Aircraft Motion Estimation under Conditions of Uncertainty ⋆

S. I. Kumkov * **S. G. Pyatko** **

* *Institute of Mathematics and Mechanics, Ural Branch RAS, S.Kovalevskaya str., 16, Ekaterinburg, 620219, Russia, Tel. (7 343) 3748332, Fax (7 343) 3742581, e-mail: kumkov@imm.uran.ru*
** *NITA, Llc., Russia, e-mail: nita@nita.ru*

Abstract: The work deals with application of the interval analysis methods to the outer estimation of the current set of states (geometrical coordinates and velocities) of aircraft, which motion is described by the standard system of ordinary differential equations of the sixth order with known geometric constraints onto controls in the longitudinal, vertical, and lateral accelerations; the current aircraft controls are not known for the observer. Estimation is performed under conditions of uncertainty about both measuring errors restricted on modulus and the chaotic disturbances that are unknown on the sign and value. The result of estimation is represented by the six–dimensional parallelepiped composed of interval estimates on each of the phase coordinates computed by special sequential procedures.

Keywords: Aircraft, motion parameters, measuring errors, chaotic disturbances, uncertainty of probability characteristics, estimation, algorithms.

1. INTRODUCTION AND PROBLEM FORMULATION

Usually, estimation of motion of dynamic systems are based mainly on obtaining a sample of sufficiently large length and application of procedures of the mathematical statistics (for example, Kalman filtration approaches). There, the probabilistic data about both the measuring errors and possible chaotic perturbations (e.g., the probability distribution laws, means, variances, correlation indices, *etc.*) have to be known.

But, in practice, for example, in air traffic control systems, the cases prevail when no such information is known, but only approximate geometric constraints can be put on the maximal values of the measuring errors, the measurement sample for processing is very short (only 5–6 sequential measurements), and the measurements are corrupted by both the usual measuring errors and chaotic perturbations with unknown characteristics. In this paper application of the interval analysis Jolin et al. (2001), Milanese and Norton (1996), Kalmykov et al. (1986) and informational sets theory Kumkov and Patsko (2001), Kumkov (2008), Patsko et al. (1999) to estimation of the current states (geometrical coordinates and velocities) of aircraft is considered.

The aircraft motion in the three-dimensional space is described by the standard system of ordinary differential equations with geometric constraints on controls in the longitudinal, vertical, and lateral accelerations. The aircraft current controls are unknown for the observer. All phase coordinates, i.e., three components of the velocity and three geometric coordinates are measured. Measurement of each phase coordinate is corrupted both by the usual measuring error of known geometric constrains (in modulus) and chaotic perturbation of unknown sign and values. No statistical (probability) data about both disturbances are known. The structure of errors of both types and their behavior in time can be arbitrary. The case is investigated when the length of the measurement sample is short, and estimation is implemented in the "sliding window" of measurements sequentially coming from some informational system (for example, Automatical Dependent Observation System–Broadcasting on the basis of GPS or GLONASS measurements, or from on–board aircraft navigational systems, *etc.*).

Problem formulation. *Having the input sample of corrupted measurements of the aircraft motion (geometrical positions and velocity components) and the given description of the aircraft dynamics, it is necessary to estimate the outer (from above) current set of its states.*

Since the simultaneous analysis in the six-dimensional phase space is hampered in practice, the problem is solved by constructing the outer box–estimation composed of interval estimations on each phase coordinate that are found in special sequential order. This guarantees the outer box-estimation to be approximately minimal in size.

The uncertainty of the measuring error is formalized in the form of the *uncertainty interval* or *uncertainty set* of measurement for each aircraft phase coordinate by the given constraint on the measuring error.

Estimation and construction of the current set of aircraft coordinates are performed in presence of possible chaotic corruption. As a result, the input sample in each coor-

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dinate can be inconsistent in the whole, can split into possible parallel subsamples (registers). These subsamples, in the turn, can split or fuse between themselves; the false subsamples generated by the chaotic disturbances do not continue are eliminated.

