TWO-LEVEL HIERARCHICAL MINIMAX PROGRAM CONTROL PROBLEM OF THE PROCESS OF TERMINAL APPROACH WITH INCOMPLETE INFORMATION FOR THE DISCRETE-TIME DYNAMICAL SYSTEM

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Abstract
In this report we consider the discrete–time dynamical system consisting from three controlled objects. The motion of all objects are described by corresponding linear or convex discrete–time recurrent vector equations and this system has two levels of a control. The first level (or I level) is dominating and the second level (or II level) is subordinate and both have different criterions of functioning and united a priori by determined information and control relations. For investigation of this dynamical system we propose the mathematical formalization in the form of realization of two-level hierarchical minimax program control problem of the process of terminal approach with incomplete information and propose the main scheme for its solving.

Key words
Hierarchical discrete-time dynamical system, approach problem.

1 Introduction

In this report we consider the discrete–time dynamical system consisting from three controlled objects. The motion of all objects are described by corresponding linear or convex discrete–time recurrent vector equations and this system has two levels of control. The first level (or I level) is dominating and the second level (or II level) is subordinate and both have different criterions of functioning and united a priori by determined information and control relations. We formulate the two-level hierarchical minimax program control problem of the process of terminal approach with incomplete information for this discrete-time dynamical system and propose the main scheme for its solving. The results obtained in this report are based on [Krasovskii, 1968]–[Bazaraa and Shetty, 1979] and can be used for computer simulation and for designing of optimal digital controlling systems for actual technical, robotics, economic, and other multilevel control processes. Mathematical models of such systems had considered, for example, in [Chernousko, 1994]–[Tarbouriech and Garcia, 1997].

2 Dynamic of objects the two-level hierarchical discrete–time dynamical system

Let on a given integer time interval $\overline{0,\mathrm{T}} = \{0, 1, \cdots, \mathrm{T}\}$ $(\mathrm{T} > 0)$ we consider a controlled multistep dynamical system consisting from three objects. The motion of object I which is a major object and controlled by the dominating player P is described by the following discrete-time recurrent vector equation

$$y(t+1) = f(t, y(t), u(t), u^{(1)}(t)),\ y(0) = y_0, \quad (1)$$

and the motion of object I_1 which is a subsidiary object and controlled by the subordinate player S is described by the following linear discrete-time recurrent vector equation

$$y^{(1)}(t+1) = A(t)y^{(1)}(t) + B(t)u(t) + C(t)u^{(1)}(t),$$

$$y^{(1)}(0) = y_0^{(1)}, \quad (2)$$

and the motion of object II controlled by the deviating player E is described by the following discrete-time recurrent vector equation

$$z(t+1) = \bar{A}(t)z(t) + \bar{B}(t)v(t),\ z(0) = z_0. \quad (3)$$

Here $t \in \overline{0,\mathrm{T}-1}$; $y \in \mathbf{R}^r$, $y^{(1)} \in \mathbf{R}^{r_1}$ and $z \in \mathbf{R}^s$ are the phase vectors of the objects I, I_1 and II, respectively ($r, r_1, s \in \mathbf{N}$; for $k \in \mathbf{N}$, $\mathbf{R}^k$ is the k-dimensional Euclidean space of column vectors); $u(t) \in \mathbf{R}^p$, $u^{(1)}(t) \in \mathbf{R}^{p_1}$ and $v(t) \in \mathbf{R}^q$ are the control vectors (the controls) of the players P, S and E, respectively, restricted by the given constraints

$$u(t) \in \mathrm{U}_1,\ u^{(1)}(t) \in \mathrm{U}_1^{(1)},\ v(t) \in \mathrm{V}_1; \qquad (4)$$

where the set U_1 is finite set in the space $\mathbf{R}^p$ and the sets $\mathrm{U}_1^{(1)}$ and V_1 are convex polyhedrons in the spaces $\mathbf{R}^{p_1}$ and $\mathbf{R}^q$, respectively (here and below, a convex polyhedron is convex cover of finite set of vectors in the corresponding Euclidean vector space); for all fixed $t \in \overline{0,\mathrm{T}-1}$ and $u(t) \in \mathrm{U}_1$ the vector-function $f : \overline{0,\mathrm{T}-1} \times \mathbf{R}^r \times \mathbf{R}^p \times \mathbf{R}^{p_1} \longrightarrow \mathbf{R}^r$ is continuous by collection of the variables $(u(t), u^{(1)}(t))$ and for all fixed $t \in \overline{0,\mathrm{T}-1}$, $y(t) \in \mathbf{R}^r$ and $u(t) \in \mathrm{U}_1$ the set $f(t, y(t), u(t), \mathbf{U}_1^{(1)}) = \{f(t, y(t), u(t), u^{(1)}(t)),\ u^{(1)}(t) \in \mathbf{U}_1^{(1)}\}$ is convex; for all $t \in \overline{0,\mathrm{T}-1}$: $A(t),\ B(t), C(t),\ \bar{A}(t)$ and $\bar{B}(t)$ are real matrices of dimensions $(r_1 \times r_1)$, $(r_1 \times p)$, $(r_1 \times p_1)$, $(s \times s)$ and $(s \times q)$, respectively, and each from matrices $A(t)$ and $\bar{A}(t)$ have inverse matrices $A^{-1}(t)$ and $\bar{A}^{-1}(t)$, respectively.

