j(\cdot) + \tilde{z}_p A_i^j(\cdot)] + M[B_{ip}(\cdot)], \tag{19}$$

$$(i = 1, \ldots, n; \ j = 1, \ldots, n; \ p = 1, \ldots, n);$$

$$\dot{\omega}_{N_i}^i = N_i M[\tilde{z}_i^{N_i - 1} A_i^j(\cdot)] + \frac{1}{2} N_i (N_i - 1) M[\tilde{z}_i^{N_i - 2} B_{ii}(\cdot)]$$

$$- \sum_{q_i = 1}^{N_i - 2} C_{N_i}^{q_i} (\omega_{q_i}^i) M[\tilde{z}_i^{N_i - q_i}],$$

$$(i = 1, \ldots, n; \ j = 1, \ldots, n; N_i = 3, 4, \ldots)$$

$$t \in [t_j, t_{j+1}], \quad (j = 0, \ldots, k),$$

$$\omega_1^i(t_0) = c_0^i, \ \omega_{11}^{ip}(t_0) = c_0^{ip}, \ \omega_{N_i}^i(t_0) = b_0^i,$$

$$\omega_1^i(t_j)_- = \omega_1^i(t_j)_+, \ \omega_{11}^{ip}(t_j)_- =$$

$$= \omega_{11}^{ip}(t_j)_+ \omega_{N_i}^i(t_j)_- = \omega_{N_i}^i(t_j)_+;$$

$$\omega_{2N_i}^i(t_f)/(\omega_2^i(t_f))^{N_i} \geq -K_{2N_i} \tag{20}$$

$$I_s(\nu) = M[F_s(X_f^k) \,|\, k \leq k_0] = c_s, \quad s \in J_1 \tag{21}$$

with the differential bonds (20) for normal differential equations of the $2n + n(n+1)/2 + (N-2)$ degree with respect to the semi-variants ω_1^i, ω_{11}^{ip}, $\omega_{N_i}^i$ of a random process $Z(t)$, the semi-variants having been obtained from the KFP-equation (10) of the logarithm of the process characteristic function (7), (8). The upper indices of the semi-invariants indicate the component numbers of the state-vector, the lower indices – the semi-invariants order. $(N-2)$ – the number of semi-invariants of higher than second order, $\tilde{z}_i = z_i - \omega_1^i$.

Relations (18) – (21) represent a problem of the theory of optimum processes being bound by inequalities and equalities. Numerical methods may be used, particularly stated in Bodner et al.(1987), to solve this problem.

5. THE PROBLEM OF INTER-ORBITAL FLIGHT

Let us consider the problem of transfer of a material point moving in the central force field from one orbit to another by the use of reactive force of the propulsion unit consisting of two simultaneously working engine sections. Unlike the known problem of the optimum transfer of a material point into a circular orbit (Krasovsky, 1968), let define the relative speed of the mass consumption of a material point $u(t)$ being subject to the additive disturbance $\xi(t)$, parametric noise $\eta(t)$ and interferences in a form of Poisson process of dotted events leading to a failure of one of the propulsion unit sections. The propulsion unit provides minimum mass consumption being defined by the functional

$$I_0(u) = \int_0^T |\,(1 + \eta)u + \xi|\, dt \rightarrow \min \tag{22}$$

while transferring a material point along the trajectory

$$\dot{X}_1^k = X_2^k;$$

$$\dot{X}_2^k = -X_1^k + X_3^k$$
$$+ 0.3(X_1^k)^2 + 0.3 X_1^k X_3^k + 0.05 (X_3^k)^2; \tag{23}$$

$$\dot{X}_3^k = a^j((1 + \eta)u + \xi) + 0.1 a^j X_1^k ((1 + \eta)u + \xi)$$

over time the T off one orbit out of position

$$X_1^k(0) = 0.1, \quad X_2^k(0) = 0, \quad X_3^k(0) = 0, \tag{24}$$

onto another orbit to a position defined by the following expressions:

$$I_1(u) = M[X_1^k(T) |\, k \leq 1] = 1.5,$$

$$I_2(u) = M[X_2^k(T) |\, k \leq 1] = 1, \tag{25}$$

$$I_3(u) = M[X_3^k(T) |\, k \leq 1] = 0,$$

where $\xi(t)$ is disturbance of jet acceleration caused by white noise of the angular traction vector; $\eta(t)$ is parametric excitation caused by erosive fuel burning in the combustion chamber. The processes $\xi(t)$ and $\eta(t)$ are not correlated, $k = 0, 1$ — realizations of faults, $j = 0, k$ and $a^0 = 1$, $a^1 = 0.5$.

The equations (23) describe the diffusion Markovian process. With respect to the semi-variants of this process and taking into account that semi-variants of the first order ω_1^i are coincident with the mathematical expectations of the state-vector components $\omega_1^i = m_i$, and that semi-invariants of the second order ω_{11}^{ij} are coincident with the correlation moments R_{ij}: $\omega_{11}^{ij} = R_{ij}$, the problem (22)–(25) is narrowed down to

$$I_0(u) = P_0 m_4^0(T) + P_1 \int_0^T m_4^1 \psi_1 dt_1 \to \min \tag{26}$$

$$\dot{m}_1^k = m_2^k; \quad \dot{m}_2^k = -m_1^k + m_3^k$$
$$+ 0,3((m_1^k)^2 + R_{11}^k) + 0,3(m_1^k m_3^k$$
$$+ R_{13}^k) + 0,05((x_3^k)^2 + R_{33}^k);$$
$$\dot{m}_3^k = a^j(u_1 - u_2)(1 + 0,1m_1^k);$$
$$\dot{m}_4^k = u_1 - u_2; \quad \dot{R}_{11}^k = 2R_{12}^k;$$
$$\dot{R}_{22}^k = 2(R_{12}^k(0,6m_1^k + 0,3m_3^k - 1)$$
$$+ R_{23}^k(0,3m_1^k + 0,1m_3^k + 1));$$

$$\dot{R}_{33}^k = 0,2a^j(u_1 - u_2)R_{13}^k + (1 + 0,2m_1^k$$
$$+ 0,01((m_1^k)^2 + R_{11}^k)(u_1 - u_2)^2 G_\eta + G_\xi); \tag{27}$$
$$\dot{R}_{12}^k = R_{11}^k(0,6m_1^k + 0,3m_3^k - 1)$$
$$+ R_{13}^k(0,3m_1^k + 0,1m_3^k + 1)) + R_{22}^k;$$
$$\dot{R}_{13}^k = a^j(u_1 - u_2)R_{11}^k + R_{23}^k; \; \dot{R}_{14}^k = R_{24}^k;$$
$$\dot{R}_{23}^k = 0,1a^j(u_1 - u_2)R_{12}^k + (0,3m_1^k + 0,1m_3^k)R_{33}^k$$
$$+ (0,6m_1^k + 0,3m_3^k - 1)R_{13}^k + 0,05\omega_3^{3k};$$
$$\dot{R}_{24}^k = (0,6m_1^k + 0,3m_3^k - 1)R_{14}^k + (0,3m_1^k$$
$$+ 0,1m_3^k + 1)R_{34}^k; \quad \dot{R}_{34}^k = 0,1a^j(u_1 - u_2)R_{14}^k;$$
$$\dot{\omega}_3^{3k} = (0,6 + 0,06m_1^k)R_{13}^k(a^j(u_1 - u_2)^2 G_\eta + G_\xi);$$

$$I_1(u) = P_0 m_4^0(T) + P_1 \int_0^T m_4^1 \psi_1 dt_1 = 1; 5$$

$$I_2(u) = P_0 m_4^0(T) + P_1 \int_0^T m_4^1 \psi_1 dt_1 = 1; \tag{28}$$

$$I_3(u) = P_0 m_4^0(T) + P_1 \int_0^T m_4^1 \psi_1 dt_1 = 0,$$

where P_0 is probability of non-failure operation of the propulsion unit: $P_0 = 1 - P_1$, P_1 is probability of failure of a propulsion unit section: $P_1 = \lambda T \exp(-\lambda T)$, $\lambda = 0,002$, $\psi = 1/T$.