2. NOTIONS AND DEFINITIONS

Aircraft motion is described by the following ordinary differential equation system (officially standardized for navigational computations):

$$\begin{aligned}
&\dot V = a, \quad \dot\theta = \alpha/V, \quad \dot\psi = \beta/V,\\
&|a(t)| \le a^{\rm ap}_{\max},\ |\,\alpha(t)\,| \le \alpha^{\rm ap}_{\max},\ |\,\beta(t)\,| \le \beta^{\rm ap}_{\max},\\
&\dot y = V_y = V \sin\theta,\\
&\dot x = V_x = V \cos\theta \cos\psi,\\
&\dot z = V_z = V \cos\theta \sin\psi.
\end{aligned} \tag{1}$$

Here, x, z, and y (in meters) are the aircraft coordinates in the standard normal ground coordinate system, in which the origin point O is located at some reference point, the OX-axis is directed to the Notrh, the OZ-axis is directed to the East, the OY-axis is directed along the local vertical, the plane XOZ coincides with the local horizontal plane; V_x, V_z, and V_y (in m/sec) are the componentns of the velocity along the coordinate axes; V (in m/sec) is the longitudinal (space) velocity; θ (in radians) is the velocity angle with respect to the local horizontal plane, the angle is count upward from this plane; ψ (in radians) is the heading angle of the projection $V_{\rm hor}$ of the velocity V into the horizontal plane, the heading is counted clockwise from the OX-axis; $a(t)$ (in m/sec^2) is the longitudinal acceleration with the *a priory* constraint $a^{\rm ap}_{\max}$ and can have arbitrary piecewise-constant structure in time, all such realizations of this control are called *admissible*; $\alpha(t)$ (in m/sec^2) is the vertical acceleration with the *a priory* constraint $\alpha^{\rm ap}_{\max}$ and can have arbitrary piecewise-constant structure in time, all such realizations of this control are called *admissible*, the acceleration $\alpha(t)$ is orthogonal to the vector of the longitudinal velocity and lieing in the vertical plane; β (in m/sec^2) is the lateral acceleration with the *a priory* constraint $\beta^{\rm ap}_{\max}$ and can have arbitrary piecewise-constant structure in time, all such realizations of this control are called *admissible*, the acceleration $\beta(t)$ is orthogonal to the vector of the longitudinal velocity and lieing in the horizontal plane.

Moreover, the following finite relations between the phase coordinates are used:

$$\begin{aligned}
&V = \sqrt{V_x^2 + V_z^2 + V_y^2},\ V_{\rm hor} = \sqrt{V_x^2 + V_z^2},\\
&\theta = \arctan\,(V_y/V), \quad \psi = \arctan\,(V_z/V_x).
\end{aligned} \tag{2}$$

Direct measurements and their uncertainty sets. The following measurements of the aircraft motion are called *direct*, since they are directly provided to the observer:

$$\begin{aligned}
&\{\,t_n,\ V_{x,n},\ V_{z,n},\ V_{y,n},\ x_n,\ y_n,\ z_n\,\}, n = 1, N,\\
&\text{with ordered instants :}\\
&\text{for all} \quad n = 1, 2, ..., \quad t_n < t_{n+1},
\end{aligned} \tag{3}$$

where n is the number of each measurement; N is the number of measurements in the sample to be processed; t_n (in seconds) is the instant of each measurement along the scale of the absolute time.

At the instant t_n, $n = 1, \ldots$, each phase coordinate is measured with the usual measuring error constrained geometrically and can be corrupted by some chaotic disturbance. The model of corruption has the form (for example, for the V_x component):

$$\begin{aligned}
&V_{x,n} = V^*_{x,n} + \varepsilon^{V_x} + \chi^{V_x},\\
&|\varepsilon^{V_x}| \le \varepsilon^{V_x}_{\max};\ \chi^{V_x} \sim \text{unknown},
\end{aligned} \tag{4}$$

where $V^*_{x,n}$ is unknown true value; ε^{V_x} is the error of measuring with the geometrical constraint $\varepsilon^{V_x}_{\max}$; χ^{V_x} is unknown possible chaotic disturbance. Measurements of other phase coordinates have the similar structure of corruption. The instants t_n of measuring are assumed to be known exactly.

