SATELLITE ORBIT ESTIMATION USING ON-LINE NEURAL NETWORKS

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Abstract: This paper presents satellite orbit estimation using artificial neural networks. A multilayer Perceptron is used to estimate the position of a low-earth orbit satellite. The main goal is to filter out noisy or incomplete data received from sensors. The algorithm is applied to the CHAMP satellite. The same orbit is estimated using the extended Kalman filter. Simulation results show superior performance of the neural network as compared to the extended Kalman filter. *Copyright © 2002 IFAC*

Keywords: Satellite orbit, Estimation, Neural networks, Extended Kalman Filter

1. INTRODUCTION

Low-Earth-Orbit (LEO) satellites circle the earth in different altitudes and inclinations. The orbit inclination is the angle between the plate of orbit and the equator. The altitude of LEO satellites is a few hundred kilometres above the earth surface (Sidi, 2000).

Different filters, such as recursive filters, batch filters, and Kalman filters have been proposed in literature for position estimation of LEO satellites (Mahy, 2001; Psiaki, 2002; Vergez et al. 2004; Yoon et al., 2000). In this paper, Neural Networks (NN) are employed for the position estimation of CHAMP LEO satellite. The real data for simulations are obtained from the following websites:

- http://www.johnstonsarchive.net/physics/sp-satellites.html
- http://www.heavens-above.com.

In order to compare the performance of the proposed method, the Extended Kalman Filter (EKF) is also used to estimate the position of the same satellite. Simulation results show superior performance of the NN as compared to the EKF.

2. DYNAMIC OF SATELLITE ORBIT

The law of planetary movements, which was discovered by Kepler about 400 years ago, is the basis of satellites rotation around the earth. According to the basic principles of these laws, if the mass of satellite is ignored as compared to the mass of the earth, and if the earth is assumed to be spherical, then according to the Newton's gravity law, the acceleration of satellite can be calculated as (Montenbruck and Gill, 2000)

$$\ddot{\boldsymbol{r}} = -\frac{GM_{\oplus}}{r^3}\boldsymbol{r} \tag{1}$$

where $M_{\oplus}$ is the mass of earth, $G = (6.67259 \pm 0.00085).10^{-11}$ $\mathrm{m^3kg^{-1}s^{-2}}$ is the gravity constant, r is the distance of satellite from the centre of earth, and $-\boldsymbol{r}/r$ is the unit vector connecting the satellite to the centre of earth. Equation (1) shows that the acceleration of satellite is inversely proportional to the distance of satellite from the centre of earth. In order to describe rotation of the satellite around the earth, the following independent parameters should be defined:

$$\mathbf{R} = [a \quad e \quad i \quad \Omega \quad \omega \quad M]^T \tag{2}$$

where a, called the semi-major axis and measured in meter or feet, is a constant defining the size of the orbit, e, called the eccentricity, is a constant defining the shape of the orbit (0=circular, less than 1=elliptical), i, called the inclination, is the angle between the equator and the orbit plane, Ω, called the right ascension of the ascending node, is the angle between the vernal equinox and the point where the orbit crosses the equatorial plane (pointing to the north), ω, called the argument of perigee, is the angle between the ascending node and the orbit's point of closest approach to the earth, and M, called the true anomaly, is the angle between the perigee and the satellite in the orbit plane.

Two orbital elements, a and e, define the shape of orbit, M defines the position of satellite on the orbit, and three other elements (i.e. i, Ω and ω) define the direction of orbit in the space. These six elements are calculated in terms of the position and the velocity vector (Tapley, 2004)

$$\mathbf{x} = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T \tag{3}$$

where x, y and z define the position in ECEF coordinate system. Both $\mathbf{R}$ and $\mathbf{x}$ can be used to define the position of satellites. In this paper, $\mathbf{x}$ is employed for estimation of LEO satellite position.