We also assume that for all instants $t \in \overline{0,\mathrm{T}}$ the phase vectors $y(t),\ y^{(1)}(t)\ \ z(t)$ of the objects I, I_1 and II, respectively, combined with the initial conditions in equations (1)–(3) are restricted by the given following constraints

$$y(t) \in \mathrm{Y}_1,\ y^{(1)}(t) \in \mathrm{Y}_1^{(1)},\ z(t) \in \mathrm{Z}_1, \qquad (5)$$

where Y_1, $\mathrm{Y}_1^{(1)}$ and Z_1 are convex polyhedrons in the spaces $\mathbf{R}^r$, $\mathbf{R}^{r_1}$ and $\mathbf{R}^s$, respectively.

All players P, S and E combined with the objects I , I_1 and II define the I level or the dominating level of the control process for considered system. The player P combined with the object I_1 define the II level or the subordinate level of the control process for considered system (which is subordinate to the I level or the dominating level of the control process).

3 Information conditions for the players

The control process in the discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

In the field of interests of the player P are the admissible phase vectors of the objects I, I_1 and II, and for any instant $\tau \in \overline{1,\mathrm{T}}$ and any time interval (simply interval) $\overline{0,\tau} \subseteq \overline{0,\mathrm{T}}$ $(0 < \tau)$ he knows the collection: $y(0) = y_0$ and $y^{(1)}(0) = y_0^{(1)}$ are the initial states of the phase vectors of objects I and I_1, respectively; $u(\cdot) = \{u(t)\}_{t \in \overline{0,\tau-1}}$ is the past realization of his control on the interval $\overline{0,\tau}$ $(\forall\ t \in \overline{0,\tau-1}$: $u(t) \in \mathrm{U}_1)$; $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{0,\tau-1}}$ is the past realization of the control of player S on the interval $\overline{0,\tau}$ $(\forall\ t \in \overline{0,\tau-1}:\ u^{(1)}(t) \in \mathrm{U}_1^{(1)})$; $\omega(\cdot) = \{\omega(t)\}_{t \in \overline{0,\tau}}$ $(\omega(t) \in \mathbf{R}^m;\ m \in \mathbf{N}, m \leq s)$ is the past realization of the information signal on the interval $\overline{0,\tau}$, and its values $\omega(t)$ $(\omega(0) = \omega_0$ is fixed) are generated for each instant $t \in \overline{0,\tau}$ by the following discrete-time vector equation

$$\omega(t) = G(y(t))z(t) + F(t)\xi(t), \qquad (6)$$

where $\xi(t)$ is a measurement error satisfying the following constraint

$$\xi(t) \in \Xi_1. \qquad (7)$$

For all $t \in \overline{0,\mathrm{T}}$ and vectors $y(t) \in \mathbf{R}^r$ we assume that $G(y(t)$ and $F(t)$ are real matrices of dimensions $(m \times s)$ and $(m \times l)$, respectively, and for all vectors $y(t) \in \mathbf{R}^r$ each matrix $G(y(t)$ has the dimension m, and it is equal to the dimension of vector ω; Ξ_1 is convex polyhedron in the space $\mathbf{R}^l$.

During this control process the player P knows the set $\mathrm{Z}(0) = \mathrm{Z}_0 \subseteq \mathrm{Z}_1$ of all admissible initial phase vectors $z(0) = z_0$ of the object II which are consistent [Krasovskii, 1968] with the initial signal ω_0 and it is a convex polyhedron in the space $\mathbf{R}^s$.

Suppose that the player P also knows a formation principle of the controls $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{\tau,\mathrm{T}-1}}$ $(\forall\ t \in \overline{\tau,\mathrm{T}-1}:\ u^{(1)}(t) \in \mathrm{U}_1^{(1)})$ by the player S on the interval $\overline{\tau,\mathrm{T}}$ which will be described below.

We also assumed that the player P knows a choice of realization of the control $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{\tau,\mathrm{T}-1}}$ $(\forall\ t \in \overline{\tau,\mathrm{T}-1}:\ u^{(1)}(t) \in \mathrm{U}_1^{(1)})$ by the player S on any interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$, and he can use it for construct of his control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau,\mathrm{T}-1}}$ on this interval $(\forall\ t \in \overline{\tau,\mathrm{T}-1} : u(t) \in \mathrm{U}_1)$.

It is assumed that in a field of interests of the player S is only admissible phase vectors of the object I_1, and for any instant $\tau \in \overline{1,\mathrm{T}}$ and the interval $\overline{0,\tau} \subseteq \overline{0,\mathrm{T}}$ $(0 < \tau)$ he knows the collection: $y^{(1)}(0) = y_0^{(1)}$ is the initial state of the phase vector of object I_1; $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{0,\tau-1}}$ is the past realization of his control on the interval $\overline{0,\tau}$ $(\forall\ t \in \overline{0,\tau-1}:\ u^{(1)}(t) \in \mathrm{U}_1^{(1)})$. We also assume that on this interval $\overline{\tau,\mathrm{T}}$ the player S knows a choice of realization of the control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau,\mathrm{T}-1}}$ $(\forall\ t \in \overline{\tau,\mathrm{T}-1}:\ u(t) \in \mathrm{U}_1)$ of the player P on this interval, which he can use for construct of his control $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{\tau,\mathrm{T}-1}}$ on the interval $\overline{\tau,\mathrm{T}}$ $(\forall\ t \in \overline{\tau,\mathrm{T}-1} : u^{(1)}(t) \in \mathrm{U}_1^{(1)})$.

