DATA-BASED ℓ_1 OPTIMAL ROBUST SYNTHESIS FOR THE FIRST ORDER PLANT

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Abstract: The problem under consideration is the ℓ_1 optimal steady-state tracking a bounded reference signal for the first order discrete-time plant. Parameters of the plant and upper bounds for perturbations and exogenous disturbance are assumed to be unknown to controller designer. It is shown that associated nonconvex problem of optimal identification is computationally tractable and can be used for data-based optimal steady-state tracking.

Keywords: Robust control, ℓ_1 optimal control, Identification, Data-based control

1. INTRODUCTION

Identification for robust control is an area of active research in the last two decades. In spite of considerable achievements, the recent special issues of Automatica (2005, V.41, No. 3) and IEEE Transactions on Automatic Control (2005, V. 50, N. 10) on system identification show that many problems related to the improvement of systems robust performance remain open. The most coherent approach to synthesis of optimal control systems based on identification is to treat the control criterion as the identification criterion. As noted in Mäkilä, Partington and Gustafsson (1995), so highly control-oriented approach to identification leads usually to extremely difficult optimization problems. A general approach to identification-based suboptimal synthesis in the ℓ_1 setup was proposed in Sokolov (1985a,1985b). The approach was based on finding a model that is not falsified by data and delivers the best (or acceptable) value of the control criterion. This approach was applied to the synthesis of adaptive control in Sokolov (1985a,1985b) and adaptive robust control in Sokolov (1996,2001b). Similar approach to data-based robust synthesis was discussed in Dahleh and Doyle (1994) at the methodological level. In Krause, Stein, and Khargonekar (1992), the idea of optimal identification for robust control in the H_∞ setup was also discussed at the methodological level under the name of preferential identification.

The purpose of the present paper is to show that the ℓ_1 optimal, within the prescribed accuracy, robust steady-state tracking is computationally tractable for unknown first order plant under bounded exogenous disturbance and the induced norm bounded perturbations in output and control.

2. PROBLEM STATEMENT

Consider the first order discrete-time plant

$$y(t) = ay(t-1) + bu(t-1) + d(t) \quad \forall t \in \mathbb{N} \quad (1)$$

where $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ is the control and $d(t) \in \mathbb{R}$ is the total disturbance. The coefficients a and b are *unknown* to controller designer. The total disturbance d is of the form

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$$d := \delta_w w + \delta_y \Delta_1 y + \delta_u \Delta_2 u \,, \tag{2}$$

where w is a normalized bounded exogenous disturbance,

$$\|w\|_{\ell_\infty} = \sup_t |w(t)| \leq 1 \,, \tag{3}$$

and Δ_1 and Δ_2 are normalized perturbations in output and control such that for all $t \in \mathbb{N}$

$$|(\Delta_1 y)(t)| \leq \sup_{t-\mu \leq s < t} |y(s)| \,, \tag{4}$$

$$|(\Delta_2 u)(t)| \leq \sup_{t-\mu \leq s < t} |u(s)| \,. \tag{5}$$

Without loss of generality, the memory of perturbations μ is assumed to be known to controller designer (see Remark 1 at the end of this section). In terms of mappings, inequalities (4) and (5) mean that $\Delta_1 : \ell_\infty \mapsto \ell_\infty$ and $\Delta_2 : \ell_\infty \mapsto \ell_\infty$ are strictly causal maps with bounded memory (see Sokolov, 2001a, for detail) and their gains are not greater than one. The nonnegative scalars $\delta_w, \delta_y, \delta_u$ represent *unknown* upper bounds for exogenous disturbance and perturbations.

Define the vector of unknown upper bounds

$$\delta := (\delta_w, \delta_y, \delta_u)^T \,,$$

the vector of unknown coefficients

$$\xi := (a, b)^T \,,$$

and the extended parameter vector

$$\theta := (\xi^T, \delta^T)^T \,.$$

The prior information about the unknow ξ is of the form

$$\xi \in \Xi \tag{6}$$

where Ξ is a known prior set.

Let $r \in \ell_\infty$ be a given reference signal and K be a causal controller of the form

$$u(t) = K(y_0^t, u_0^{t-1}, r) \quad \forall t \in \mathbb{N} \tag{7}$$

where

$$y_0^t = (y(0), \cdots, y(t)) \,,$$
$$u_0^{t-1} = (u(0), \cdots, u(t-1))$$

are measurement data by the time instant t.

