LOCAL SYNCHRONIZATION OF CYCLIC COUPLED HYPERCHAOTIC SYSTEMS AND ITS CIRCUIT IMPLEMENTATION

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Abstract

Most of the available research works on cyclic coupling of chaotic systems focussed on either analytical and numerical results or numerical and experimental results. This research paper, investigates synchronization of two cyclic coupled hyperchaotic systems using analytical, numerical and experimental techniques. Based on Routh-Hurwitz criterion, analytical condition for stable synchronization of the hyperchaotic systems are derived. The results obtained from MultiSIM and analog circuit confirm the effectiveness and feasibility of the analytical results. It is worthy of note that the cyclic coupling synchronization scheme gives several synchronization options, save synchronization time and cost. Moreover, cyclic coupling synchronization scheme has potential applications in biological information transmission networks.

Key words

Cyclic coupling, Routh-Hurwitz criterion, Hyperchaotic oscillator, MultiSIM, Analog implementation.

1 Introduction

Synchronization phenomenon is one of the most exciting behaviour of coupled nonlinear dynamical systems due to its significant and relevant applications in physical, chemical, biological and complex systems in general [Strogatz, 1994; Alligood et al., 1997; Njah and Ojo, 2010; Ma et al., 2012; Okpabi et al., 2017;

Tong et al., 2016]. Several types of synchronization [Rosenblum et al., 1997; Pecora and Carroll, 1990; Ojo et al., 2016a; Ojo et al., 2011; Ojo et al., 2013b; Gasri et al., 2018], methods of synchronization [Yang, 2012; Njah, 2011; Lu et al., 2013; Ojo et al., 2013a; Adegoke et al., 2013; Yang, 2012], and schemes [Ouannas, 2014; Zhang and Deng, 2014; Dongmo et al., 2018; Ojo et al., 2016b; Yu et al., 2013; Adelakun et al., 2017] have been developed in order to obtain most efficient applications of synchronization to natural and artificial systems. The search for the best coupling techniques to achieve excellent synchronization efficiency has led to the discovery of different coupling techniques such as unidirectional coupling [Adelakun et al., 2014; Rulkov et al., 1995; Ojo et al., 2013c], bidirectional coupling [Kumar et al., 2016; Khan and Poria, 2012], cross coupling [Zhang and Deng, 2014; Mengfei et al., 2015], cyclic coupling [Bera et al., 2016; Olusola et al., 2013; Egunjobi et al., 2018] and others [Liu et al., 2011; Iqbal et al., 2018; Belykh et al., 2005; Kandel et al., 2000]. The types of coupling technique and topology determine the level of stability or instability of synchronization of coupled nonlinear systems.

Having established that several coupling techniques are available for synchronization of nonlinear systems, our particular interest is on analytical, numerical and experimental implementation of synchronization via cyclic coupling technique. The choice of cycling coupling for synchronization is due to the following

reasons: (i) many synchronization choices (topologies) are available between each pair of variables depending on the dimension of the systems (ii) the available choices (topologies) can be optimized based on your particular objective of synchronization (iii) synchronization cost can be saved based on available choices (topologies)(iv) synchronization time can also be saved based on available choices (topologies) (v) the pair of variables which cannot be synchronization can easily be detected via the analytical criterion. The details of the available choices (topologies) is given in section 3 of this paper.

Cyclic coupled synchronization is a process whereby one system transmits an information to another system using one pair of its variables and then decode the transmitted information via a different pair of variables [Olusola et al., 2013]. It is worthy of note that the cyclic coupling configuration gives a better synchronization results where the diffusive bidirectional coupling configuration failed [Olusola et al., 2013]. Despite several advantages of cyclic coupling synchronization, not many research works have reported in this direction. [Bera et al., 2016; Olusola et al., 2013; Egunjobi et al., 2018; Adelakun et al., 2018].

Investigation on synchronization of cyclic coupled chaotic systems via analytical and numerical simulations was carried out in [Olusola et al., 2013]. It was discovered in the paper that cyclic coupling technique achieves stable synchronization in some chaotic systems where bidirectional mutual coupling technique failed. Also, synchronization of cyclic coupled Sprott systems was investigated and implemented using only electronic experiment [Egunjobi et al., 2018]. To best of our knowledge, synchronization of hyperchaotic systems via cyclic coupling has not been reported. Also, there is no single paper on synchronization of cyclic coupled chaotic systems that combines analytical, numerical and experimental simulations results. Motivated by the above discussion, this research work provides analytical, numerical and experimental evidence of synchronization of cyclic coupled hyperchaotic systems. The choice of hyperchatic system is as result of high complex dynamical behaviour and its attractive application in chaos based secure communication.

This paper is organized as follows. Section 2, provides description of analog circuit for the hyperchaotic system. Section 3, presents the theory of cyclic coupling. Section 4 deals with derivation of synchronization criteria for cyclic coupled hyperchaotic systems. Section 5 deals with results and discussion. Finally, section 6 concludes the paper.