Linear differential equations like $\dot{\mathbf{x}} = \mathbf{F}.\mathbf{x}$ are not appropriate for discrete estimation as in Kalman filter. For instance, transition matrix in Kalman filter is considered generally as $\overline{\mathrm{x}}_{\mathrm{k}} = \varphi.\overline{\mathrm{x}}_{\mathrm{k-1}}$, which can be written as

$$\mathbf{x}_k = \frac{\partial \mathbf{x}_k}{\partial \mathbf{x}_{k-1}}.\mathbf{x}_{k-1} \tag{4}$$

where the transition matrix φ is equal to

$$\frac{\partial \mathbf{x}_k}{\partial \mathbf{x}_{k-1}} = \begin{bmatrix} \frac{\partial x_k}{\partial x_{k-1}} & \frac{\partial x_k}{\partial y_{k-1}} & \frac{\partial x_k}{\partial z_{k-1}} & \cdots \\ \frac{\partial y_k}{\partial x_{k-1}} & \frac{\partial y_k}{\partial y_{k-1}} & \frac{\partial y_k}{\partial z_{k-1}} & \cdots \\ \frac{\partial z_k}{\partial x_{k-1}} & \frac{\partial z_k}{\partial y_{k-1}} & \frac{\partial z_k}{\partial z_{k-1}} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{5}$$

which can be calculated as

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \mathbf{F}.\mathbf{x} \quad \dot{\overline{\mathrm{x}}} = \frac{d\overline{\mathrm{x}}}{dt} = \mathbf{F}.\overline{\mathrm{x}} \qquad \Rightarrow \frac{d\overline{\mathrm{x}}}{\overline{\mathrm{x}}} = \mathbf{F}.\mathrm{dt}$$

$$\Rightarrow \int_{\mathrm{k-1}}^{\mathrm{k}} \frac{d\overline{\mathrm{x}}}{\overline{\mathrm{x}}} = \int_{\mathrm{k-1}}^{\mathrm{k}} \mathbf{F}.\mathrm{dt} = \mathbf{F}.\int_{\mathrm{k-1}}^{\mathrm{k}} \mathrm{dt} \tag{6}$$

$$\Rightarrow \ln \overline{\mathrm{x}}_{\mathrm{k}} - \ln \overline{\mathrm{x}}_{\mathrm{k-1}} = \ln \frac{\overline{\mathrm{x}}_{\mathrm{k}}}{\overline{\mathrm{x}}_{\mathrm{k-1}}} = \mathbf{F}.(\mathrm{t}_{\mathrm{k}} - \mathrm{t}_{\mathrm{k-1}})$$

$$\Rightarrow \frac{\overline{\mathrm{x}}_{\mathrm{k}}}{\overline{\mathrm{x}}_{\mathrm{k-1}}} = \mathrm{e}^{\mathbf{F}.(\mathrm{t}_{\mathrm{k}} - \mathrm{t}_{\mathrm{k-1}})}.$$

$$\Rightarrow \overline{\mathrm{x}}_{\mathrm{k}} = \mathrm{e}^{\mathbf{F}.(\mathrm{t}_{\mathrm{k}} - \mathrm{t}_{\mathrm{k-1}})}.\overline{\mathrm{x}}_{\mathrm{k-1}}$$

Using the expansion series

$$e^{\mathbf{F}\Delta t} = \mathbf{I} + \mathbf{F}\Delta t + \frac{1}{2!}\mathbf{F}^2\Delta t^2 + \frac{1}{3!}\mathbf{F}^3\Delta t^3 + \cdots \tag{7}$$

and omitting the nonlinear terms yields

$$\mathbf{\Phi} = \mathbf{I} + \mathbf{F}\Delta t \tag{8}$$

Using the equation of motion of satellites in (1), the following equations are obtained:

$$\frac{dx}{dt} = \dot{x} \quad , \quad \frac{dy}{dt} = \dot{y} \quad , \quad \frac{dz}{dt} = \dot{z}$$

$$\begin{aligned} \frac{d\dot{x}}{dt} &= \ddot{x} = -\frac{GM}{r^3}x \\ \frac{d\dot{y}}{dt} &= \ddot{y} = -\frac{GM}{r^3}y \\ \frac{d\dot{z}}{dt} &= \ddot{z} = -\frac{GM}{r^3}z \end{aligned} \tag{9}$$