We also assume that at every instant $t \in \overline{0,\mathrm{T}-1}$ of main interval $\overline{0,\mathrm{T}}$ a choice of the control $u^{(1)}(t)$ by the player S depend not only from the restriction (4) but it also depend from a choice of the control $u(t) \in \mathrm{U}_1$ by the player P on the base of a priori mapping Ψ_1.

Let the mapping Ψ_1 is defined by the following description

$$\Psi_1 : \mathrm{U}_1 \to \mathrm{comp}(\mathrm{U}_1^{(1)});\ \forall\, t \in \overline{0, \mathrm{T}-1},\ \forall\, u(t) \in \mathrm{U}_1,$$

$$u^{(1)}(t) \in \Psi_1(u(t)) \in \mathrm{comp}(\mathrm{U}_1^{(1)}), \qquad (8)$$

where $\Psi_1(u(t))$ is a convex polyhedron in the space $\mathbf{R}^{p_1}$ for all $u(t) \in \mathrm{U}_1$. Therefore, it mean that a choice of admissible realization of the control $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ by the player S on the interval $\overline{\tau, \mathrm{T}}$ at every instant $t \in \overline{\tau, \mathrm{T}-1}$ constrained not only by condition (4), but also constrained by the admissible realization of the control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\mathrm{T}-1}}$ of the player P, which is communicated to the player S, and values of the control $u(t)$ at every instant $t \in \overline{\tau, \mathrm{T}-1}$ define the corresponding condition (8). It mean that the constraint (8) is a condition that define a behavior of the player S for achievement of his aim in the control process (it will be formulate below) and obviously depend from a behavior of the player P.

It is also assumed that in this control process for all instant $t \in \overline{0, \mathrm{T}}$ the player P knows all equations and constraints (1)–(8) and the player S knows (2), (4), (5) and (8).

We assumed that in this control process the player E can be fully informed about of all parameters of the discrete-time dynamical system (1)–(8) and about realizations of the phase vectors of the objects I, I_1 and II on the interval $\overline{0, \mathrm{T}}$.

4 Definitions and auxiliary properties of the dynamical system

For a strict mathematical formulation of the two-level hierarchical minimax program control problem of the process of terminal approach with incomplete information for the discrete-time dynamical system (1)–(8) we introduce some definitions.

For any fixed number $k \in \mathbf{N}$ and interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau \le \vartheta)$, we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of the functions $\varphi : \overline{\tau, \vartheta} \longrightarrow \mathbf{R}^k$ of an integer argument with the metric ρ_k defined by the relation

$$\rho_k(\varphi_1(\cdot), \varphi_2(\cdot)) = \max_{t\in\overline{\tau,\vartheta}} \| \varphi_1(t) - \varphi_2(t) \|_k$$

$$((\varphi_1(\cdot), \varphi_2(\cdot)) \in \mathbf{S}_k(\overline{\tau, \vartheta}) \times \mathbf{S}_k(\overline{\tau, \vartheta})),$$

and by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact (in this metric) subsets in the space $\mathbf{S}_k(\overline{\tau, \vartheta})$. Here $\| \cdot \|_k$ is the Euclidean norm in $\mathbf{R}^k$.

Using the constraint (4) we define the set $\mathbf{U}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_p(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\vartheta-1}}$ of the player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$ by the following relation

$$\mathbf{U}(\overline{\tau, \vartheta}) = \{u(\cdot) :\ u(\cdot) \in \mathbf{S}_p(\overline{\tau, \vartheta - 1}),$$

$$\forall\, t \in \overline{\tau, \vartheta - 1},\ u(t) \in \mathrm{U}_1\}.$$

Similarly, using the constraints (4) and (7) we define the following sets: $\mathbf{U}^{(1)}(\overline{\tau, \vartheta})$ is the set of all admissible program controls of player S; $\mathbf{V}(\overline{\tau, \vartheta})$ is the set of all admissible program controls of player E; $\mathbf{\Xi}(\overline{\tau, \vartheta})$ is the set of all admissible program errors of simulating a realization of information signal which is described by the relations (6), (7) and all of these sets defined on the interval $\overline{\tau, \vartheta}$.

Using the constraints (4) and (8) for fixed admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P we define the set $\mathbf{\Psi}_{\overline{\tau,\vartheta}}(u(\cdot)) \in \mathrm{comp}(\mathbf{S}_{p_1}(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u^{(1)}(\cdot) \in \mathbf{U}^{(1)}(\overline{\tau, \vartheta})$ of player S on the interval $\overline{\tau, \vartheta}$, corresponding to admissible program control $u(\cdot)$ of player P by the following relation

$$\mathbf{\Psi}_{\overline{\tau,\vartheta}}(u(\cdot)) = \{u^{(1)}(\cdot) :\ u^{(1)}(\cdot) \in \mathbf{U}^{(1)}(\overline{\tau, \vartheta}),$$

$$\forall\, t \in \overline{\tau, \vartheta - 1},\ u^{(1)}(t) \in \Psi_1(u(t))\}.$$

Now, by virtue of (1)–(7), we define the set $\hat{\mathbf{\Omega}}(\overline{\tau, \vartheta}) \subset \mathbf{S}_m(\overline{\tau + 1, \vartheta})$ of all admissible program realizations of information signal $\omega(\cdot) = \{\omega(t)\}_{t\in\overline{\tau+1,\vartheta}}$ on the interval $\overline{\tau, \vartheta}$.