For each measurement of each phase coordinate, the *uncertainty inteval* or *uncertainty set* is put in correspondence:

$$\begin{aligned}
&H^{V_x}_n = [\underline{h}^{V_x}_n,\, \overline{h}^{V_x}_n] :\\
&\underline{h}^{V_x}_n = V_{x,n} - \varepsilon^{V_x}_{\max};\ \overline{h}^{V_x}_n = V_{x,n} + \varepsilon^{V_x}_{\max},
\end{aligned} \tag{5}$$

where $\varepsilon^{V_x}_{\max}$ is the geometrical constraint onto the measuring error in V_x; $\underline{h}^{V_x}_n$ and $\overline{h}^{V_x}_n$ are the lower and the upper boundaries of the uncertatinty interval. For other phase coordinates, the similar uncertainty sets are computed.

Indirect measurements and their uncertainty sets. As it was mentioned above, the simultaneous consistency analysis of the measurements (3) in the six-dimensional phase space of the dynamic system (1) is hampered in practice. So, for reasonable computational implementation, the sequential analysis of the measurements samples in each phase coordinate is performed on the first steps for the variables V, θ, and ψ, which values at the instants t_n are regarded as the auxiliary or *indirect* measurements.

By values of the direct measurements (3), the values of the indirect measurements are calculated evidently by relations (2), and the similar relations are used for calculations of the uncertainty intervals of the indirect measurements for $n = 1, \ldots$:

$$\begin{aligned}
&H^V_n = [\underline{h}^V_n,\, \overline{h}^V_n], \quad H^\theta_n = [\underline{h}^\theta_n,\, \overline{h}^\theta_n],\\
&H^\psi_n = [\underline{h}^\psi_n,\, \overline{h}^\psi_n].
\end{aligned} \tag{6}$$

Forecast sets of the phase coordinates. Let we have an uncertainty interval, for example, H^V_n at the instant t_n and the dynamics of this phase coordinate is described by a corresponding equation from (1).

Then the *forecast interval*, *forecast set*, or *attainability set* of possible values of this coordinate at the instant t_{n+1} is the interval constructed in the following special way:

$$\begin{aligned}
&G^V_{n+1} = [\underline{g}^V_{n+1},\, \overline{g}^V_{n+1}] :\\
&\delta T = t_{n+1} - t_n,\\
&\underline{g}^V_{n+1} = \overline{h}^V_n - a^{\rm ap}_{\max}\delta T;\ \overline{g}^V_{n+1} = \underline{h}^V_n + a^{\rm ap}_{\max}\delta T.
\end{aligned} \tag{7}$$

From (7) it is seen that the construction is implemented only from the marginal points of the set H^V_n under the marginal values $\pm a^{\rm ap}_{\max}$ of control $a(t)$. The similar attainability sets can be similarly constructed for other phase coordinates.

Consistency and inconsistency of pair of measurements. If the forecast set, for example, interval G_{n+1}^V at the instant t_{n+1} *completely includes* the uncertainty interval H_{n+1}^V at this instant

$$H_{n+1}^V \subset G_{n+1}^V, \tag{8}$$

then the nth and $(n+1)$th masurements of this coordinate are called *consistent*. Otherwise, they are called *inconsistent*.

The uncertainty interval of a consistent measurement is used to call the *current informational interval* I_n^V (or *informational set*) of this coordinate.

Remark 1. *From the engineering point of view, inclusion* (8) *means that by an admissible control from any point in* H_n^V *it is possible to achieve the whole interval* H_{n+1}^V *and otherwise. Such type of consistency is used to call strong.*

Consistency of a sample of measurements. A sample of measurements comprised of the consequently consistent (in the given sense) measurements is called *consistent*.

The tube of admissible trajectories in each phase coordinate. Let we have a pair of consistent measurements with informational intervals $I_n^V = H_n^V$ and $I_{n+1}^V = H_{n+1}^V$. The totality of all trajectories, for example of velocity, $\{V(t)\}$ beginning with the left ends at I_n^V at the instant t_n and right ends at I_{n+1}^V at the instant t_{n+1}, generated by all admissible controls $a(t)$ from (1) is called the *tube of admissible trajectories* $\mathrm{Tb}(V, n, n+1)$. The tubes of the admissible trajectories in other phase coordinates are defined similarly.