The steady-state robust performance of the closed loop system (1) and (7) will be described by the control criterion

$$J(K, \xi, \delta, \mu) := \sup_{\Delta_1, \Delta_2} \sup_w \limsup_{t \to \infty} |\, y(t) - r(t) \,| \tag{8}$$

where the suprema are taken on the sets of disturbances w and perturbations Δ_1, Δ_2 satisfying the constraints (3), (4), and (5), respectively.

Let the desired controller $K_0(\xi)$ for the plant with the *known* coefficients ξ be of the form

$$u(t) = -\frac{a}{b} y(t) + \frac{1}{b} r \quad \forall t \in \mathbb{N}. \tag{9}$$

Denote the controller (9) by . This controller provides the equality

$$y(t) - r = d(t) \quad \forall t \in \mathbb{N} \tag{10}$$

for the output y of the plant (1) and turns out to be optimal with respect to the control criterion (8) (see Remark 2 at the end of this section).

Let $\|r\|_{ss} := \limsup_{t\to\infty} |r(t)|$ and $\|a(q^{-1})r\|_{ss} := \limsup_{t\to\infty} |r(t) - ar(t-1)|$. The steady-state robust performance of the closed loop system (1) and (9) is described in the next theorem.

Theorem 1. The controller $K_0(\xi)$ described by the equation (9) ensures the inequality

$$J(K_0(\xi), \xi, \delta, \mu) \leq J(K_0(\xi), \xi, \delta, +\infty) = \tag{11}$$

$$\frac{\delta_w + \delta_y \|r\|_{ss} + \delta_u \frac{\|a(q^{-1})r\|_{ss}}{|b|}}{1 - \delta_y - \delta_u \frac{|a|}{|b|}} \,. \tag{12}$$

If the signals r and $a(q^{-1})r$ get into neighborhoods of their *ss*-limits uniformly often (see Sokolov, 2001a for detail), then

$$J(K, \xi, \delta, \mu) \nearrow J(K_0(\xi), \xi, \delta, +\infty) \tag{13}$$

as $\mu \to +\infty$.

The inequality (11) and the representation (12) follow from Theorem 8 and the monotone convergence in (13) follows from Theorem 6 in in Sokolov (2001a).

Note also that the positiveness of the denominator in (12) is the necessary and sufficient condition of robust stability against infinite memory perturbations ($\mu = +\infty$).

Problem formulation. The problem of data-based suboptimal robust synthesis is to find a controller K of the form (7) that ensures, with the prescribed accuracy, the inequality

$$I(K, \xi, \delta, \mu) \leq J(K_0(\xi), \xi, \delta, +\infty) \tag{14}$$

for the output y of the plant (1) with the *unknown* vector $\theta = (\xi^T, \delta^T)^T$.

Remark 1. The classical robust analysis and synthesis in Khammash and Pearson (1991,1993) and in Khammash, Salapaka, and Vanvoorhis (2001) deals with known systems under zero initial conditions, the class of infinite memory perturbations associated with $\mu = +\infty$ and the uniform control criterion of the form

$$I(K, \xi, \delta, +\infty) := \sup_{\Delta_1, \Delta_2} \sup_{w} \sup_{t \in \mathbb{N}} |\, y(t)\, | \quad (15)$$

For control of unknown plant, we have to replace the uniform control criterion (15) by the steady-state control criterion (8) to have a time for estimating unknown parameters. All the results on the robust stability and robust performance obtained in Khammash and Pearson (1991,1993) remain true for the steady-state control criterion (8) and nonzero initial conditions, if the perturbations Δ_1 and Δ_2 are additionally restricted to be of finite or fading memory (see Khammash, 1995 for detail). However, both of these properties are not verifiable by measurement data. In order to have an opportunity to verify the prior assumptions about perturbations, we consider perturbations with bounded memory of the form (4) and (5). In view of (13), the more the memory of perturbations μ, the lesser the conservatism associated with considering bounded memory perturbations will be.

Remark 2. One can prove that the controller $K_0(\xi)$ is optimal with respect to the control criterion (8) against the class of fading or finite memory perturbations, if $|b| \geq \delta_u$ for the plant with $|a| < 1$. Since models with $|b| < \delta_u$ are of no practical interest, they are excluded from consideration.

Remark 3. The controller K of the form (7) is generally impractical in view of possible unbounded controller's memory and the requirement of bounded memory must be taken into account under solving the stated problem.