Then for any instant $\tau \in \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ let $\hat{\mathbf{W}}(\tau) = \{\tau\} \times \mathbf{R}^r \times \mathbf{R}^{r_1} \times \mathrm{comp}(\mathbf{R}^s)$ $(\hat{\mathbf{W}}(0) = \hat{\mathbf{W}}_0 = \{w(0) = w_0 :\ w_0 = \{0, y_0, y_0^{(1)}, Z_0\} \in \{0\} \times \mathbf{R}^r \times \mathbf{R}^{r_1} \times \mathrm{comp}(\mathbf{R}^s)\})$ is the set of all admissible τ-positions $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \overline{0, \mathrm{T}} \times \mathbf{R}^r \times \mathbf{R}^{r_1} \times \mathrm{comp}(\mathbf{R}^s)$ of the player P in discrete-time dynamical system (1)–(8) (where $Z(\tau)$ is the set of admissible phase vectors $z(\tau) \in \mathbf{R}^s$ of the object II at instant τ, $w(0) = w_0 = \{0, y_0, y_0^{(1)}, Z_0\}$, $w^*(0) = w_0^* = \{0, y_0, y_0^{(1)}, Z_0^*\}$), where the nonempty set Z_0^* defined, by virtue (6), (7), by the following relation

$$Z_0^* = \{z_0 :\ z_0 \in Z_0,\ \exists\, \xi_0 \in \Xi_1,$$

$$\omega_0 = G(y_0) z_0 + F(0)\xi_0\}.$$

And let $\hat{\mathbf{W}}^{(1)}(\tau) = \{\tau\} \times \mathbf{R}^{r_1}$ ($\hat{\mathbf{W}}^{(1)}(0) = \hat{\mathbf{W}}^{(1)}_0 = \{w^{(1)}(0) = w^{(1)}_0 : w^{(1)}_0 = \{0, y^{(1)}_0\} \in \{0\} \times \mathbf{R}^{r_1}\}$) is the set of all admissible τ-positions $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \overline{0,\mathrm{T}} \times \mathbf{R}^{r_1}$ $(w^{(1)}(0) = w^{(1)}_0 = \{0, y^{(1)}_0\})$ of the player S in discrete-time dynamical system (1)–(8).

Let for any interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \vartheta)$, and for fixed admissible realizations, by virtue of (1)–(7), the τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ of the player P, and the controls $u(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ and $u^{(1)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau,\vartheta}}(u(\cdot))$ of players P and S, respectively, and the information signal $\omega(\cdot) \in \hat{\mathbf{\Omega}}(\overline{\tau,\vartheta})$ we denote by $\mathbf{R}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot))$ the set of all collections $(\tilde{z}(\tau), \tilde{v}(\cdot)) \in Z(\tau) \times \mathbf{V}(\overline{\tau,\vartheta})$ consistent (see [Krasovskii, 1968], [Kurzhanskii, 1977], [Shorikov, 1997]) with this information on the interval $\overline{\tau,\vartheta}$, by the following relation

$$\mathbf{R}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) = \{(\tilde{z}(\tau), \tilde{v}(\cdot)) :$$

$$(\tilde{z}(\tau), \tilde{v}(\cdot)) \in Z(\tau) \times \mathbf{V}(\overline{\tau,\vartheta}),\ \forall\, t \in \overline{\tau+1,\vartheta},$$

$$\exists\, \xi_*(t) \in \Xi_1,\ \omega(t) = G(y(t))\tilde{z}(t) + F(t)\xi_*(t)$$

$$(y(t) = y_t(\overline{\tau,\vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$\tilde{z}(t) = z_t(\overline{\tau,\vartheta}, \tilde{z}(\tau), \tilde{v}(\cdot)))\}, \qquad (9)$$

where by $y(t) = y_t(\overline{\tau,\vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot))$ and $\tilde{z}(t) = z_t(\overline{\tau,\vartheta}, \tilde{z}(\tau), \tilde{v}(\cdot))$ we denoted the sections at instant $t \in \overline{\tau+1,\vartheta}$ of the motions of objects I and II, respectively, on the interval $\overline{\tau,\vartheta}$. By virtue of (1) and (3), the motions of objects I and II are generated by the collections $(y(\tau), u(\cdot), u^{(1)}(\cdot))$ and $(\tilde{z}(\tau), \tilde{v}(\cdot))$, respectively.