3. THE MAIN PROCEDURES OF ANALYSIS AND ESTIMATION (ON EXAMPLE OF VELOCITY CHANNEL)

Elaborated algorithms for building the current set of admissible states of aircraft includes the following main procedures.
1. Elimination of measurements inconsistent in time (formalization of corresponding engineering criterion).
2. Analysis of consistency of each next coming measurement with each accumulated subsample.
3. "Splitting" the current inconsistent sample into consistent (in itself) parallel subsamples (registers).
4. Elimination of subsamples that can not be continued.
5. "Fusion" of parallel subsamples.
6. Shift of the "sliding window" on each subsample.
7. Constructing the output informational set or a collection of informational sets in the case of unique or several parallel subsamples, correspondingly.

1.Elimination of measurements inconsistent in time implements simple engineering criterion of minimal admissible time–tempo $\tau_{\min}$ of coming the next measurement (Fig.1).

2. Analysis of consistency of each next coming measurement with each accumulated subsample. The next coming measurement is analyzed on consistency with the last measurement of each subsample (Fig.2). In the case of

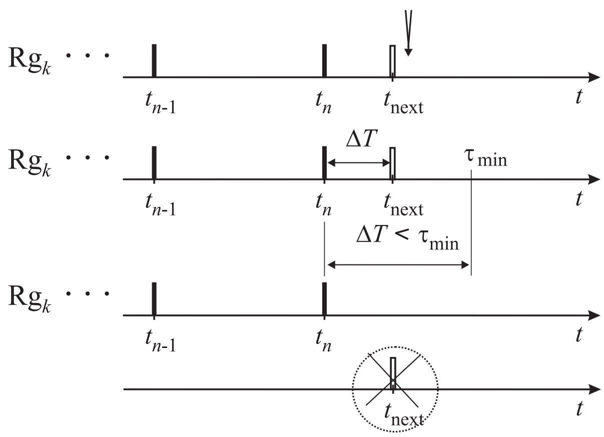

Fig. 1. Elimination of time–inconsistent measurement

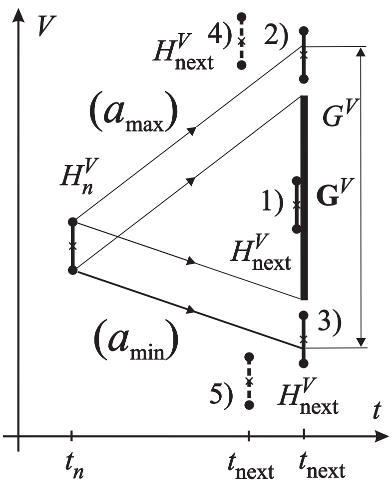

Fig. 2. Analysis consistency of pair of measurements

inconsistency of the next measurement with the last and the pre–last measurements of the accumulated subsamples, this measurement is taken as a beginning of a new subsample (register Rg) (Fig.3).

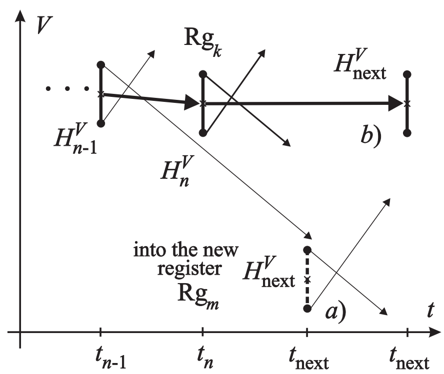

Fig. 3. Consistency and inconsistency of the next measurement with accumulated subsamples

3. "Splitting" the current inconsistent sample into consistent (in itself) parallel subsamples. Operation of this procedure is illustrated in Fig.4.