Calculating the partial derivatives with respect to the positions and speeds, the dynamic equation of satellite movements are

$$\mathbf{F}_{Sat} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -(1-3\frac{x_k^2}{r_k^2})\frac{GM}{r^3} & 3\frac{x_k y_k}{r_k^5}GM & 3\frac{x_k z_k}{r_k^5}GM & 0 & 0 & 0 \\ 3\frac{x_k y_k}{r_k^5}GM & -(1-3\frac{y_k^2}{r_k^2})\frac{GM}{r^3} & 3\frac{y_k z_k}{r_k^5}GM & 0 & 0 & 0 \\ 3\frac{x_k z_k}{r_k^5}GM & 3\frac{y_k z_k}{r_k^5}GM & -(1-3\frac{z_k^2}{r_k^2})\frac{GM}{r^3} & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

When the position of satellite (i.e. the range, the azimuth and the elevation angles) is continually available, one can estimate the position of satellite with respect to the earth tracking station. Then, using the position of the earth-tracking station, the measured positions can be calculated with respect to the earth centre.

In this paper, the orbital estimation of the CHAMP satellite is considered. The orbital specification of this satellite is shown in Table 1.

Table 1: Orbit specification of CHAMP satellite

Orbit inclination (i)	87.2346°
Argument of perigee (ω)	81.5653°
Right Ascension of Ascending Node(Ω)	303.3713°
Semi-major axis (a)	361 Km
Eccentricity (e)	0.00033
The closest distance from the orbit	358 Km
The far distance from the orbit	364 Km
The time for one complete rotation	91.58 min

3. ORBIT ESTIMATION USING EXTENDED KALMAN FILTER

Since the relationship between the measurements of the satellite position is nonlinearly related to the state of the system, this violates the linear assumption of the Kalman filter. The Extended Kalman Filter (EKF) is an ad hoc technique to provide a way to use the standard Kalman filter on non-linear process or measurement models resulting in sub-optimal estimates. The measurement model and process model are linearized about the mean and covariance at every iteration, and then, the standard Kalman filter is applied to the linearized models.

In the linear discrete Kalman filter, the state of the system can be updated with a straightforward matrix multiplication $\mathbf{F}(n+1,n)$. Similarly, converting from the state to measurement space is accomplished with another matrix multiplication $\mathbf{C}(n)$. Both of these matrices are approximated in the EKF using a first order Taylor expansion. To accomplish this, the Jacobian matrix of both the process model and the measurement model need to be calculated. Since the process model is already linear, the calculation is trivial,

$$\mathbf{F}(n,\mathbf{x})=\begin{bmatrix} p_x+v_x\Delta t \\ p_y+v_y\Delta t \\ p_z+v_z\Delta t \\ v_x \\ v_y \\ v_z \end{bmatrix} \tag{11}$$

$$\frac{\partial \mathbf{F}(n,\mathbf{x})}{\partial \mathbf{x}}=\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

But, the Jacobian matrix of the nonlinear measurement model is nontrivial

$$\mathbf{C}(n,\mathbf{x})=\begin{bmatrix} \sqrt{p_x^2+p_y^2+p_z^2} \\ \arctan(\frac{p_y}{p_x}) \\ \arccos(\frac{p_z}{\sqrt{p_x^2+p_y^2+p_z^2}}) \end{bmatrix} \tag{13}$$

$$\frac{\partial \mathbf{C}(n,\mathbf{x})}{\partial \mathbf{x}}=\begin{bmatrix} \frac{p_x}{r} & \frac{p_y}{r} & \frac{p_z}{r} \\ \frac{-p_y}{-p_z^2+r^2} & \frac{-p_x}{-p_z^2+r^2} & 0 \\ \frac{p_z p_x}{r^2\sqrt{-p_z^2+r^2}} & \frac{p_z p_y}{r^2\sqrt{-p_z^2+r^2}} & \frac{-\sqrt{-p_z^2}}{r^2} \end{bmatrix} \tag{14}$$

These linearized matrices are incorporated into the EKF using the following equations.