Let

$$\mathbf{Z}^{(e)}_{\vartheta}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) =$$

$$\{z^{(e)}(\vartheta) :\ z^{(e)}(\vartheta) \in \mathbf{R}^s,$$

$$z^{(e)}(\vartheta) = z_{\vartheta}(\overline{\tau,\vartheta}, z(\tau), v(\cdot)),\ (z(\tau), v(\cdot)) \in$$

$$\in \mathbf{R}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot))\} \qquad (10)$$

be the information set of player P by relatively of the player E and of object II (see [Krasovskii, 1968], [Kurzhanskii, 1977], [Shorikov, 1997]) for a posteriori minimax filtering process in the discrete-time dynamical system (1)–(7) on the interval $\overline{\tau,\vartheta}$, corresponding to the instant ϑ and admissible collection $(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \in \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau,\vartheta}) \times \mathbf{\Psi}_{\overline{\tau,\vartheta}}(u(\cdot)) \times \hat{\mathbf{\Omega}}(\overline{\tau,\vartheta})$. Note that it is the set of all admissible realizations of the phase vectors of object II at the instant ϑ, which are consistent with all information about this system and known the player P on the interval $\overline{\tau,\vartheta}$ about a behavior of the player II.

For any fixed interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \vartheta)$, the τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ of player P and the controls $u(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ and $u^{(1)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau,\vartheta}}(u(\cdot))$ of players P and S, respectively, we define the following sets

$$\mathbf{\Omega}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot)) = \{\omega(\cdot) :\ \omega(\cdot) \in \hat{\mathbf{\Omega}}(\overline{\tau,\vartheta}),$$

$$\forall\, t \in \overline{\tau+1,\vartheta},\ \omega(t) = G(y(t))z(t) + F(t)\xi(t),$$

$$y(t) = y_t(\overline{\tau,\vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$z(t) = z_t(\overline{\tau,\vartheta}, z(\tau), v(\cdot)),$$

$$\xi(t) \in \Xi_1,\ (z(\tau), v(\cdot)) \in Z(\tau) \times \mathbf{V}(\overline{\tau,\vartheta})\}; \qquad (11)$$

$$\mathbf{W}(\tau, w(\tau), \vartheta, u(\cdot), u^{(1)}(\cdot)) =$$

$$= \{w(\vartheta) :\ w(\vartheta) \in \hat{\mathbf{W}}(\vartheta),$$

$$w(\vartheta) = \{\vartheta, y(\vartheta), y^{(1)}(\vartheta), Z(\vartheta)\},$$

$$y(\vartheta) = y_{\vartheta}(\overline{\tau,\vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$y^{(1)}(\vartheta) = y_{\vartheta}(\overline{\tau,\vartheta}, y^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$Z(\vartheta) = \mathbf{Z}^{(e)}_{\vartheta}(\overline{\tau,\vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)),$$

$$\omega(\cdot) \in \mathbf{\Omega}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot))\}, \qquad (12)$$

which will be called the sets of all admissible information signals on the interval $\overline{\tau, \vartheta}$ and all admissible ϑ-positions of player P, respectively, corresponding to the τ-position $w(\tau)$ of player P, and the controls $u(\cdot)$ and $u^{(1)}(\cdot)$ of players P and S, respectively.

It is known [Shorikov, 1997] that the information set $\mathbf{Z}^{(e)}_{\vartheta}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot))$ of a posteriori minimax filtering process for discrete-time dynamical system (1)–(8) is convex and may be approximate by convex polyhedron in the space $\mathbf{R}^s$ and construct by way to realization of a finite sequence of one-step operations only. Note that this information set will be use for formalization and solving of main approach problem in this report.

5 Criterions of a quality for the two-level hierarchical minimax program control problem of the process of terminal approach with incomplete information

Then for estimate a quality of this control process by player P on the I level of control we define the following terminal functional

$$\alpha : \ \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{U}^{(1)}(\overline{\tau, \mathrm{T}}) \times \hat{\mathbf{\Omega}}(\overline{\tau, \mathrm{T}}) =$$

$$= \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \qquad (13)$$

and its values for all admissible on the interval $\overline{\tau, \mathrm{T}}$ the collections of realizations $(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \in \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \alpha)$ are defined by the following concrete relation

$$\alpha(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) =$$

$$= \mu_1 \cdot \beta(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)) +$$

$$+ \mu \cdot \gamma(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)). \qquad (14)$$

Here the functional β is defined by the relation

$$\beta : \ \hat{\mathbf{W}}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{U}^{(1)}(\overline{\tau, \mathrm{T}}) =$$

$$= \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \beta) \longrightarrow \mathbf{E} \qquad (15)$$

and its values for all admissible on the interval $\overline{\tau, \mathrm{T}}$ the collections of realizations $(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)) \in \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \beta)$ are defined by the following relation

$$\beta(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)) =$$

$$= \| \{y^{(1)}(\mathrm{T})\}_{k_1} - \{y^{(1)}_*\}_{k_1} \|_{k_1}, \qquad (16)$$

where $\{y^{(1)}(\mathrm{T})\}_{k_1} = \{y^{(1)}_{\mathrm{T}}(\overline{\tau, \mathrm{T}}, y^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot))\}_{k_1}$ is k_1-projection $(k_1 \leq r_1)$ of the sections at terminal instant T of motion of object I_1 on the interval $\overline{\tau, \mathrm{T}}$, and corresponding by the collection $(y^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot))$.