4. Elimination of subsamples that can not be continued. This procedure implements simple engineering criterion of

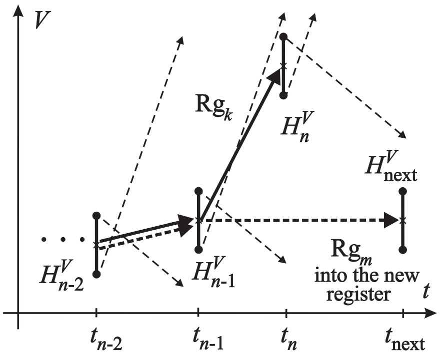

Fig. 4. Splitting of a sample into parallel subsamples

maximal admissible time–delay $\tau_{\max}$ of coming the next measurement (Fig.5).

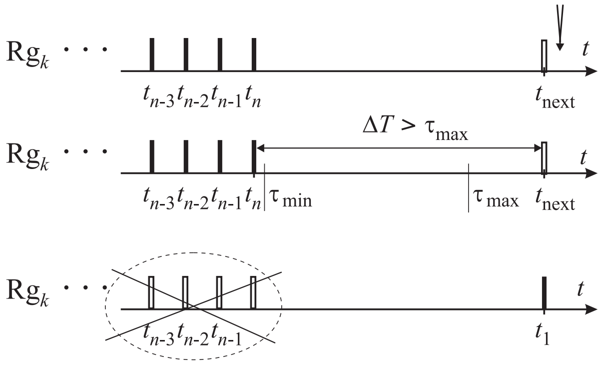

Fig. 5. Elimination of a non–continuable subsample

5. "Fusion" of parallel subsamples. The procedure detects fusion of a pair of subsamples of the next coming consistent measurement. As a result, one of the subsamples is taken for further consideration by additional engineering criterion: a) by the maximal length, b) by the minimal "summary expenditure of control". The other subsample is deleted (Fig.6, solid arrows).

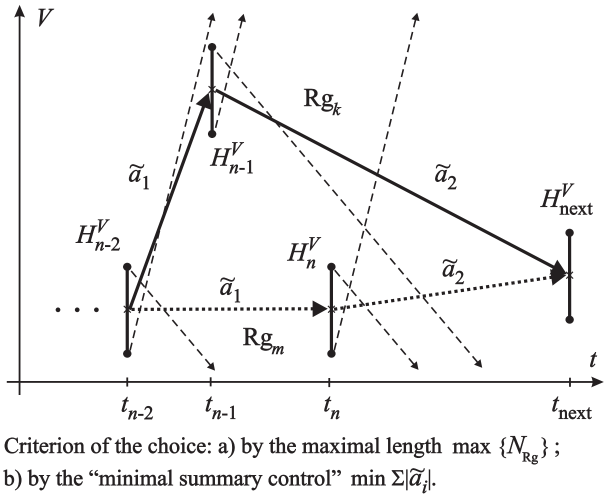

Fig. 6. Fusion of subsamples

6. Shift of the "sliding window" on each subsample, if the next coming measurement is consistent with some subsample. The standard time–shift is performed in the case of complete previous occupation of the window by this sample.

7. Constructing the output informational set or a collection of informational sets in the case of unique or several parallel subsamples. Formation of the output informational set (interval) in the case of unique consistent sample is shown in Fig.7. The case with two parallel subsamples is given in Fig.8.

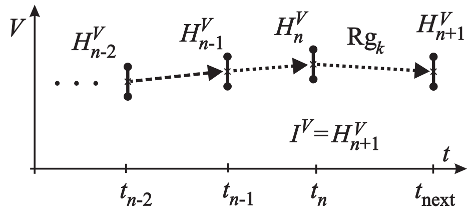

Fig. 7. Output informational interval (for unique sample

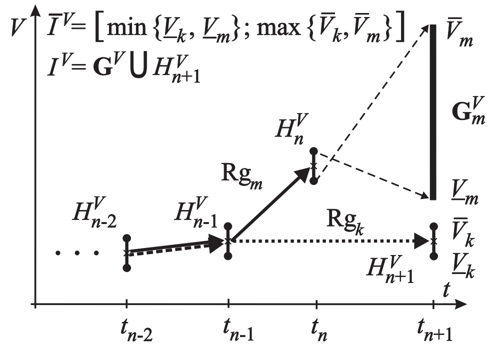

Fig. 8. Output informational intervals (for several subsamples

The typical tube $\mathrm{Tb}(V, n-1, n)$ of the admissible trajectories between informational intervals I_n^V and I_{n+1}^V of two consistent measurements is shown in Fig.9. Here, the upper $\overline{V}(t)$ and the lower $\underline{V}(t)$ boundary trajectories are performed on corresponding marginal values of the control $a(t)$.