3. OPTIMAL IDENTIFICATION FOR CONTROL

Let y_0^t be the collection of the plant outputs under some control actions u_0^{t-1} by the time instant t.

Definition. An extended parameter vector $\hat{\theta} = (\hat{\xi}^T, \hat{\delta}^T)^T$ is said to be *unfalsified* by the data y_0^t, u_0^{t-1}, if there exist an exogenous disturbance w and perturbations Δ_1, Δ_2 satisfying constraints (3), (4), and (5), respectively, and such that the equation (1) associated with the extended parameter vector $\hat{\theta}$ is satisfied on the time interval $[0, t]$.

The set of all unfalsified vectors $\hat{\theta}$ is clearly

$$\Theta_t := \{\ \hat{\theta} \mid |y(\tau) - \hat{a}y(\tau-1) - \hat{b}u(\tau-1)| \leq \hat{\delta}_w + \quad (16)$$

$$\hat{\delta}_y \sup_{\tau-\mu \leq s < \tau} |y(\tau)| + \hat{\delta}_u \sup_{\tau-\mu \leq s < \tau} |u(\tau)|,\ \tau = 0, \ldots, t\}$$

and is described by the system of $2(t+1)$ linear inequalities. Introduce simplifying notation

$$J(\theta) := J(K_0(\xi), \xi, \delta, +\infty)\,.$$

The problem of *optimal identification* for the data y_0^t, u_0^{t-1} is defined as

$$\min_{\hat{\theta} \in \Theta_t} J(\hat{\theta}) \quad (17)$$

and the minimizer in (17) is the best unfalsified vector $\hat{\theta}$.

According to Dahleh and Doyle (1994), "the basic control problem for a given process can be stated as follows: *Given some prior information about the process and a set of finite data, design a feedback controller that meets given performance specifications*". The problem (17) is the cited basic control problem meeting the strongest performance specification in the form of the optimality of controller to be designed.

The application of (17) in on-line identification faces with the following problems:
1. The computational complexity of the nonconvex programming in (17).
2. The requirement of bounded memory noted in Remark 3 above.
3. What is the behavior of the closed loop system with the minimizers in (17) as current estimates?

Problem 1 is discussed in this section and the problems 2 and 3 in the next one.

Approximate solution of the problem (17) is based on solution of optimal errors quantification problems on a grid in the prior set Ξ. Without loss of generality, we assume that the prior set Ξ is of the form

$$\Xi = [\, \underline{a}\, , \bar{a}\,] \times [\, \underline{b}\, , \bar{b}\,]$$

with known upper and lower bounds $\underline{a}, \bar{a}, \underline{b}, \bar{b}$ and $\underline{b}\,\bar{b} > 0$.

Choose arbitrary $\varepsilon_1 > 0$ and define

$$\xi_{i,j} = (a_i, b_j)^T := (\, \underline{a} + i\varepsilon_1\, , \underline{b} + j\varepsilon_1\,)^T\,,$$

$i = 0, 1, \ldots, n$, $a_n \in [\overline{a} - \varepsilon_1, \overline{a}]$, $j = 0, 1, \ldots, m$, $b_m \in [\overline{b} - \varepsilon_1, \overline{b}]$, so that the grid step is ε_1. For each grid point $\xi_{i,j}$ consider the linear-fractional program

$$J^{eq}(\hat{\xi}) := \min_{\{\hat{\delta} \mid\ \hat{\theta} \in \Theta_t\}} J(\hat{\xi}, \hat{\delta})\,, \quad (18)$$

which is called in Sokolov (2005) the problem of *optimal errors quantification.* It is well known that this linear fractional problem is reducible to linear programming by introducing a new variable (see, e.g., Boyd and Vandenberghe, 2003 for detail).

Approximate solution of the problem (17)

$$\theta_{\varepsilon_1} := (\xi_{\varepsilon_1}^T, \delta_{\varepsilon_1}^T)^T \quad (19)$$

is defined as follows :

$$\xi_{\varepsilon_1} := \operatorname*{argmin}_{\xi_{i,j}} \quad J^{eq}(\xi_{i,j}) \tag{20}$$

and δ_{ε_1} is the solution to the problem (18) for the coefficient vector ξ_{ε_1} :

$$J(\xi_{\varepsilon_1}, \delta_{\varepsilon_1}) = J^{eq}(\xi_{\varepsilon_1}) .$$

It must be noted that the minimizations on a fine grid by itself can not ensure approximate minimization of a nonconvex cost function with the prescribed tolerance for all linear constraints. The tolerance of the solution (19) to the problem (17) is possible to assess due to a specific interconnection between the cost function $J(\hat{\theta})$ and the linear constraints in this problem.