Note that the functional β is defined a value of distance between admissible realizations of k_1-projection $(k_1 \leq r_1)$ a section at terminal instant T of a motion the object I_1 and analogous projection of given and fixed vector $y^{(1)}_* \in \mathbf{R}^{r_1}$, and make possible by the players P and S to estimate a quality in the program control problem of the process of terminal approach on the I and II levels of this two-level hierarchical system of a control on the interval $\overline{\tau, \mathrm{T}}$.

In the formula (14) the functional γ is defined by relation

$$\gamma : \ \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{U}^{(1)}(\overline{\tau, \mathrm{T}}) \times \hat{\mathbf{\Omega}}(\overline{\tau, \mathrm{T}}) =$$

$$= \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \gamma) \longrightarrow \mathbf{E}, \qquad (17)$$

and its values for all admissible on the interval $\overline{\tau, \mathrm{T}}$ the collections of realizations $(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \in \mathbf{\Gamma}(\overline{\tau, \mathrm{T}}, \gamma)$ are defined by the following relation

$$\gamma(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) =$$

$$= \max_{\{z(\mathrm{T})\}_k \in \{Z^{(e)}(\mathrm{T})\}_k} \| \{y(\mathrm{T})\}_k - \{z(\mathrm{T})\}_k \|_k, \quad (18)$$

where the set $Z^{(e)}(\mathrm{T}) = \mathbf{Z}^{(e)}_{\mathrm{T}}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \neq \emptyset$, and $\{Z^{(e)}(\mathrm{T})\}_k$ is its k-projection.

Note that the functional γ is defined a value of distance between admissible realizations of k-projection $(k \leq r;\ k \leq s)$ a section at terminal instant T of a motion the object I and analogous projection a section at terminal instant T of a motion the object II and make possibility by the player P to estimate a quality in the program control problem of the process of terminal approach on the I level of this two–level hierarchical system of a control on the interval $\overline{\tau, \mathrm{T}}$.

In the formula (14), $\mu^{(1)} \in \mathbf{R}^1$ and $\mu \in \mathbf{R}^1$ are any fixed numerical parameters which satisfy the following condition: $\mu + \mu_1 = 1$.

Also note that on the I level of this control process we not exclude situation when the parameter $\omega(\cdot) \in \mathbf{\Omega}(\overline{\tau, \mathrm{T}})$ may be realized on the interval $\overline{\tau, \mathrm{T}}$ by worst form for the player P, namely, when it determine a maximal admissible value of the functional γ.

For estimate a quality by the player S in the program control problem of the terminal approach process on the II level of this two–level hierarchical system of a control on the interval $\overline{\tau,\mathrm{T}}$ we will be use the functional β which defined by the relations (15), (16).

We also note that if on the base of functionals β and γ let we consider the vector-functional $\delta = (\beta, \gamma)$ such that it defined by the relation

$$\delta:\ \mathbf{\Gamma}(\overline{\tau,\mathrm{T}},\beta) \times \mathbf{\Gamma}(\overline{\tau,\mathrm{T}},\gamma) \longrightarrow \mathbf{E} \times \mathbf{E}, \tag{19}$$

and its values for admissible on the interval $\overline{\tau,\mathrm{T}}$ realizations of all arguments are defined according to the relations (14)–(18). Then we can assert that the functional α is a convolution the vector-functional δ after using the scalar's method (see, for example, [Fradkov, 2003]).

6 The optimization in program control problems of the process of terminal approach

On the base of the above the aim of the player S, which combined with the object I_1 defines the II level of the two-level hierarchical control system for discrete-time dynamical system (1)–(8) on any fixed interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$, may be formulate by the following way. The player S using his information and control possibilities has interest in such result on the II level of this control system on the interval $\overline{\tau,\mathrm{T}}$ when the functional β which determined by the relations (15), (16) for each admissible realizations of his τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ and the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of player P on it time interval, has minimal admissible value by means way of using to choice of his admissible program control $u^{(1)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$.

Then for realization of the aim of player S on the II level of this two-level hierarchical control system for the discrete-time dynamical system described by the relations (1)–(8) we can formulate the following multistep optimal program control problem of the process of terminal approach for the object I_1.

Problem 1. For any fixed time interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$, and for any admissible on the II level of two-level hierarchical control system for the discrete-time dynamical system (1)–(8) the realization of the τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S and any admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of player P on the I level of this control system it is required to find the set $\mathbf{U}^{(1,e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ of optimal program controls $u^{(1,e)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ of player S, corresponding the control $u(\cdot)$ of player P which is determined by the following relation

$$\mathbf{U}^{(1,e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot)) =$$

$$= \{u^{(1,e)}(\cdot):\ u^{(1,e)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot)),$$

$$c_\beta^{(e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot)) =$$

$$= \beta(w^{(1)}(\tau), u(\cdot), u^{(1,e)}(\cdot)) =$$

$$= \min_{u^{(1)}(\cdot) \in\ \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))} \beta(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot))\}, \tag{20}$$

where the functional β is defined by the relations (15) and (16).

