

# Wavelet Analysis of Rotor Vibration

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**Abstract:** The aim of this paper is to introduce a method of dynamic analysis for rotor systems based on wavelet transformation of oscillations. Comparing introduced and traditional analysis methods, advantages of wavelet analysis at the random impulsive loading and quasi-stationary motion regimes are opened. As the starting system an unbalanced rotor on oil - film journal bearings is considered.

**Keywords:** rotor, oil - film journal bearing, vibration, wavelet analysis

## 1. INTRODUCTION

Amplitude detection and spectral analysis are well-known methods in order to analyze the vibrosignals. By amplitude spectrum the researchers fix an existence of resonant, impact-excited, transient and unstable rotor vibration. Sometimes this way is called by «trajectories method» (Komarov, 2005). However the conclusion concerning vibration type through signal level history is unobvious if a decay or rising of motion amplitude is quasi-stationary. As a rule by frequency spectrum the researchers appreciate a technical state of system (Rusov, 1996). In particular it may be done in order to discover next defects: unbalance, impact excitation and «oil whirl»<sup>1</sup> of rotor (Oravsky, 2001). However spectral analysis based on Fourier transformation is not effective for investigation of the random impulsive loading and defects development and transient processes. Disadvantages of traditional analysis methods may be overcome using the wavelet analysis penetrated deep into practice of digital signal processing, for example look through the language of the computing MatLab software system.

## 2. ESSENCE OF WAVELET ANALYSIS

As is well known Fourier transformation replaces the time representation of vibrosignal  $f(t)$  by its frequency representation  $f(\omega)$  that is expansion in infinite sinusoid with different frequency:

$$f(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt,$$

where  $f(t)$  is virgin signal,  $\omega$  is frequency,  $t$  is time,  $f(\omega)$  is spectral expansion of virgin signal. Wavelet transformation of function  $f(t)$  consists in its expansion in set of scalable and shifted versions of small wave:

$$C(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right) dt,$$

<sup>1</sup> Oil whirl is oil-induced subsynchronous forward whirl with rotor half-speed typically varying from  $0.46\omega$  to  $0.49\omega$ .

where  $f(t)$  is virgin signal,  $\psi$  is basic (small) wave function or wavelet dependent on parameters  $a$  and  $b$ , which accordingly define scale and shift of time.

For signal  $f(t)$  having pronounced oscillatory form it is quite reasonable to choose wavelet close to sinusoid. For example look at the Morlet's wavelet (Fig. 1). If to interpret wavelet simplistically as the modulated sinusoid, then the sinusoid frequency will equal average wavelet frequency.

In the general case (when the time dependence of wavelet is well nonharmonic) determination of average frequency requires the signal processing and it is realized by iteration methods (Djakonov, 2002).

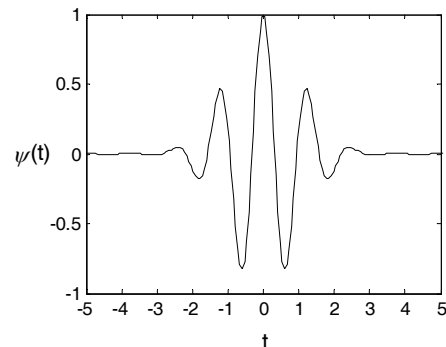


Fig.1. Morlet Wavelet  $\psi(t)=\exp(-t^2/2)\cos 5t$

The wavelet transformation result for vibrosignal is two-dimensional array of coefficients in coordinate terms «time scale  $a$ , time localization  $b$ ». It is named wavelet spectrum and contains information about a change of frequency and amplitude for different components of signal in time.

The wavelet spectrum for function  $f(t)$  is a surface in the three-dimensional space. More often two-dimensional visualization as a projection of coefficients  $C(a,b)$  on the plane  $(a,b)$  is used by color shades. The highlights corresponds to great coefficients of wavelet transformation  $C(a,b)$ . They are located near features of function  $f(t)$  what allows to discover all features of vibrosignal.

### 3. MODEL OF ROTOR SYSTEM

Two examples of wavelet analysis for rotor oscillations will be described later. For a simple let's consider the vertical symmetrical rigid rotor with single disc on the oil-film journal bearings (Fig. 2). Also let's take into account the most widely distributed operational conditions, which are unbalance, impact excitation and oil whirl of rotor.

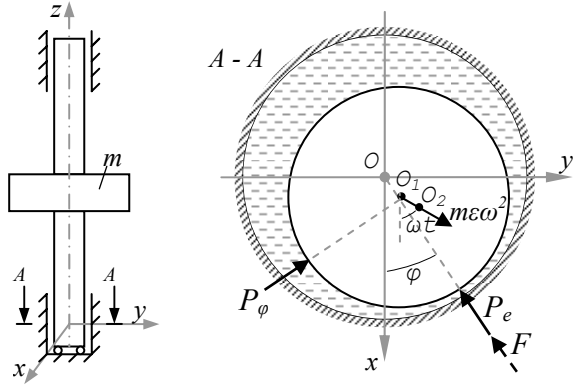


Fig.2. Used model for rotor-bearings system

Involved nomenclature includes:  $x_1, y_1$  are coordinates of rotor journals  $O_1$ ;  $x_2, y_2$  are coordinates of centre of rotor gravity  $O_2$ ;  $\varepsilon=O_1O_2$  is rotor unbalance,  $\varphi$  is position angle for rotor displacement  $e=OO_1$ ;  $P_e$  and  $P_\varphi$  are hydrodynamic forces of oil-film.