Theorem 2. There exists $C > 0$ such that for any data y_0^t, u_0^{t-1}

$$J(\theta_{\varepsilon_1}) \leq \min_{\hat{\theta} \in \Theta_t} J(\hat{\theta}) + C\varepsilon_1 . \tag{21}$$

The proof of Theorem 2 is omitted.

So to solve the problem (17) with the ε_1 tolerance, the number of the linear fractional problems (18) to be solved is in the order of $1/\varepsilon_1^2$.

4. OPTIMAL STEADY-STATE TRACKING

Solution of Problems 2 and 3 mentioned in the previous section is based on the use of outer approximations of the sets Θ_t. The estimation algorithm below is similar to that in Sokolov (2001b) with the only difference in the use of the control criterion associated with the tracking problem instead of the regulation problem.

The initial set estimate is of the form

$$E(-1) := \{ \ \hat{\theta} = (\hat{\xi}^T, \hat{\delta}^T)^T | \ \hat{\xi} \in \Xi , \hat{\delta} \geq 0 \ \}$$

where the inequality $\hat{\delta} \geq 0$ is taken componentwise. Initial point estimate is of the form

$$\theta(-1) := (\xi^T(-1), 0, 0, 0)^T$$

with arbitrary $\xi^T(-1) \in \Xi$. Let $\theta(t-1) = (\xi(t-1)^T, \delta(t-1)^T)^T$ be a point estimate and $E(t-1)$ be a set estimate of the unknown θ at the time instant $t-1$. Define auxiliary notation

$$\psi(t-1) := (y(t-1), u(t-1))^T ,$$

$$\eta(t) := \operatorname{sign}(y(t) - \xi(t-1)^T \psi(t-1)) ,$$

$$m_y(t) := \sup_{t-\mu \leq s < t} |y(s)|, \ m_u(t) := \sup_{t-\mu \leq s < t} |u(s)| ,$$

$$\phi(t-1) := (\eta(t)\psi(t-1)^T, 1, m_y(t), m_u(t))^T .$$

$$\Omega(t) := \{ \ \hat{\theta} \mid \hat{\theta}^T \phi(t-1) \geq y(t)\eta(t) \ \} .$$

Chose a pair of positive scalars ε_1 and ε_2, parameters of the estimation algorithm. Then the set estimate $E(t)$ is defined as follows. If

$$\theta(t-1)^T \phi(t-1) \geq y(t)\eta(t) - \varepsilon_2 |\phi(t-1)|$$

then $E(t) := E(t-1)$; otherwise

$$E(t) := E(t-1) \cap \Omega(t) . \tag{22}$$

If the set estimate $E(t-1)$ was not updated then the point estimate $\theta(t-1)$ is not updated too. Otherwise $\theta(t)$ is defined as the approximate solution of the problem

$$\min_{\hat{\theta} \in E(t)} J(\hat{\theta}) \tag{23}$$

obtained by the algorithm described in section 3 with the grid step ε_1.

The set estimation algorithm has a simple geometric interpretation. At the time instant $t-1$ the set estimate $E(t-1)$ is described by a part of the linear inequalities in (16). New information about the unknown vector θ at the time instant t is of the form

$$|y(\tau) - \hat{a}y(\tau-1) - \hat{b}u(\tau-1)| \leq \hat{\delta}_w + \hat{\delta}_y m_y(t) + \hat{\delta}_u m_u(t)$$

and can be rewritten as a pair of linear in $\hat{\theta}$ inequalities. Only one of these inequalities, describing the half-space $\Omega(t)$, can be violated for the point estimate $\theta(t-1)$. The set estimate $E(t-1)$ is updated if and only if the distance of the current point estimate $\theta(t-1)$ to the half-space $\Omega(t)$ is greater than ε_2. The update of $E(t-1)$ according to (22) with deleting redundant inequalities can be effectively performed by the algorithm proposed in Walter, Piet-Lahanier (1989).

Finally, the control action $u(t)$ at the time instant t is realized by the controller $K(\xi(t))$:

$$u(t) = -\frac{a(t)}{b(t)} y(t) + \frac{1}{b(t)} r . \tag{24}$$

The next theorem shows that the controller (24) solves the stated problem with the prescribed accuracy.