The set $\mathbf{U}^{(1,e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot)) = \{u^{(1,e)}(\cdot)\} \subseteq \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ $(w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$, $w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ which constructed from solving of the problem 1 we shall call the set of optimal program controls of player S on II level of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) and corresponding to it the number's value $c_\beta^{(e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ we shall call the value of optimal result for the player S on the II level of this control system. Note that both this elements correspond to the fixed and admissible the interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$, and the τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S on the II level of this control system, and the control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of player P on the I level of this control system.

We also note that the solution of problem 1 on the interval $\overline{\tau,\mathrm{T}}$ determine the principle of forming the set $\mathbf{U}^{(1,e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ of the optimal program controls $u^{(1,e)}(\cdot)$ by player S on the II level of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) corresponding to the realizations of his τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ and subordinating to a choice of the admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ by player P on I level of this control system.

According to the definitions and assumptions made above about all parameters and information relations for the discrete-time dynamical systems (1)–(8), the aim of player P on the I level of this control system and any fixed interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$, and admissible his τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = w^*(0) = \{0, y_0, y_0^{(1)}, Z_0^*\} = w_0^* \in \hat{\mathbf{W}}_0)$ may be formulate by the following way. The player P using his information and controls possibilities has interest in such result on the I level of this two-level hierarchical control system on the interval $\overline{\tau,\mathrm{T}}$ when the functional α determined by the relations (13)–(18) has minimal admissible value by means way of using to choice of his admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ and from solving of the problem 1 using the optimal program control $u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau,\mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ of player S which subordinate of the player P, where the τ-position $w^{(1)}(\tau) =$

$\{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S on the II level of this control system is formed from the τ-position $w(\tau)$.

We note that on the I level of this control system we not exclude situation when the parameters $v(\cdot) \in \mathbf{V}(\overline{\tau, \mathrm{T}})$ and $\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau, \mathrm{T}})$ which influence on a change of values of the information signal $\omega(\cdot) \in \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), u^{(1)}(\cdot))$, may be realized on the interval $\overline{\tau, \mathrm{T}}$ by worst form for the player P, namely, when it determine maximal admissible value of the functional α for fixed realizations of the elements $w(\tau), u(\cdot)$ and $u^{(1)}(\cdot)$.

Then for realization of the aim of player P on the I level of this two-level hierarchical control system for the discrete-time dynamical system described by the relations (1)–(8) we can formulate the following multi-step minimax program control problem of the process of terminal approach with incomplete information for objects I and II.

Problem 2. For any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, and admissible on the I level of two-level hierarchical control system for the discrete-time dynamical system (1)–(8) the realization of τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P it is required to find the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ of minimax program controls of player P which is determined by the following relation

$$\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{u^{(e)}(\cdot) : \ u^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}}),$$

$$c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \min_{u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u^{(e)}(\cdot))} \{$$

$$\max_{\omega(\cdot) \in \mathbf{\Omega}_1(u^{(e)}(\cdot))} \alpha(w(\tau), u^{(e)}(\cdot), u^{(1,e)}(\cdot), \omega(\cdot))\} =$$

$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})} \min_{u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))} \{$$

$$\max_{\omega(\cdot) \in \mathbf{\Omega}_1(u(\cdot))} \alpha(w(\tau), u(\cdot), u^{(1,e)}(\cdot), \omega(\cdot))\}\}. \quad (21)$$

Here the functional α is defined by the relations (13)–(18), and the τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w(0) = \{0, y_0^{(1)}\} = w_0 \in \hat{\mathbf{W}}_0^{(1)})$ of player S is formed from the τ-position $w(\tau)$ of player P and defined at instant τ the realization of phase vector of the object I_1 on II level of this control system, and the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) = \{u^{(1,e)}(\cdot)\} \subseteq \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of optimal program controls of the player S on II level of this control system for any realizations of the τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ of player S and the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P is constructed from solving of the problem 1, and the set $\mathbf{\Omega}_1(u(\cdot)) = \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), u^{(1,e)}(\cdot))$, and the set $\mathbf{\Omega}_1(u^{(e)}(\cdot)) = \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, w(\tau), u^{(e)}(\cdot), u^{(1,e)}(\cdot))$.

The set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ which constructed from solving of the problem 2 we shall call the set of minimax program controls of player P on I level of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) and corresponding to it the number's value $c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ we shall call the value of minimax result for the player P on the I level of this control system. Note that both this elements correspond to the fixed and admissible the interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ and the τ-position $w(\tau) \in \hat{\mathbf{W}}(\tau)$ $(w(0) = w_0 \in \hat{\mathbf{W}}_0)$ of player P on I level of this control system.

Note that the solution of the problem 2 on the interval $\overline{\tau, \mathrm{T}}$ determine the principle of forming the minimax program controls $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ by player P on I level of this control system, corresponding to the realizations of his τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$.

On the base of the solutions formulated problems 1 and 2 we consider the following problem.