To find the solution to problem (26)–(28), the graded projection method (Bodner et al., 1987) with the algorithm convergence to the necessary optimum conditions of theorem 1 is used. The solution to a deterministic problem (Krasovsky, 1977) with control $u(t) = 0,025\mathrm{Sgn}(\cos t)$, $t \in [0, T]$, $T = 2\pi$ was taken as an initial approximation. Under this control, a material point firstly accelerates during time $0 \le t \le \pi/2$. At time-point $t = \pi/2$ a control switching takes place and during time-segment $\pi/2 \le t \le 3\pi/2$ the material point slows down. At time-point $t = 3\pi/2$ a new switching takes place, the material point accelerates again, and at $t = 2\pi$ the material point goes to a desired orbit along a tangent line. At this moment, the control is released. While solving problems (26)–(28) the following alternatives were explored.

Alternative 1. Non-failure functioning of the propulsion unit with probability $P_0 = 1 - P_1$. A failure of one section with probability P_1 over time-segment $[0, 2\pi]$.

Alternative 2. A failure of the propulsion unit section with probability P_1 over time-segment $[0, 2\pi]$.

Alternative 3. Non-failure functioning of the propulsion unit over time-segment $[0,\ \pi/2]$. A failure of a section with probability P_1 over time-section $[\pi/2,\ 2\pi]$.

Alternative 4. Non-failure functioning of the propulsion unit over time-section $[0,\ 3\pi/2]$. A failure of a section with probability P_1 over time-section $[3\pi/2,\ 2\pi]$.

Approximations of the solutions to problem (26)–(28) by using criterion (26) are given in Tab. 1 and Tab. 2.

Table 1. Values of functional

Iter. number	Approximations of $I_0(u)$ by alternatives of failures			
	1	2	3	4
1	0,157	0,157	0,157	0,157
2	0,151	0,157	0,155	0,157
3	0,144	0,157	0,153	0,156
4	0,138	0,157	0,151	0,156
5	0,131	0,157	0,149	0,156
6	0,125	0,157	0,147	0,155
7	0,119	0,157	0,144	0,155
8	0,113	0,157	0,142	0,154
9	0,107	0,157	0,139	0,154

Table 2. Optimum control

Time	Optimum control u^* by alternatives of failures			
	1	2	3	4
0	0,025	0,0343	0,0425	0,0360
0,628	0,025	0,0343	0,0429	0,0360
1,257	0,025	0,0342	0,0424	0,0360
1,885	$-0,025$	$-0,0158$	$-0,0248$	$-0,0141$
2,513	$-0,025$	$-0,0159$	$-0,0250$	$-0,0142$
3,142	$-0,025$	$-0,0159$	$-0,0251$	$-0,0144$
3,770	$-0,025$	$-0,0159$	$-0,0252$	$-0,0145$
4,398	$-0,025$	$-0,0159$	$-0,0252$	$-0,0146$
5,027	0,025	0,0342	0,0249	0,0269
5,655	0,025	0,0342	0,0251	0,0268
6,283	0,025	0,0344	0,0251	0,0266

The values of the average state–vector variances $X = (X_1, X_2, X_3)$ for optimum problem solution to each failure alternative are presented in Tab. 3.