The Kalman gain is calculated as (Haykin, 2001)

$$\mathbf{G}_f(n)=\mathbf{K}(n,n-1)\mathbf{C}^H(n)(\mathbf{C}(n)\mathbf{K}(n,n-1)\mathbf{C}^H(n)+\mathbf{Q}_2(n))^{-1} \tag{15}$$

where $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{F}$ and $\mathbf{C}$ are the covariance process-noise, the covariance measurement noise, the linearized state transition, and the linearized measurement matrices, respectively. The estimation error is equal to

$$\boldsymbol{\alpha}(n)=\mathbf{y}(n)-\mathbf{C}(n,\hat{\mathbf{x}}(n|Y_{n-1})) \tag{16}$$

where $\mathbf{y}(n)$ is the vector containing the actual measurements and $\mathbf{C}(n,\mathbf{x})$ is the measurement model function at time n. The new state estimates are calculated as

$$\hat{\mathbf{x}}(n|Y_n)=\hat{\mathbf{x}}(n|Y_{n-1})+\mathbf{G}_f(n)\boldsymbol{\alpha}(n) \tag{17}$$

The covariance error matrix is equal to

$$\mathbf{K}(n)=(\mathbf{I}-\mathbf{G}_f(n)\mathbf{C}(n))\mathbf{K}(n,n-1) \tag{18}$$

Then, the next estimates of states are calculated as

$$\hat{\mathbf{x}}(n+1|Y_n)=\mathbf{F}(n,\mathbf{x}(n|Y_n)) \tag{19}$$

And the error covariance for the next iteration is predicted,

$$\mathbf{K}(n+1,n)=\mathbf{F}(n+1,n)\mathbf{K}(n)\mathbf{F}^H(n+1,n)+\mathbf{Q}_1(n) \tag{20}$$

where $\mathbf{F}(n+1,n)$ is the Jacobian matrix evaluated at the current state estimate.

Figs. 1-5 show the simulation results for the EKF for estimating the position of the CHAMP satellite in one complete rotation around the earth. The measurement noise is 7% of the actual values. The x, y and z variables are shown in km, measured from the surface of the earth and in the direction of the earth centre. Next, it is assumed that there exists packet loss in data. That is, in the time interval $50 < t < 60$ min, no data is received for estimation. Fig. 4 shows the estimated position in this case. In addition, Fig. 5 shows the noisy measured data and the estimated states in 3D.