Theorem 3. Assume the unknow extended parameter vector θ satisfies the prior constraints

$$\delta_y + \delta_u \frac{|a|}{|b|} < 1 - \kappa , \quad 0 \leq \delta_w \leq \bar{\delta}_w \tag{25}$$

with known $\kappa > 0$ and $\bar{\delta}_w$. Then there exist positive C_1, C_2, $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$ such that for all $\varepsilon_1 \leq \bar{\varepsilon}_1$ and $\varepsilon_2 \leq \bar{\varepsilon}_2$ the estimation algorithm (22) and (23) and the controller (24) ensure the following properties of the closed loop system:

1. for all sufficiently large t

$$E(t) = E_\infty := \lim_{t \to \infty} E(t) ,$$

$$\theta(t) = \theta_\infty = (\theta_{c,\infty}^T, \delta_\infty^T)^T := \lim_{t \to \infty} \theta(t) ,$$

2. the output of the closed loop system satisfies the inequality

$$\limsup_{t\to\infty} |\, y(t) - r| \leq \qquad (26)$$
$$J(K_0(\xi), \xi, \delta, +\infty) + C_1\varepsilon_1 + C_2\varepsilon_2 \,.$$

The proof of Theorem 3 is omitted.

The inequality (26) means that the stated problem (14) can be solved with the desired accuracy. The prior constraints (25) are nonrestrictive. Indeed, the first of them is a slightly stronger condition of robust stability for the plant (1) controlled by the optimal controller (9). The arbitrary small constant κ and the arbitrary large upper bound $\bar{\delta}_w$ simply exclude models with large values of $J(\theta)$.

5. SIMULATIONS AND CONCLUDING REMARKS

The controlled plant is modeled by the equation

$$y(t) = 0.983y(t-1) + 0.52u(t-1) + d(t) \quad (27)$$

where the total disturbance d is of the form

$$d(t) = 0.1\,\rho_0(t) + 0,06\,\rho_1(t) \max_{t-5\leq<t} |y(s)| +$$
$$0,11\,\rho_2(t) \max_{t-5\leq<t} |u(s)|$$

and $\rho_j(t)$, $t \in \mathbb{N}$, $j = 0,1,2$ are independent uniformly distributed on $[-1,1]$ random sequences so that the unknown vector of upper bounds is of the form $\delta = (0.1,\, 0.06,\, 0.11)^T$. The prior information about the unknown coefficient vector $\xi = (0.983,\, 0.52)^T$ is of the form

$$\xi \in [\,0.7\,,\, 1.2\,] \times [\,0.2\,,\, 1\,]\,.$$

The initial controller of the form (9), associated with the prior initial estimate

$$\xi(-1) = (\,0.95\,;\, 0.4\,)^T,$$

is not updated on the time interval [0, 300]. The control objective is in tracking the set point $y = 50$ so that the reference signal is

$$\forall\, t \quad r(t) = 50\,.$$

The corresponding error signal $z(t) := y(t) - r(t)$ on the time interval [0,300] is presented on the upper plot in the Fig. 1. One can see a considerable deflection from zero generated by the inaccuracy of the initial estimate $\xi(-1)$.

The approximate solution of the problem of optimal identification (17) for the data (y_0^{300}, u_0^{299}) was computed using the relatively large grid step $\varepsilon_1 = 0.05$. Solution of the associated 11×17 linear fractional problems of errors quantification (18) for the grid points $\xi_{i,j}$ took 2.671 sec. on the PC with AMD Athlon 64 3000+ processor.

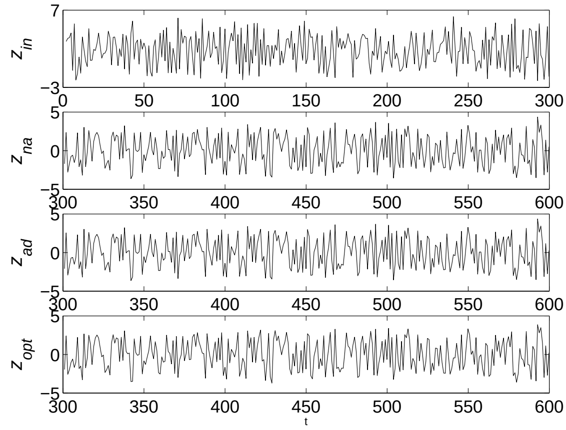

Fig. 1. Error signals $z(t) = y(t) - 50$: z_{in} – for initial controller; z_{na} – for nonadaptive controller, associated with $\xi(300)$; z_{ad} – for adaptive controller; z_{opt} – for optimal controller for known model.