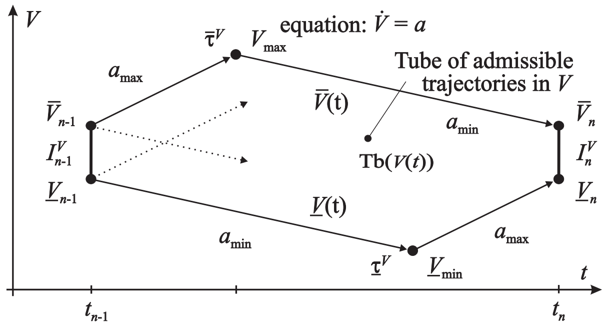

Fig. 9. Tube of admissible trajectories in V

In the main procedures, the analysis of consistency in θ–channel repeats the procedures described above for the V channel. Recall, that processes in the phase coordinate $\theta(t)$ are stipulated by the special form of its differential equation in (1)

$$\dot{\theta} = \alpha / V, \tag{9}$$

where the control acceleration (unknown to the observer) is constrained geometrically $-\alpha^{\rm ap}_{\max} \le \alpha(t) \le \alpha^{\rm ap}_{\max}$ and can have arbitrary piecewise-constant structure in time.

Equation (9) is nonlinear with respect to the phase coordinate V, and in the general case of realization $V(t)$ it is impossible to obtain some forecast trajectory $\theta(t,\alpha(t),V(t))$ in the finite form. But in the forecast we are only interested in the extremal lower and upper boundary points of the forecast interval in θ

$$\underline{\theta}(t_{n+1}) = \overline{\theta}_n + \min\{\int\limits_{t_n,t_{n+1}} \alpha(t)dt/V(t)\},$$
$$\overline{\theta}(t_{n+1}) = \underline{\theta}_n) + \max\{\int\limits_{t_n,t_{n+1}} \alpha(t)dt/V(t)\}. \tag{10}$$

The extremal forecast trajectories in Eq.(10) are acheived on the corresponding extremal values $\pm\alpha^{\rm ap}_{\max}$ of accelerations and the lower extremal trajectory $\underline{V}(t)$ of the tube in V in this interval. Recall that for aircraft motions everywhen $\underline{V}(t) \gg 0$. This trajectory can consist of (see Fig.9): only one segment with constant value of the acceleration $a(t) \equiv -a^{\rm ap}_{\max}$ or $a(t) \equiv a^{\rm ap}_{\max}$, or maximum two segments with the sequence of constant values of the acceleration $a(t) = -a^{\rm ap}_{\max} \to a^{\rm ap}_{\max}$. It allows to obtain the boundary points of the forecast set $G^\theta_{n+1}) = [\underline{\theta}_{n+1},\ \overline{\theta}_{n+1}]$ in very simple finite forms.

Having the forecast interval G^θ_{n+1}, the analysis of consistency of each next coming measurement θ_n and all other procedures in the θ–channel are performed similarly to such operations in the V–channel. As a result, we obtain the sequence of informational intervals I^θ_n and tubes $\mathrm{Tb}(\theta,n,n+1)$ of admissible trajectories in θ.

The similar procedures are implemented in the ψ–channel resulting in obtaining the sequence of informational intervals I^ψ_n in ψ and corresponding tubes $\mathrm{Tb}(\psi,n,n+1)$.

4. PROCESSING THE CHANNELS OF GEOMETRIC COORDINATES

Channel of the vertical coordinate y. Now, having the tube $\mathrm{Tb}(V,n,n+1)$ in V and tube $\mathrm{Tb}(\theta,n,n+1)$ in θ, and dynamics (1) for vertical coordinate y

$$\dot y = V\sin\theta, \tag{11}$$

it becomes possible to implement the consistency analysis of the sample of measurements in this coordinate on the basis of uncertainy intervals H^y_n, $n = 1,\ldots$.