2 Description of analog circuit for the hyperchaotic system

The four-dimensional hyperchaotic system proposed in this work can be expressed as:

$$\dot{x} = a(x - y) - yz + w \qquad (1)$$
$$\dot{y} = -by + xz$$
$$\dot{z} = -cz + dx + xy$$
$$\dot{w} = -e(x + y)$$

where x, y, z, w are the state variables and the values of the constants a, b, c, d and e are respectively 0.98, 9.00, 50.00, 0.06 and 0.90. The circuit components of the hyperchatic system consists of adder, inverter, integrator and multiplier as shown in Fig. 1. The differential equation from the circuit is as follows

$$\frac{dx}{d\tau} = \frac{1}{RC}\left(\frac{R}{R_1}x - \frac{R}{10R_3}yz + \frac{R}{R_4}w\right) \qquad (2)$$
$$\frac{dy}{d\tau} = \frac{1}{RC}\left(-\frac{R}{R_5}y + \frac{R}{10R_6}xz\right)$$
$$\frac{dz}{d\tau} = \frac{1}{RC}\left(-\frac{R}{R_7}z + \frac{R}{R_8}x + \frac{R}{10R_9}xy\right)$$
$$\frac{dw}{d\tau} = \frac{1}{RC}\left(-\frac{R}{R_{10}}x - \frac{R}{R_{11}}y\right)$$

If time scale transformation $\tau = 10RCt$ and considering the variable compression $x \to 0.1x$, $y \to 0.1y$ and $z \to 0.1z$ on Eqn.2, taking $R_1 = R_2$ and $R_{10} = R_{11}$ then we have:

$$\frac{dx}{d\tau} = 10\frac{R}{R_1}x - y - \frac{R}{10R_3}yz + 10\frac{R}{R_4}w \qquad (3)$$
$$\frac{dy}{d\tau} = -10\frac{R}{R_5}y + \frac{R}{10R_6}xz$$
$$\frac{dz}{d\tau} = -\frac{R}{R_7}z + 10\frac{R}{R_8}x + \frac{R}{10R_9}xy$$
$$\frac{dw}{d\tau} = -10\frac{R}{R_{10}}x - 10\frac{R}{R_{11}}y$$

Comparing Eqn.(1) with Eqn. (3), we have: $a = 10\frac{R}{R_1}$, $b = 10\frac{R}{R_5}$, $c = 10\frac{R}{R_7}$, $d = 10\frac{R}{R_8}$ $e = 10\frac{R}{R_{10}}$, where $R = R_4 = R_5 = R_6 = R_7 = R_{10} = R_{11} = R_{12} = R_{13} = R_{17} = R_{18} = R_{19} = R_{20} = R_{22} = R_{23} = 10k\Omega, R_1 = R_2 = R_{14} = R_{16} = 10k\Omega, R_3 = 102k\Omega$, $R_9 = 1.1k\Omega, R_15 = 1.67M\Omega$ and $R_{21} = 111k\Omega$. Capacitors $(C_1) - (C_4) = 10nF$ The analog circuit is shown in Fig. 1 and phase portraits of the hyperchaotic attractor obtained via MultiSIM and analog circuit are shown in Fig. 2