Problem 3. For any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, and admissible on the I level of two-level hierarchical control system for the discrete-time dynamical system (1)–(8) the realization of τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{W}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P it is required to find the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of optimal minimax program controls of player P which is determined by the following relation

$$\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{\hat{u}^{(e)}(\cdot) :$$

$$\hat{u}^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)),$$

$$\min_{u^{(1)}(\cdot) \in \ \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))} \beta(w^{(1)}(\tau), \hat{u}^{(e)}(\cdot), u^{(1)}(\cdot)) =$$

$$= \min_{u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))} \min_{u^{(1)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(u^{(e)}(\cdot))} \{$$

$$\beta(w^{(1)}(\tau), u^{(e)}(\cdot), u^{(1)}(\cdot))\}\}; \quad (22)$$

and for any admissible on the II level of this control system the τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) =$

$w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)}$) of player S, which is formed from the τ-position $w(\tau)$, and any admissible realization of the optimal minimax program control $\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on the I level of this control system and formed from solving of the problem 2 and the problem described by the relation (22), it is required to find the set $\hat{\mathbf{U}}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) \subseteq \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) \subseteq \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))$ of optimal minimax program controls $\hat{u}^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$ of player S on the II level and the number $c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$, which is the value of optimal result for player S on the II level of this control system and corresponding to the control $\hat{u}^{(e)}(\cdot)$ of player P, and its are determined by the following relations

$$\hat{\mathbf{U}}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) = \{\hat{u}^{(1,e)}(\cdot) :$$

$$\hat{u}^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)),$$

$$c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \max_{\omega(\cdot) \in \mathbf{\Omega}_2(\hat{u}^{(1,e)}(\cdot))} \{$$

$$\alpha(w(\tau), \hat{u}^{(e)}(\cdot), \hat{u}^{(1,e)}(\cdot), \omega(\cdot))\} =$$

$$= \min_{u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))} \max_{\omega(\cdot) \in \mathbf{\Omega}_2(u^{(1,e)}(\cdot))} \{$$

$$\alpha(w(\tau), \hat{u}^{(e)}(\cdot), u^{(1,e)}(\cdot), \omega(\cdot))\}; \qquad (23)$$

$$c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) =$$

$$= \beta(w^{(1)}(\tau), \hat{u}^{(e)}(\cdot), \hat{u}^{(1,e)}(\cdot)) =$$

$$= \min_{u^{(1)}(\cdot) \in \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))} \{$$

$$\beta(w^{(1)}(\tau), \hat{u}^{(e)}(\cdot), u^{(1)}(\cdot))\}. \qquad (24)$$

Here the functional α is defined by the relations (13)–(18), and the functional β is defined by the relations (15), (16), and the τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ ($w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)}$) of player S is formed from the τ-position $w(\tau)$ of player P and defined at instant τ the realization of phase vector of the object I_1 on the II level this control system for discrete-time dynamical system (1)–(8), and $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) = \{u^{(1,e)}(\cdot)\} \subseteq \mathbf{\Psi}_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))$ is the set of optimal program controls of player S on the II level of this control system and corresponding by any fixed and admissible on the interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ ($\tau < \mathrm{T}$) the realizations of τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ ($w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)}$) of player S on the II level of this control system and of optimal minimax program control $\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on the I level of this control system and it is formed from the solution of the problem 1; the set $\mathbf{\Omega}_2(u^{(1,e)}(\cdot)) = \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, w(\tau), \hat{u}^{(e)}(\cdot), u^{(1,e)}(\cdot))$ and the set $\mathbf{\Omega}_2(\hat{u}^{(1,e)}(\cdot)) = \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, w(\tau), \hat{u}^{(e)}(\cdot), \hat{u}^{(1,e)}(\cdot))$.

7 The main scheme for realization of this two-level hierarchical control system

Note that we may consider solutions of formulated problems 1–3, which combined determined the two-level hierarchical minimax programm control problem of the process of terminal approach with incomplete information for the discrete-time dynamical system (1)–(8).

Then the main scheme for realization of this two-level hierarchical control system for any fixed and admissible time interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ ($\tau < \mathrm{T}$), and realizations τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ ($w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0$) of the player P on the I level of this control system, and τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ ($w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)}$) of the player S on the II level of this control system we can describe in the form of following sequence of actions:

1) for any admissible control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of the player P on the I level of this control system, from solving of the corresponding problem 1 we construct the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ of optimal program controls of player S on the II level of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) and the number $c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$, which is the value of optimal result for the player S on the II level of this control system and corresponding the control $u(\cdot)$ and satisfying the relation (20);

2) from solving of the problem 2 we construct the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of minimax program controls of player P on the I level of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) satisfying the relation (21);

3) from solving of the problem 3 we construct the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of optimal minimax program controls of player P on the I level of this control system;

4) for any optimal minimax program control $\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on the I level of

this control system, from solving of the problem 3 we construct the set $\hat{\mathbf{U}}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) \subseteq \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$ of optimal minimax program controls of player S on the level of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) and the number $c_{\beta}^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$, which is the value of optimal result for the player S on the II level of this control system corresponding the control $\hat{u}^{(e)}(\cdot)$ and satisfying the relations (23) and (24).

8 Conclusion

In conclusion we note that the concrete algorithm for realization of this two-level hierarchical control system for the discrete-time dynamical system (1)–(8) can be described on the base of the algorithms for solving of the program terminal control problem with incomplete information, which are proposed in works [Shorikov, 1997] and [Shorikov, 2005].

The results obtained in this report can be used for computer simulation and for designing of optimal digital controlling systems for actual technical, robotics, economic, and other multilevel control processes. Mathematical models of such systems had considered, for example, in [Chernousko, 1994]–[Tarbouriech and Garcia, 1997].

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