Table 3. The average state-vector variances

Variables	Values of variables by alternatives of failures			
	1	2	3	4
$m_1(T)$	1,46	1,56	1,51	1,57
$m_2(T)$	0,87	1,12	1,06	1,16
$m_3(T)$	0,064	0,063	0,059	0,058
$R_{11}(T)$	3,50	4,61	4,50	4,80
$R_{22}(T)$	1,12	1,76	1,63	1,88
$R_{33}(T)$	0,067	0,069	0,068	0,069

The optimum control $u^*(t)$, which carries out an inter-orbital flight of a material point by first failure alternative, puts into effect the following regime of flight. Over time-segment $0 \leq t \leq 1,257$ the material point accelerates with relative mass consumption $u = 0,025$.

Then, over the time-segment $1,257 \leq t \leq 1,885$, a switching is made by the linear law; over time-segment $1,885 \leq t \leq 4,398$ a slowing-down takes place with the relative mass consumption $u = 0,025$. Over time-segment $4,398 \leq t \leq 5,027$ a new switching takes place by the linear law; over time-segment $5,027 \leq t \leq 6,283$ the material point accelerates again with relative mass consumption $u = 0,025$. At time $t = 6,283$ the control is released. The relative mass consumption during such flight amounts to value 0,107.

The optimum control $u^*(t)$, which carries out an interorbital flight of a material point by the second failure alternative, puts into effect the following regime of flight. Over time-segment $0 \leq t \leq 1,257$ the material point accelerates with the relative mass consumption $u = 0,0343$.

Then, over the time-segment $1,257 \leq t \leq 1,885$ a switching is made by the linear law; over time-segment $1,885 \leq t \leq 4,398$ a slowing-down takes place with the relative mass consumption $u = 0,0159$. Over time-segment $4,398 \leq t \leq 5,027$ a new switching takes place by the linear law; over time-segment $5,027 \leq t \leq 6,283$ the material point accelerates again with relative mass consumption $u = 0,0342$. At time $t = 6,283$ the control is released. The relative mass consumption during such flight amounts to value 0,157.

The optimum control $u^*(t)$, which carries out an interorbital flight of a material point by the third failure alternative, puts into effect the following regime of flight. Over time-segment $0 \leq t \leq 1,257$ the material point accelerates with relative mass consumption $u = 0,0425$.

Then, over the time-segment $1,257 \leq t \leq 1,885$, a switching is made by the linear law; over time-segment $1,885 \leq t \leq 4,398$ a slowing-down takes place with relative mass consumption $u = 0,0252$. Over time-segment $4,398 \leq t \leq 5,027$ a new switching takes place by the linear law; over time-segment $5,027 \leq t \leq 6,283$ the material point accelerates again with relative mass consumption $u = 0,0251$. At time $t = 6,283$ the control is released. The relative mass consumption during such flight amounts to value 0,139.

The optimum control $u^*(t)$, which carries out an interorbital flight of a material point by fourth failure alternative, puts into effect the following regime of flight. Over time-segment $0 \leq t \leq 1,257$ the material point accelerates with relative mass consumption $u = 0,036$. Then, over the time-segment$1,257 \leq t \leq 1,885$, a switching is made by the linear law; over time-segment $1,885 \leq t \leq 4,398$ a slowing-down takes place with relative mass consumption $u = 0,0141 \div 0,0146$. Over time-segment $4,398 \leq t \leq 5,027$ a new switching takes place by the linear law; over time-segment $5,027 \leq t \leq 6,283$ the material point accelerates again with relative mass consumption $u = 0,0266 \div 0,0269$.

At time $t = 6,283$ the control is released. The relative mass consumption during such flight amounts to value 0,154.

Thus, designing a flight program of a material point being subjected to parametric and additive disturbances as well as possible failures of the propulsion unit, it is necessary to foresee the relative mass consumption of 0. 157.

6. CONCLUSION

To find the optimum control for non-linear stochastic systems of random abrupt nature, the approach presented here lets build, at the average, the optimum program control for sufficiently large range of expected conditions of functioning of flying vehicles and their subsystems, parametric disturbances, additive disturbances and failures being conditioned, in particular, by Poisson stream.

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