3 Theory of Cyclic Coupling

The cyclic coupling process involves engagement of a pair of variable of one system with a different pair of variable from another system in a bidirectional manner.
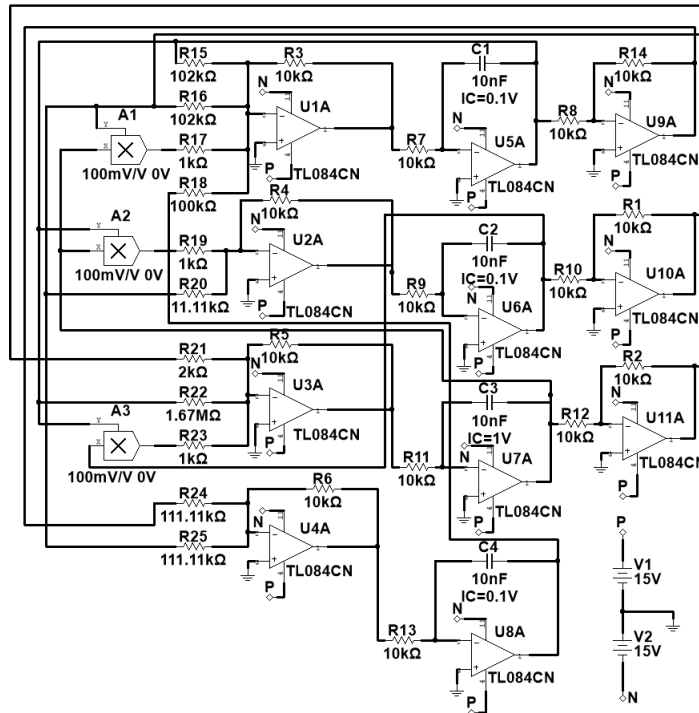
Figure 1.   Analog circuit of 4D hyperchaotic system with corresponding power supply
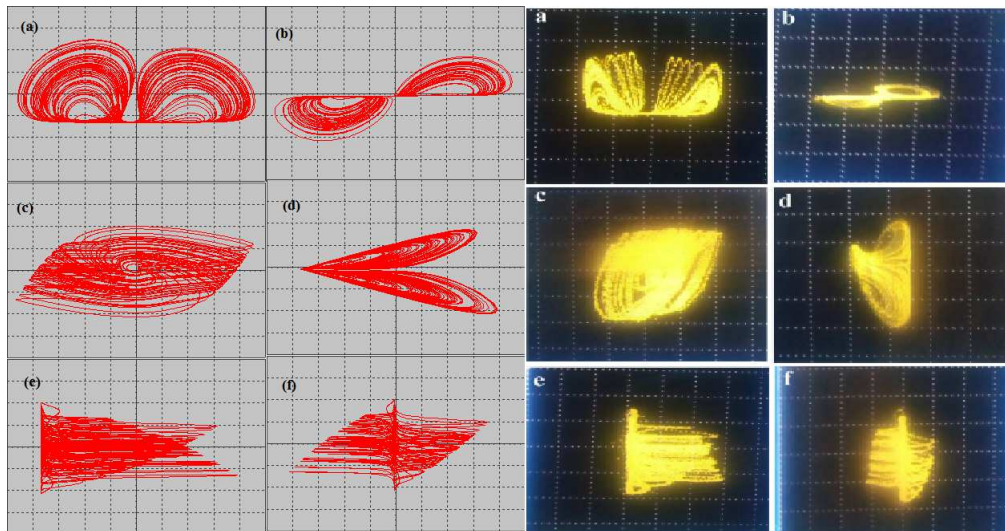
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Figure 2.   Two dimensional phase portraits of the hyperchaotic attractor for (a)$x - y$, (b) $x - z$, (c) $x - w$, (d) $y - z$, (e) $y - w$ and (f) $z - w$ axes obtained through MultiSIM software (left hand side) and analog circuit (right hand side)

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Consider cyclic coupling for two systems each of n dimension. The two systems are:

$$\dot{x} = Bx + f(x) + \beta_i H_{i,j}(x, y)$$
$$\dot{y} = By + g(y) + \beta_j H_{i,j}(x, y) \qquad (4)$$

where $x = (x_1, x_2, x_3, ...x_n)^T$, $y = (y_1, y_2, y_3, ...y_n)^T$ are the dynamical state variables of the systems. Matrix A represents the linear part of the system. f and g are non linear part of the system. $\beta_i, \beta_j (1, j = 1, 2, 3, ...n)$ are coupling parameters, where β_i is the coupling parameter on the first system and β_j is the coupling parameter on the second system. $H_{i,j}(x, y)$ is the output matrix of each pair of the system that are involved in the cyclic coupling process.

For better understanding, we consider cyclic cou-

pling using the following pair of systems. In this example, we shall use $x_1, x_2, x_3, ...x_n$ for the dynamical variables of the first system and $y_1, y_2, y_3, ...y_n$ for the dynamical variables of the second system. Now we look at possible independent coupling choices for systems of different dimensions: (a) cyclic coupling between two systems of two dimensions each gives only one independent coupling choice $x_1 \rightarrow y_1$ and $x_2 \leftarrow y_2$; (b) cyclic coupling between two systems of three dimensions each gives three independent coupling choices (i) $x_1 \rightarrow y_1$ and $x_2 \leftarrow y_2$ (ii) $x_1 \rightarrow y_1$ and $x_3 \leftarrow y_3$ (iii) $x_2 \rightarrow y_2$ and $x_3 \leftarrow y_3$; (c) cyclic coupling between two systems of four dimensions each gives six independent coupling choices (i) $x_1 \rightarrow y_1$ and $x_2 \leftarrow y_2$ (ii) $x_1 \rightarrow y_1$ and $x_3 \leftarrow y_3$ (iii) $x_1 \rightarrow y_1$ and $x_4 \leftarrow y_4$ (iv) $x_2 \rightarrow y_2$ and $x_3 \leftarrow y_3$ (v) $x_2 \rightarrow y_2$ and $x_4 \leftarrow y_4$ (vi) $x_3 \rightarrow y_3$ and $x_4 \leftarrow y_4$. We noticed that the number of independent choices for cyclic coupling between two n dimension system is $^nC_2$. For example: $n = 2$, gives 1 independent choice; $n = 3$ gives 3 independent choices; $n = 4$ gives 6 independent choices; $n = 5$ gives 10 independent choices and $n = n$ gives $^nC_2 = n(n-1)/2!$ independent choices.