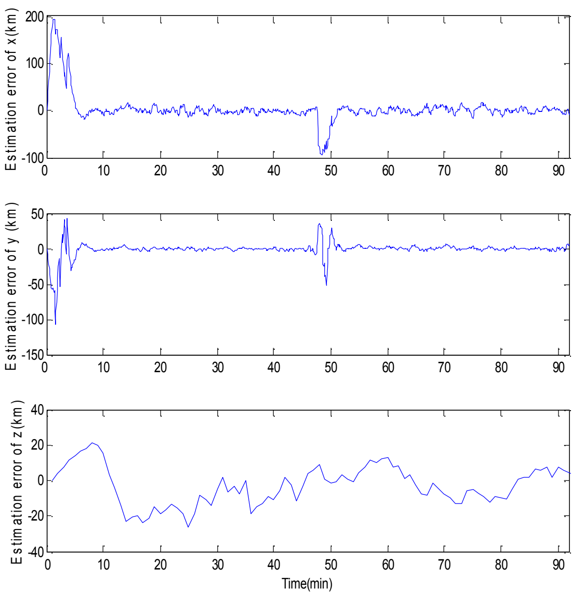

Fig.1. Estimation errors along the x, y and z axis using EKF

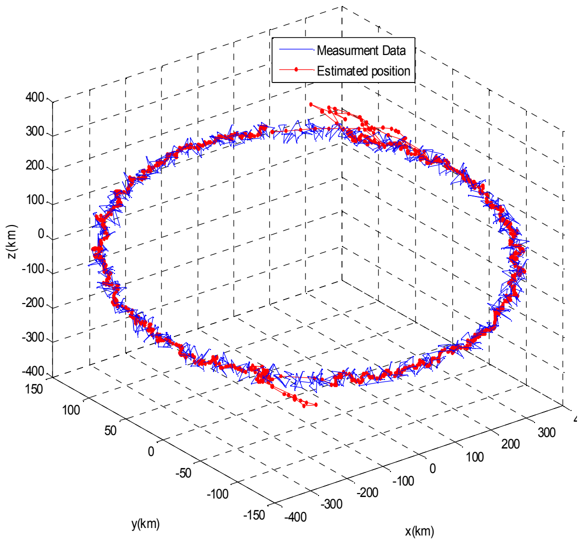

Fig. 2. Position estimation of the CHAMP satellite in 3D, using the EKF

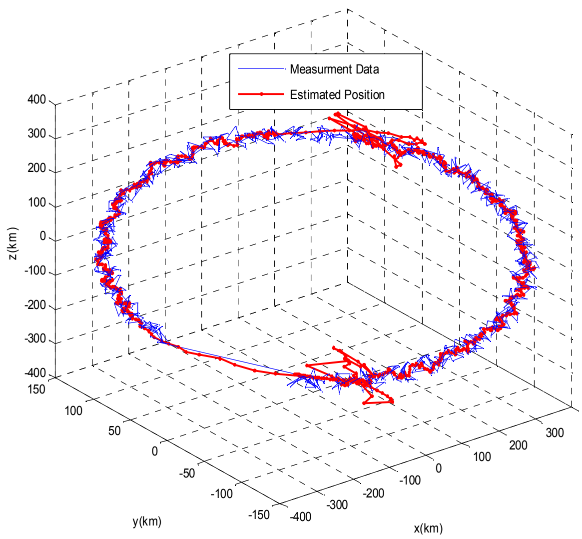

Fig. 5. Position estimation in 3D, when there is data packet loss

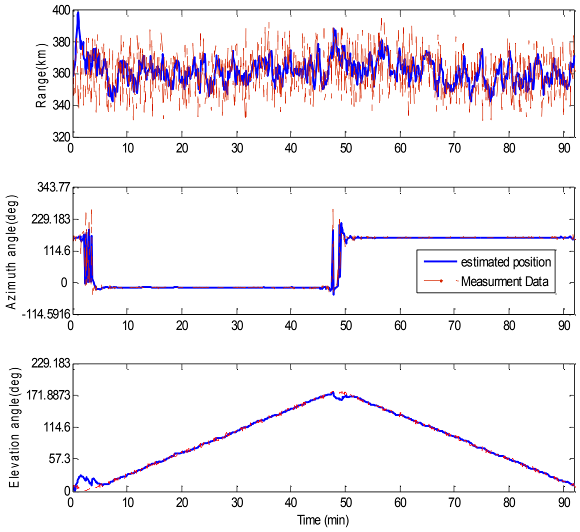

Fig. 3. Position estimation in polar coordinates

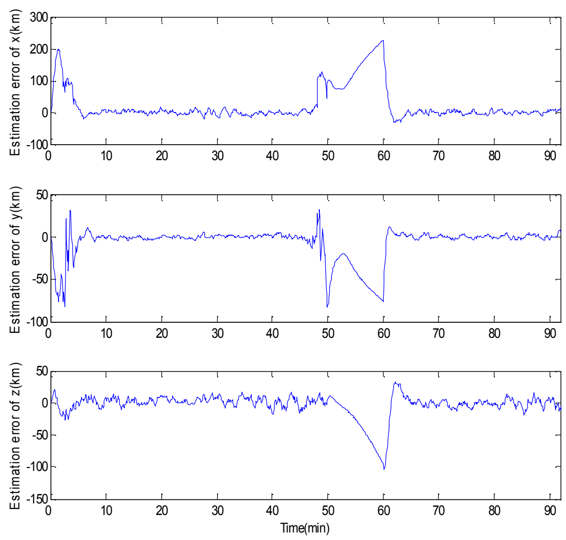

Fig. 4. Estimation error using the EKF with packet loss in data

4. ORBIT ESTIMATION USING NEURAL NETWORKS

It is a well-known fact that neural networks are capable of estimation. Different neural networks are employed for estimation in engineering applications. In this paper, a multilayer perceptron with error backpropagation algorithm is used for estimation of satellite position. The goal of this algorithm, which is based on the gradient descent method, is to minimize the instantaneous estimation error (Haykin, 1999)

$$E(k) = \left(y_o(k) - y(k)\right)^2 = \left(F(\mathbf{x}^T;\mathbf{w}) - y(k)\right)^2 \quad (21)$$

where $y_o(k)$ is the output of the neural network and $y(k)$ is the desired output at *k*th iteration step, respectively. The vector $\mathbf{w}$ contains the adjustable weights (synaptic and bias) of the network.

The network employed in this paper, has three layers: the input layer, which contains the input nodes, with seven inputs, the hidden layer, which contains neurons with nonlinear activation function (sigmoidal functions), and the output layer, which contains neurons with nonlinear activation function (sigmoidal functions) and provides the estimated output of the network. For satellite estimation, in this paper, there are 7 inputs, 50 neurons in the hidden layer, and 1 output in the neural networks. Hence, three neural networks are used to estimate *x*, *y* and *z*, respectively. The inputs to the neural networks are $[y_d(i+1) \;\; y_d(i) \;\; y_d(i-1) \;\; y_d(i-2) \;\; y_m(i-1) \;\; y_m(i-2) \;\; y_m(i-3)]$, where y_d is the desired orbit and the y_m is the measurement data. In order to avoid saturation in the neurons during training of the network, the inputs and outputs are normalized between zero and one (Haykin, 1999). Weights are initialized randomly using small numbers. The learning rate is equal to 0.9. Adaptations of weights are carried out on-line. That is, there is no stop in training of the network.

Figs. 6-9 show the simulation results. As these Figures show, the neural network can estimate the position of the satellite much better than the EKF, even when there is packet loss in data.