Equations for lateral oscillations of system (Fig. 2) have been defined in the Cartesian coordinates, taken into account an impulsive loading expressing the random contacts between rotor and frame in the form of external shock  $F$ , and only the constant rotor speed  $\omega$ . The latter means that a drive of infinite power is supposed behind. On basis of these assumptions and expressions  $x_2=x_1+\varepsilon\cos\omega t$ ,  $y_2=y_1+\varepsilon\sin\omega t$  the corresponding motion equations of system are as follows:

$$\begin{aligned} m\ddot{x}_1 + 2(P_e \cos\varphi + P_\varphi \sin\varphi) + F \cos\varphi &= m\varepsilon\omega^2 \cos\omega t, \\ m\ddot{y}_1 + 2(P_e \sin\varphi - P_\varphi \cos\varphi) + F \sin\varphi &= m\varepsilon\omega^2 \sin\omega t. \end{aligned} \quad (1)$$

Hydrodynamic forces  $P_e$  and  $P_\varphi$  have been described by well known hypothesis (Keljzon et al., 1982):

$$P_e = \frac{12\pi\mu LR^3 \dot{e}}{\delta^3 [1-(e/\delta)^2]^{3/2}}, \quad P_\varphi = \frac{12\pi\mu LR^3 (\omega - 2\dot{\varphi})e}{\delta^3 [2+(e/\delta)^2] \sqrt{1-(e/\delta)^2}}, \quad (2)$$

where  $\mu$  is viscosity of oil-film,  $\delta$  is value of radial clearance,  $L$  is length of journal bearing,  $R$  is journal radius.

Function  $F=F(t)$  characterizing type of shock loading (Banakh et al., 2008) have been described in the form of three half-sinusoidal impulses (Fig.3) i.e. for contact force the conditions were fulfilled:

$$F = \begin{cases} 0, \\ H \sin(\pi t/\tau) \text{ for } t_1 < t < (t_1 + \tau), \\ t_2 < t < (t_2 + \tau), t_3 < t < (t_3 + \tau) \end{cases} \quad (3)$$

where  $H$  is impulses' level,  $t$  is time,  $\tau$  is impulses' length.

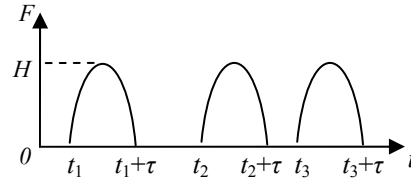


Fig.3. Used impulsive loading

In order to equations (1) and (2) and (3) have organized the closed loop system, the displacement  $e$  and position angle  $\varphi$  must be expressed through coordinates of rotor centreline  $x_1, y_1$ :

$$\begin{aligned} \cos\varphi &= \frac{x_1}{e}, \quad \sin\varphi = \frac{y_1}{e}, \quad e = \sqrt{x_1^2 + y_1^2}, \\ \dot{e} &= \frac{x_1\dot{x}_1 + y_1\dot{y}_1}{e}, \quad \dot{\varphi} = \frac{x_1\dot{y}_1 - y_1\dot{x}_1}{e^2} \end{aligned} \quad (4)$$

Integration of combined equations (1) - (4) at the system parameters  $m=1.5\text{kg}$ ,  $\varepsilon=60\mu\text{m}$ ,  $\delta=100\mu\text{m}$ ,  $L=10\text{mm}$ ,  $R=15\text{mm}$ ,  $\omega=1000\text{rpm}$ ,  $\mu=8\cdot 10^{-3}\text{N}\cdot\text{s}/\text{m}^2$  without and with random impulsive loading ( $H=10m\varepsilon\omega^2$ ,  $t_1=0.6\text{s}$ ,  $t_2=0.77\text{s}$ ,  $t_3=0.89\text{s}$ ,  $\tau=0.002\text{s}$ , look at the Fig. 3) by Runge-Kutta method has allowed obtaining typical trajectories, sweeps, frequency spectrums and wavelet spectrums for rotor oscillations corresponding different operational conditions (Fig.4 and Fig.5).

### 4. SPECTRAL AND AMPLITUDE ANALYSES AND WAVELET ANALYSIS OF ROTOR VIBRATION

Analyzing classical frequency spectrum, the conclusion may be done about an existence of unbalanced effect and oil whirl for rotor only.

Qualitatively obtained spectrums are very similar against each other, there are typical peak near to rotor speed  $\omega$  and peak near to frequency  $1/2\omega$ .