In order to establish the stability criteria and the coupling strength threshold for synchronization of cyclic coupled systems (4) is written such that e_1 and e_2 are are taken as deviations of the system from synchronized state. Then the variation equation of the deviation of e_1 and e_2 can be written as

$$\dot{e}_1 = Be_1 + f'(x)e_1 + \beta_i H_{i,j}(e_2 - e_1)$$
$$\dot{e}_2 = Be_2 + g'(y)e_2 + \beta_j H_{i,j}(e_1 - e_2) \quad (5)$$

where B is a linear matrix, $f'(x)$ and $g'(y)$ are non linear functions. Using the approximation in [Khan and Poria, 2012], the time average of $f'(x)$ and $g'(y)$ is denoted by α. Now, synchronization error is defined as

$$e = e_2 - e_1 \quad (6)$$

Substitution of (5) into time derivative of (6) yields

$$\dot{e} = (B + \alpha I - \beta_i H_{i,j} - \beta_j H_{i,j})e \quad (7)$$

the error dynamics is stable if

$$P = B + \alpha I - \beta_i H_{i,j} - \beta_j H_{i,j} < 0 \quad (8)$$

otherwise, it is not stable. Therefore, P is a significant equation in determination of the stability of cyclic coupled systems. Moreover, the stability criteria for complete synchronization are derived from the negative real parts of the eigenvalues λ of P according to Routh-Hurwitz stability criterion. Also, β_i, β_j and α are real value and non-negative.

4 Derivation of Synchronization Criteria

This section deals with derivation of analytical criteria for the cyclic coupled hyperchaotic system. The cyclic coupling requires that two pairs of variables are engaged in the coupling such that there are twelve possible topologies of cyclic coupling for two four dimensional systems, out of which six are independent while the other six topologies are symmetric for identical oscillators.

Now, considering 2 pairs of variables for the 4–dimensional systems, 6 independent options are;

$(i) x_1 \rightarrow x_2, y_1 \leftarrow y_2$ when $H_{1,2} = diag(1,1,0,0)$
$(ii) x_1 \rightarrow x_2, z_1 \leftarrow z_2$ when $H_{1,3} = diag(1,0,1,0)$
$(iii) x_1 \rightarrow x_2, w_1 \leftarrow w_2$ when $H_{1,4} = diag(1,0,0,1)$
$(iv) y_1 \rightarrow y_2, z_1 \leftarrow z_2$ when $H_{2,3} = diag(0,1,1,0)$
$(v) y_1 \rightarrow y_2, w_1 \leftarrow w_2$ when $H_{2,4} = diag(0,1,0,1)$
$(vi) z_1 \rightarrow z_2, w_1 \leftarrow w_2$ when $H_{3,4} = diag(0,0,1,1)$

The arrows in the equations above indicate the direction of coupling. To establish the concept of cyclic coupling we use the stability criteria in (8)

4.1 Cyclic Coupling Between x and y Variables

When $H_{1,2} = diag(1,1,0,0)$, we obtain

$$\dot{x}_1 = a(x_1 - y_1) - y_1 z_1 + w_1 \quad (9)$$
$$\dot{y}_1 = -by_1 + x_1 z_1 + \beta_2(y_2 - y_1)$$
$$\dot{z}_1 = -cz_1 + dx_1 + x_1 y_1$$
$$\dot{w}_1 = -e(x_1 + y_1)$$
$$\dot{x}_2 = a(x_2 - y_2) - y_2 z_2 + w_2 + \beta_1(x_1 - x_2)$$
$$\dot{y}_2 = -by_2 + x_2 z_2$$
$$\dot{z}_2 = -cz_2 + dx_2 + x_2 y_2$$
$$\dot{w}_2 = -e(x_2 + y_2)$$

Using the stability condition (8) in (9), we have

$$P = \begin{pmatrix} a & -a & 0 & 1 \\ 0 & -b & 0 & 0 \\ d & 0 & -c & 0 \\ -e & -e & 0 & 0 \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix} - \begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} \alpha + a - \beta_1 & -a & 0 & 1 \\ 0 & \alpha - b - \beta_2 & 0 & 0 \\ d & 0 & \alpha - c & 0 \\ -e & -e & 0 & \alpha \end{pmatrix}$$

The eigenvalues of matrix P are,
$\lambda_1 = \alpha - c$
$\lambda_2 = \alpha - b - \beta_2$
$\lambda_{3,4} = \frac{-(\beta_1 - a - 2\alpha) \pm \sqrt{(\beta_1 - a - 2\alpha)^2 - 4[\alpha^2 + (a - \beta_1)\alpha - e]}}{2}$

So that the required stability conditions are:
(1) For $\lambda_1 = \alpha - c$, we have $\alpha < c$