The extremal forecast of the lower and upper points the forecast interval G^y_{n+1} in y is implemented similarly to (10) as follows:

$$\underline{y}(t_{n+1}) = \overline{h}^y_n + \min_{\mathrm{Tb}(V,n,n+1),\mathrm{Tb}(\theta,n,n+1)}\{ \int\limits_{t_n,t_{n+1}} V(t)\sin\theta(t)dt\};$$
$$\overline{y}(t_{n+1}) = \underline{h}^y_n + \max_{\mathrm{Tb}(V,n,n+1),\mathrm{Tb}(\theta,n,n+1)}\{ \int\limits_{t_n,t_{n+1}} V(t)\sin\theta(t)dt\}. \tag{12}$$

Unfortunately, in general case, these extremal forecast points can be calculated only numerically. Moreover, the crucial fact in optimization of the integrals can appear: the optima could be acheived **on the internal trajectories of the tubes** $\mathrm{Tb}(V,n,n+1)$ and $\mathrm{Tb}(\theta,n,n+1)$.

Thus, for practical implementation, special numerical algorithms were elaborated that give the approximate estimate of $\underline{y}(t_{n+1})$ from below and the approximate estimate of $\overline{y}(t_{n+1})$ from above.

Estimation of informational intervals I^y_n is carried out (on the basis of the constructed forecast interval G^y_{n+1}) by operations, similar to ones in V–, θ–, and ψ–channels.

Channel of the coordinates x and z. Having the tube $\mathrm{Tb}(V,n,n+1)$ in V, tube $\mathrm{Tb}(\theta,n,n+1)$ in θ, tube $\mathrm{Tb}(\psi,n,n+1)$ in ψ, and dynamics (1) for the coordinate x

$$\dot x = V\cos\theta\cos\psi, \tag{13}$$

it becomes possible to implement the consistency analysis of the sample of measurements in this coordinate by uncertainty intervals H^x_n, $n = 1,\ldots$.

The extremal forecast lower and upper points of the forecast interval G^x_{n+1} in x are computed

$$\underline{x}(t_{n+1}) = \overline{h}_{x,n} + \min_{\mathrm{Tb}(V,n,n+1),\mathrm{Tb}(\theta,n,n+1),\mathrm{Tb}(\psi,n,n+1)}\{ \int\limits_{t_n,t_{n+1}} V(t)\cos\theta\cos\psi dt\ \},$$
$$\overline{y}(t_{n+1}) = \underline{h}_{y,n} + \max_{\mathrm{Tb}(V,n,n+1),\mathrm{Tb}(\theta,n,n+1),\mathrm{Tb}(\psi,n,n+1)}\{ \int\limits_{t_n,t_{n+1}} V(t)\cos\theta\cos\psi dt\ \}. \tag{14}$$

Unfortunately, just as in constructing the extremal forecast points of the forecast interval in the coordinate y, the points in (14) can be calculated only numerically. Moreover, the same crucial fact in optimization of the integrals can appear: the optima could be acheived **on the internal trajectories of the tubes** $\mathrm{Tb}(V,n,n+1)$, $\mathrm{Tb}(\theta,n,n+1)$, and $\mathrm{Tb}(\psi,n,n+1)$.

Estimation of informational intervals I^x_n is carried out (on the basis of the constructed forecast interval G^x_{n+1}) by operations, similar to ones in V–, θ–, ψ–, and y–channels.

For practical implementation, special numerical algorithms were elaborated that give the approximate estimate of $\underline{x}(t_{n+1})$ from below and the approximate estimate of $\overline{x}(t_{n+1})$ from above.

The channel of the coordinate z is processed similarly to the x–channel with its own dynamics (1).

5. CONCLUSION

The approach based on ideas of the interval analysis and informational sets showed its effectiveness in applications to processing measurements of the aircraft motion corrupted by measuring errors and chaotic perturbations.

Under conditions of uncertainty and absence of any statistical characteristics of disturbances, it is succeeded to check out and eliminate both outliers and false subsamples generated by outliers, and to construct the output interval box–estimation of the current phase coordinates of the aircraft motion. Numerical experiments with model and real information have shown stable work of the elaborated algorithms.

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