Analyzing trajectories and sweeps of vibration, the conclusion may be done about an existence of impulsive loading. At second case (Fig.5) there are typical straight-line portions of rotor motion.

However it is impossible to conclude anything concerning development dynamics of these processes by the amplitude spectrum (on basis of trajectories and sweeps) and by frequency spectrum (on basis of Fourier transformation of vibration) especially. On the contrary it is possible by means of wavelet transformation of rotor oscillations performed along one lateral direction.

Any wavelet spectrum marks the signal length  $b$  or time by horizontal axis. The scales  $a$ , which are inversely proportional to frequency actually, are marked by vertical axis there. Color of wavelet spectrum show values of wavelet coefficients  $C(a,b)$  or amplitude of harmonics in other words.

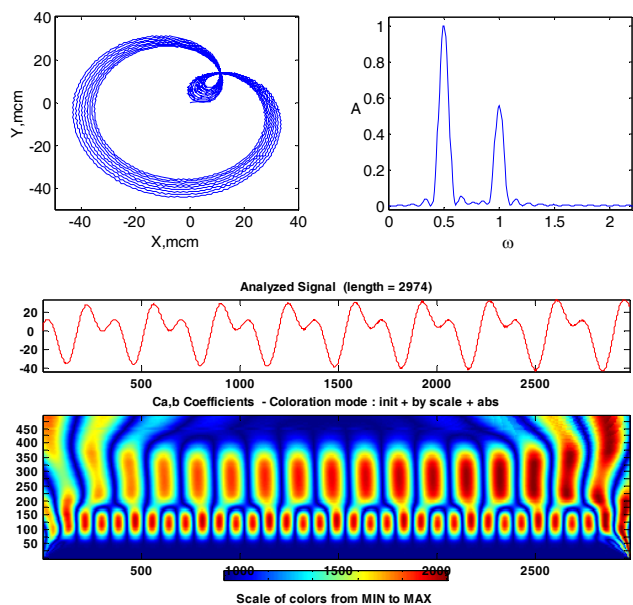


Fig. 4. Trajectory, frequency spectrum, sweep and wavelet spectrum of rotor vibration at the constant unbalance and development of process «oil whirl»

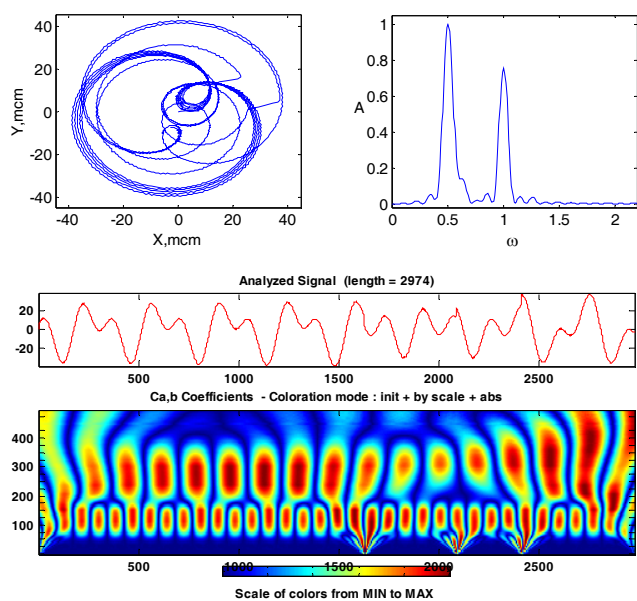


Fig. 5. Trajectory, frequency spectrum, sweep and wavelet spectrum of rotor vibration at the constant unbalance and development of process «oil whirl» and three random impacts

Analyzing obtained wavelet spectrums by colour intensity, one can see from Fig. 4 and Fig. 5 that the amplitude of subharmonic with frequency  $\frac{1}{2}\omega$  (at the 280<sup>th</sup> scale approximately) rises in time. It means the development of process «oil whirl». Consequently analyzed rotor motion is unstable. Opposite if the highlights would be constant, as in the case of fundamental harmonic with frequency  $\omega$  (at the 140<sup>th</sup> scale approximately), or if the highlights would decrease, then it would testify to stability or disappearance of process accordingly.

Note that at the shock in wavelet spectrum (Fig.5) a high-frequency component appears in the form of triangles at the impact moments. It allows determining the number and periodicity of impacts exactly.

## 5. CONCLUSIONS

1. One can define by wavelet spectrum the frequency content of vibrosignal as well as the time action domains for each of frequency components. And so one can state wavelet transformation and Fourier transformation supplement each other usefully.

2. Wavelet-analysis of vibration in the practice by means of human observation or on the basis of image identification methods allows to judge dynamic stability of rotor in the pseudo-stationary (quasi-stationary) case.

## 6. ACKNOWLEDGEMENTS

The financial support provided by the Russian Foundation for Basic Research and RFBR Grant No 09-01-90421-ykr\_φ\_a award scheme is gratefully acknowledged.

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