(2) For $\lambda_2 = \alpha - b - \beta_2$, we have $\beta_2 > \alpha - b$

(3) If $(\beta_1 + \beta_2 - a - 2\alpha)^2 < 4[(\alpha - \beta_2)(\alpha - \beta_1 + a) - e]$, then $\lambda_{3,4}$ are complex and stability condition is $\beta_1 + \beta_2 > (2\alpha + a)$

(4) If $(\beta_1 + \beta_2 - a - 2\alpha)^2 > 4[(\alpha - \beta_2)(\alpha - \beta_1 + a) - e]$, then $\lambda_{3,4}$ are real and stability condition is $\beta_1 + \beta_2 > (2\alpha + a)$ and $[(\alpha - \beta_2)(\alpha - \beta_1 + a) - e] > 0$

Choosing a suitable value of $\beta_1 = 1.1$ and $\beta_2 = 2.98$, complete synchronization was obtained.

4.2 Cyclic Coupling Between x and z Variables

When $H_{1,3} = diag(1, 0, 1, 0)$, we obtain

$$\dot{x}_1 = a(x_1 - y_1) - y_1 z_1 + w_1 \qquad (10)$$
$$\dot{y}_1 = -by_1 + x_1 z_1$$
$$\dot{z}_1 = -cz_1 + dx_1 + x_1 y_1 + \beta_2(z_2 - z_1)$$
$$\dot{w}_1 = -e(x_1 + y_1)$$
$$\dot{x}_2 = a(x_2 - y_2) - y_2 z_2 + w_2 + \beta_1(x_1 - x_2)$$
$$\dot{y}_2 = -by_2 + x_2 z_2$$
$$\dot{z}_2 = -cz_2 + dx_2 + x_2 y_2$$
$$\dot{w}_2 = -e(x_2 + y_2)$$

Using the stability condition (8) in (10), we have

$$P = \begin{pmatrix} a & -a & 0 & 1 \\ 0 & -b & 0 & 0 \\ d & 0 & -c & 0 \\ -e & -e & 0 & 0 \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix} -$$
$$\begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$P = \begin{pmatrix} \alpha + a - \beta_1 & -a & 0 & 1 \\ 0 & \alpha - b & 0 & 0 \\ d & 0 & \alpha - c - \beta_2 & 0 \\ -e & -e & 0 & \alpha \end{pmatrix}$$

The eigenvalues of matrix P are,
$\lambda_1 = \alpha - c - \beta_2$
$\lambda_2 = \alpha - b$
$\lambda_{3,4} = \frac{-(\beta_1 - a - 2\alpha) \pm \sqrt{(\beta_1 - a - 2\alpha)^2 - 4[(\alpha - \beta_2)(\alpha - \beta_1 + a) - e]}}{2}$

So that the required stability conditions are:
(1) For $\lambda_1 = \alpha - c$, we have $\alpha < c$
(2) For $\lambda_2 = \alpha - b$, we have $\alpha < b$
(3) If $(\beta_1 - a - 2\alpha)^2 < 4[\alpha^2 + (a - \beta_1)\alpha - e]$, then $\lambda_{3,4}$ are complex and stability condition is $\beta_1 > (2\alpha + a)$
(4) If $(\beta_1 - a - 2\alpha)^2 > 4[\alpha^2 + (a - \beta_1)\alpha - e]$, then $\lambda_{3,4}$ are real and stability condition is $\beta_1 > (2\alpha + a)$ and $[\alpha^2 + (a - \beta_1)\alpha - e] > 0$

Choosing a suitable value of $\beta_1 = 1.1$ and $\beta_2 = 2.98$, complete synchronization was obtained.

4.3 Cyclic Coupling Between x and w Variables

When $H_{1,4} = diag(1, 0, 0, 1)$, we obtain

$$\dot{x}_1 = a(x_1 - y_1) - y_1 z_1 + w_1 \qquad (11)$$
$$\dot{y}_1 = -by_1 + x_1 z_1$$
$$\dot{z}_1 = -cz_1 + dx_1 + x_1 y_1$$
$$\dot{w}_1 = -e(x_1 + y_1) + \beta_2(w_2 - w_1)$$
$$\dot{x}_2 = a(x_2 - y_2) - y_2 z_2 + w_2 + \beta_1(x_1 - x_2)$$
$$\dot{y}_2 = -by_2 + x_2 z_2$$
$$\dot{z}_2 = -cz_2 + dx_2 + x_2 y_2$$
$$\dot{w}_2 = -e(x_2 + y_2)$$

Using the stability condition (8) in (11), we have

$$P = \begin{pmatrix} a & -a & 0 & 1 \\ 0 & -b & 0 & 0 \\ d & 0 & -c & 0 \\ -e & -e & 0 & 0 \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix} -$$
$$\begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 \end{pmatrix}$$
$$P = \begin{pmatrix} \alpha + a - \beta_1 & -a & 0 & 1 \\ 0 & \alpha - b & 0 & 0 \\ d & 0 & \alpha - c & 0 \\ -e & -e & 0 & \alpha - \beta_2 \end{pmatrix}$$