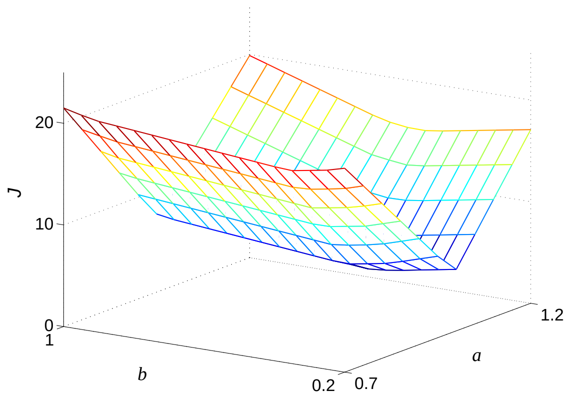

Fig. 2. The values of $J^{eq}(\xi_{i,j})$ for the grid step $\varepsilon_1 = 0.05$.

The values of $J^{eq}(\xi_{i,j})$ are presented in the Fig. 2. The minimal value of $J^{eq}(\xi_{i,j})$ was 4.6847 for the grid point $\xi_{i,j} = (1, 0.45)^T$. Then the Matlab function *fmincon* was exploited for solution of (17) using the coefficients $(1, 0.5)^T$ as initial points. The approximate solution of (17) was

$$\theta(300) = (0.9845,\ 0.4697,\ 0,\ 0.0732,\ 0)^T\,.$$

Notice that the total disturbance was prescribed entirely to the perturbation in output p_y. Then

$$J(\theta(300)) = 3.9501 < 4.2022 = J^{eq}(\xi) <$$
$$4.6219 = J(\theta) < 6.1604 = J^{eq}(\xi(-1))\,.$$

Note that the middle values of $J^{eq}(\xi)$ and $J(\theta)$ are unknown to controller designer because the vector θ is unknown. But they illustrate the properties of the initial estimate $\xi(-1)$ and the obtained estimate $\theta(300)$.

The other 3 plots in Fig. 1 represent the following signals. The signal $z_{na} = y(t) - 50$ is the error signal when the model was controlled by the controller of the form (9) associated with the coefficient vector $\xi(300)$. The signal $z_{ad} = y(t) - 50$ is the error signal when the model was controlled by the adaptive controller (24). Note that identification problems (23) were solved using the *fmincon* function instead of computations on grids for all $t > 300$. Finally, the signal $z_{opt} = y(t) - 50$ is the error signal when the model was controlled by the optimal controller (9) in the case of the known coefficient vector ξ. The simulations in all three cases used identical samples of $\rho_k(t)$.

The final estimate under the adaptive control (24) was

$$\theta(600) = (0.9843,\ 0.5037,\ 3.5425,\ 0,\ 0.0576)^T$$

and the final best unfalsified value of the control criterion was $J(\theta(600)) = 4,0929$, which is less than $J(\theta)$.

The set estimates $E(300)$ and $E(600)$ computed with the dead zone parameter $\varepsilon_2 = 0.0001$ were described both by 23 inequalities. The time of computations for the time interval [0, 300] was 2.671 sec. and of all other computations for the time interval [301, 600] – 4.828 sec.

Finally, the ranges $\max_t z(t) - \min_t z(t)$ for all signals presented in Fig. 1 are 8.2926, 7.9889, 7.9889, 7.57 and the values of $\max_t |z(t)|$ are 6.1604, 4.3593, 4,3593, 3.8571, respectively. These figures mean that both the fixed controller associated with the estimate $\theta(300)$ and the on-line identification based controller provide equally good robust performance, which is close to the robust performance of the optimal controller for the known model. Note that the difference between the signals z_{na} and z_{ad} ($\max_t |z_n a(t) - z_a d(t)| = 0.3817$) is difficult to see from their graphs.

The main merits of the identification algorithm in this paper are as follows.

1. The identification algorithm is adequate to the ℓ_1 robust control theory and involves no other assumptions about the uncertainty and the exogenous disturbance.

2. All prior information regarding the plant model, the uncertainty and the exogenous disturbance is validated on-line and models falsified by data are discarded.

3. The on-line optimal identification allows to achieve the best possible result in the form of the optimality of closed loop system against the considered class of perturbations. The price for this result is the necessity of large computations.

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