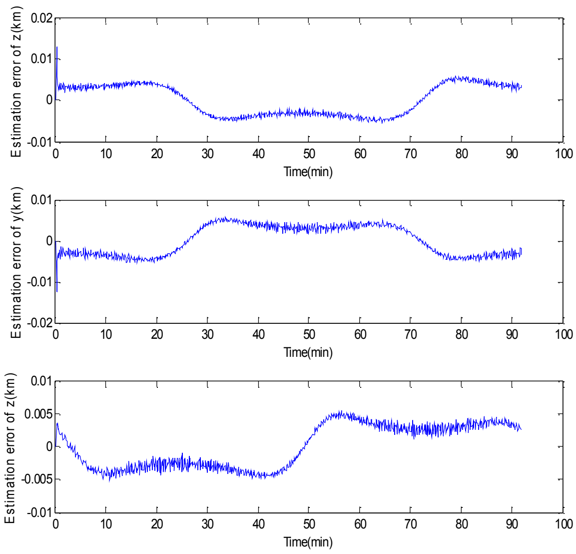

Fig. 6. Estimation errors along the *x*, *y* and *z* axis, using neural network

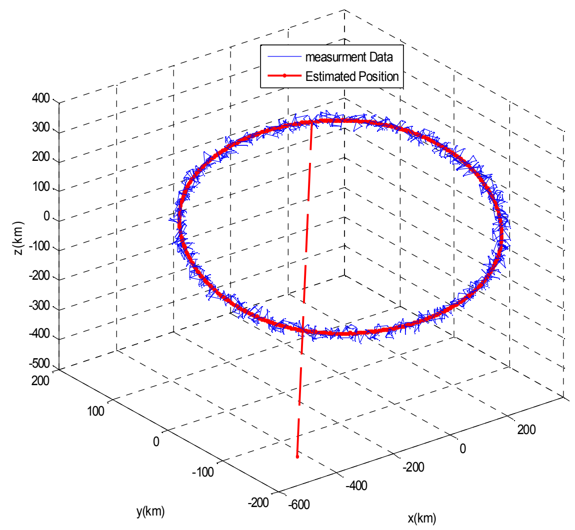

Fig 7. Position estimation of the CHAMP satellite in 3D, using the neural network

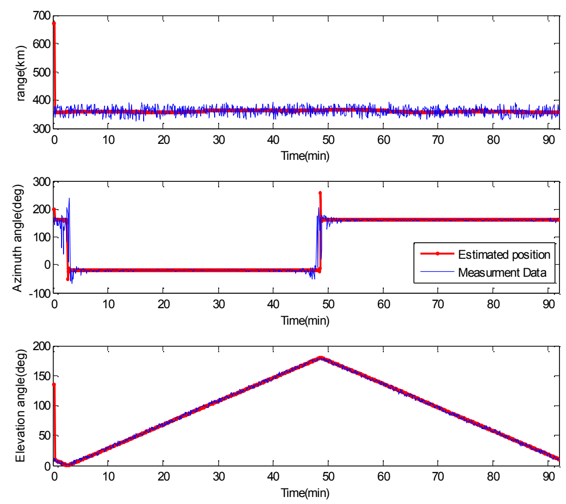

Fig. 8. Position estimation in polar coordinates

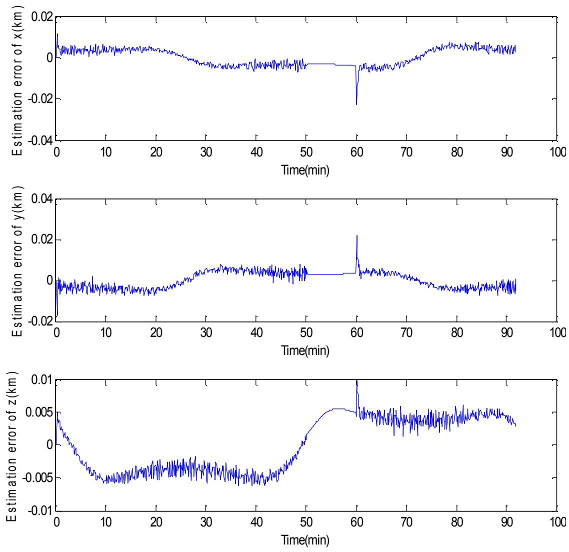

Fig. 9. Estimation error using the neural network, when there is packet loss in data.

5. CONCLUSION

In this paper, the extended Kalman filter and neural networks were employed for estimation of LEO satellites orbit. In the case of the extended Kalman filter, the initial values of states must be defined properly; otherwise states of the filter can diverge, yielding instability. On the other hand, this problem does not exist in neural network, since initial values of weights are selected randomly. The RMSE[1] for estimating the range along the *x*, *y* and *z* axis using the extended Kalman filter are 31, 11 and 7.2 Km, respectively, and for the neural network these errors are 0.094, 0.0092 and 0.0076 Km, respectively. In other words, the on-line trained neural network could estimate the orbital position of the satellite much better the extended Kalman filter. Moreover, both methods could cope well with data loss. Nevertheless, the neural network still shows less estimation error in this case. One important fact for the neural network is that, it must be trained on-line with no stopping in the training.

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