The eigenvalues of matrix P are,
$\lambda_1 = \alpha - c-$
$\lambda_2 = \alpha - b$
$\lambda_{3,4} = \frac{-(\beta_1 + \beta_2 - a - 2\alpha) \pm \sqrt{(\beta_1 + \beta_2 - a - 2\alpha)^2 - 4[\alpha^2 + (a - \beta_2)\alpha - e]}}{2}$

So that the required stability conditions are:
(1) For $\lambda_1 = \alpha - c - \beta_2$, we have $\beta_2 < \alpha - c$
(2) For $\lambda_2 = \alpha - b$, we have $\alpha < b$
(3) If $(\beta_1 + \beta_2 - a - 2\alpha)^2 < 4[(\alpha - \beta_2)(\alpha - \beta_1 + a) - e]$, then $\lambda_{3,4}$ are complex and stability condition is $\beta_1 > (2\alpha + a)$
(4) If $(\beta_1 + \beta_2 - a - 2\alpha)^2 > 4[(\alpha - \beta_2)(\alpha - \beta_1 + a) - e]$, then $\lambda_{3,4}$ are real and stability condition is $\beta_1 > (2\alpha + a)$ and $[(\alpha^2 + (a - \beta_1)\alpha - e] > 0$

Choosing a suitable value of $\beta_1 = 20$ and $\beta_2 = 20$, complete synchronization was obtained.

4.4 Cyclic Coupling Between y and z Variables

When $H_{2,3} = diag(0, 1, 1, 0)$, we obtain

$$\dot{x}_1 = a(x_1 - y_1) - y_1 z_1 + w_1 \qquad (12)$$
$$\dot{y}_1 = -by_1 + x_1 z_1$$
$$\dot{z}_1 = -cz_1 + dx_1 + x_1 y_1 + \beta_2(z_2 - z_1)$$
$$\dot{w}_1 = -e(x_1 + y_1)$$
$$\dot{x}_2 = a(x_2 - y_2) - y_2 z_2 + w_2$$
$$\dot{y}_2 = -by_2 + x_2 z_2 + \beta_1(y_1 - y_2)$$
$$\dot{z}_2 = -cz_2 + dx_2 + x_2 y_2$$
$$\dot{w}_2 = -e(x_2 + y_2)$$

Using the stability condition (8) in (12), we have

$$P = \begin{pmatrix} a & -a & 0 & 1 \\ 0 & -b & 0 & 0 \\ d & 0 & -c & 0 \\ -e & -e & 0 & 0 \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix} -$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} \alpha + a & -a & 0 & 1 \\ 0 & \alpha - b - \beta_1 & 0 & 0 \\ d & 0 & \alpha - c - \beta_2 & 0 \\ -e & -e & 0 & \alpha \end{pmatrix}$$

The eigenvalues of matrix P are,
$$\lambda_1 = \alpha - c - \beta_2$$
$$\lambda_2 = \alpha - b - \beta_1$$
$$\lambda_{3,4} = \frac{2\alpha + a) \pm \sqrt{(2\alpha + a)^2 - 4[\alpha^2 + a - e]}}{2}$$

So that the required stability conditions are:
(1) For $\lambda_1 = \alpha - c - \beta_2$, we have $\beta_2 > \alpha - c$
(2) For $\lambda_2 = \alpha - b - \beta_1$, we have $\beta_1 > \alpha - b$
(3) If $(2\alpha + a)^2 < 4[(\alpha^2 + a - e]$, then $\lambda_{3,4}$ are complex and stability condition is $2\alpha < a$
(4) If $(2\alpha + a)^2 > 4[(\alpha^2 + a - e]$, then $\lambda_{3,4}$ are real and stability condition is $2\alpha < a$ and $[\alpha^2 + a - e] > 0$

The analytical synchronization criterion obtained shows that synchronization cannot be achieved between y and z variables.

4.5 Cyclic Coupling Between y and w Variables

When $H_{2,4} = diag(0, 1, 0, 1)$, we obtain

$$\dot{x}_1 = a(x_1 - y_1) - y_1 z_1 + w_1 \qquad (13)$$
$$\dot{y}_1 = -by_1 + x_1 z_1$$
$$\dot{z}_1 = -cz_1 + dx_1 + x_1 y_1$$
$$\dot{w}_1 = -e(x_1 + y_1) + \beta_2(w_2 - w_1)$$
$$\dot{x}_2 = a(x_2 - y_2) - y_2 z_2 + w_2$$
$$\dot{y}_2 = -by_2 + x_2 z_2 + \beta_1(y_1 - y_2)$$
$$\dot{z}_2 = -cz_2 + dx_2 + x_2 y_2$$
$$\dot{w}_2 = -e(x_2 + y_2)$$

Using the stability condition (8) in (13), we have

$$P = \begin{pmatrix} a & -a & 0 & 1 \\ 0 & -b & 0 & 0 \\ d & 0 & -c & 0 \\ -e & -e & 0 & 0 \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix} -$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 \end{pmatrix}$$

$$P = \begin{pmatrix} \alpha + a & -a & 0 & 1 \\ 0 & \alpha - b - \beta_1 & 0 & 0 \\ d & 0 & \alpha - c & 0 \\ -e & -e & 0 & \alpha - \beta_2 \end{pmatrix}$$

The eigenvalues of the matrix P are,
$$\lambda_1 = \alpha - b - \beta_1$$
$$\lambda_2 = \alpha - c$$
$$\lambda_{3,4} = \frac{(\beta_2 - 2\alpha - a) \pm \sqrt{(\beta_2 - 2\alpha - a)^2 - 4[\alpha^2 + (a - \beta_2)\alpha - a\beta_2 + e]}}{2}$$

So that the required stability conditions are:
(1) For $\lambda_1 = \alpha - b - \beta_1$, we have $\beta_1 < \alpha - b$
(2) For $\lambda_2 = \alpha - c$, we have $\alpha < c$
(3) If $(\beta_2 - a - 2\alpha)^2 < 4[(\alpha^2 + (a - \beta_2)\alpha - a\beta_2 + e]$, then $\lambda_{3,4}$ are complex and stability condition is $\beta_2 > 2\alpha + a$
(4) If $(\beta_2 - a - 2\alpha)^2 > 4[(\alpha^2 + (a - \beta_2)\alpha - a\beta_2 + e]$, then $\lambda_{3,4}$ are real and stability condition is $\beta_2 > 2\alpha + a$ and $[(\alpha^2 + (a - \beta_2)\alpha - a\beta_2 + e] > 0$

Choosing a suitable value of $\beta_1 = 45$ and $\beta_2 = 70$, complete synchronization was obtained.

4.6 Cyclic Coupling Between z and w Variables

When $H_{3,4} = diag(0, 0, 1, 1)$, we obtain

$$\dot{x}_1 = a(x_1 - y_1) - y_1 z_1 + w_1 \qquad (14)$$
$$\dot{y}_1 = -by_1 + x_1 z_1$$
$$\dot{z}_1 = -cz_1 + dx_1 + x_1 y_1$$
$$\dot{w}_1 = -e(x_1 + y_1) + \beta_2(w_2 - w_1)$$
$$\dot{x}_2 = a(x_2 - y_2) - y_2 z_2 + w_2$$
$$\dot{y}_2 = -by_2 + x_2 z_2$$
$$\dot{z}_2 = -cz_2 + dx_2 + x_2 y_2 + \beta_1(z_1 - z_2)$$
$$\dot{w}_2 = -e(x_2 + y_2)$$

Using the stability condition (8) in (14), we have

$$P = \begin{pmatrix} a & -a & 0 & 1 \\ 0 & -b & 0 & 0 \\ d & 0 & -c & 0 \\ -e & -e & 0 & 0 \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix} -$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 \\ 0 & 0 & 0 & \beta_2 \end{pmatrix}$$

$$P = \begin{pmatrix} \alpha + a & -a & 0 & 1 \\ 0 & \alpha - b & 0 & 0 \\ d & 0 & \alpha - c - \beta_1 & 0 \\ -e & -e & 0 & \alpha - \beta_2 \end{pmatrix}$$

The eigenvalues of matrix P are,

$\lambda_1 = \alpha - c - \beta_1$

$\lambda_2 = \alpha - b$

$\lambda_{3,4} = \frac{-(\beta_2 - 2\alpha - a) \pm \sqrt{(\beta_2 - 2\alpha - a)^2 - 4[(\alpha + a)(\alpha - \beta_2) - e]}}{2}$

So that the required stability conditions are:

(1) For $\lambda_1 = \alpha - c - \beta_1$, we have $\beta_1 < \alpha - c$

(2) For $\lambda_2 = \alpha - b$, we have $\alpha < b$

(3) If $(\lambda_2 - a - 2\alpha)^2 < 4[(\alpha + a)(\alpha - \lambda_2) - e]$, then $\lambda_{3,4}$ are complex and stability condition is $\lambda_2 > 2\alpha + a$

(4) If $(\lambda_2 - a - 2\alpha)^2 > 4[(\alpha + a)(\alpha - \lambda_2) - e]$, then $\lambda_{3,4}$ are real and stability condition is $\lambda_2 > 2\alpha + a$ and $[(\alpha + a)(\alpha - \lambda_2) - e] > 0$

Choosing a suitable value of $\beta_1 = 1$ and $\beta_2 = 1$, complete synchronization was obtained.

5 Results and Discussion

Having established the analytical criterion for synchronization in each pair of the cyclic coupled variables, we shall confirm each of the derived analytical criterion using MultiSIM software and analog circuit. Now, we implement the analytical criterion obtained for each pair of the hyperchaotic system with each of system with different initial conditions $(1, 0, 1, 5)$ and $(5.5, 6, 7.2, 8)$ on MultiSIM software. The results from MultiSIM software shown in Fig 3 confirm the correctness and effectiveness of the derived analytical criterion. The results in Fig. 3 show that five out of the six topology synchronized except topology that involves y and z variables. The results in Fig. 3 is in perfect consonance with the analytical criterion obtained in section 4. On the other hand, the circuit realization involves integrators and summers built with operational amplifiers , TL084CN (U_{iA}), multipliers $(A1 - A6)$, power supply unit, resistors and capacitors. The two analog circuits captured in Fig. 4, with initial conditions $(1, 0, 1, 5)$ and $(5.5, 6, 7.2, 8)$ used for different cyclic synchronization paths. The coupling constant which determine the small or large window of the phase portrait of the attractor can be generated from k_x, k_y, k_z and k_w depending on the choice of the trajectory of the cyclic path, where $k_x = k(x_1 \rightarrow x_2)$ or $k(x_1 \leftarrow x_2)$, $k_y = k(y_1 \rightarrow y_2)$ or $k(y_1 \leftarrow y_2)$, $k_z = k(z_1 \rightarrow z_2)$ or $k(z_2 \leftarrow z_1)$ and $k_w = k(w_1 \rightarrow w_2)$ or $k(w_2 \leftarrow w_1)$. To achieve the desired goal, the coupling constant is varied according to analytical value obtained in section 4 and the results obtained are shown in Fig.

5. The results in depict in Fig. 5 show that synchronization is observed in each of the five pair of the variable considered. The analog circuit implementation of synchronization between cyclic coupling variables y and z variables is not shown in Fig. 5 because the analytical and MultiSIM software simulation already confirm there is no synchronization in y and z topology.

6 Conclusions

Cyclic coupling synchronization scheme for two identical 4D hyperchaotic systems has been proposed in this research paper. The results from numerical and experimental simulations confirm the effectiveness and the feasibility of the derived analytical criterion. In other word, the analytical, numerical and analog implementation confirm synchronization in five out of six available topologies. The main advantage of cyclic coupling is that it gives several choices for synchronization, as a result, synchronization choices can be optimized. It help to limit the choice of synchronization to a particular pair of variables or any desired pair of variables that would give the least synchronization time and synchronization cost. Hence, cyclic coupling synchronization scheme can be used to save synchronization time and synchronization cost.

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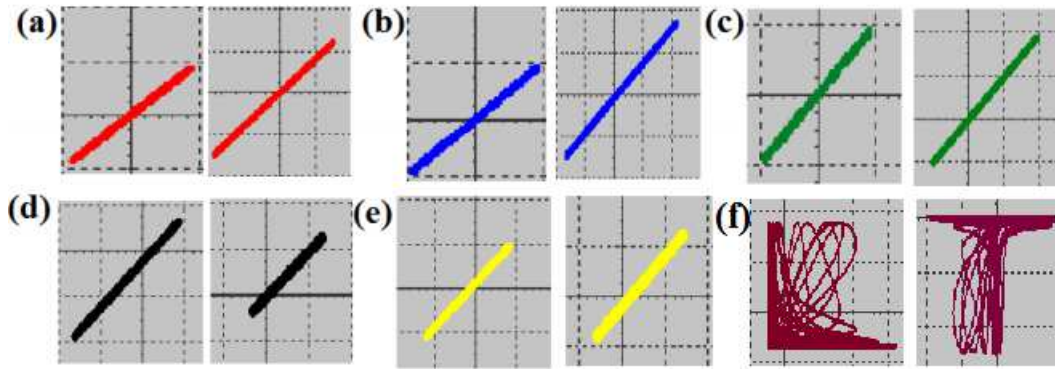Dongmo, E. D., Ojo, K. S., Woafo, P., and Njah, A. N. (2018). Difference synchronization of identical and

Figure 3. Comfirmation of derived synchronization criterion for the following pair of cyclic coupled variables: (a)$x_1 \rightarrow x_2, y_1 \leftarrow y_2$ (b)$x_1 \rightarrow x_2, z_1 \leftarrow z_2$, (c)$x_1 \rightarrow x_2, w_1 \leftarrow w_2$ (d)$y_1 \rightarrow y_2, w_1 \leftarrow w_2$ and(e) $z_1 \rightarrow z_2, w_1 \leftarrow w_2$ (f)$y_1 \rightarrow y_2, z_1 \leftarrow z_2$ through MultiSIM
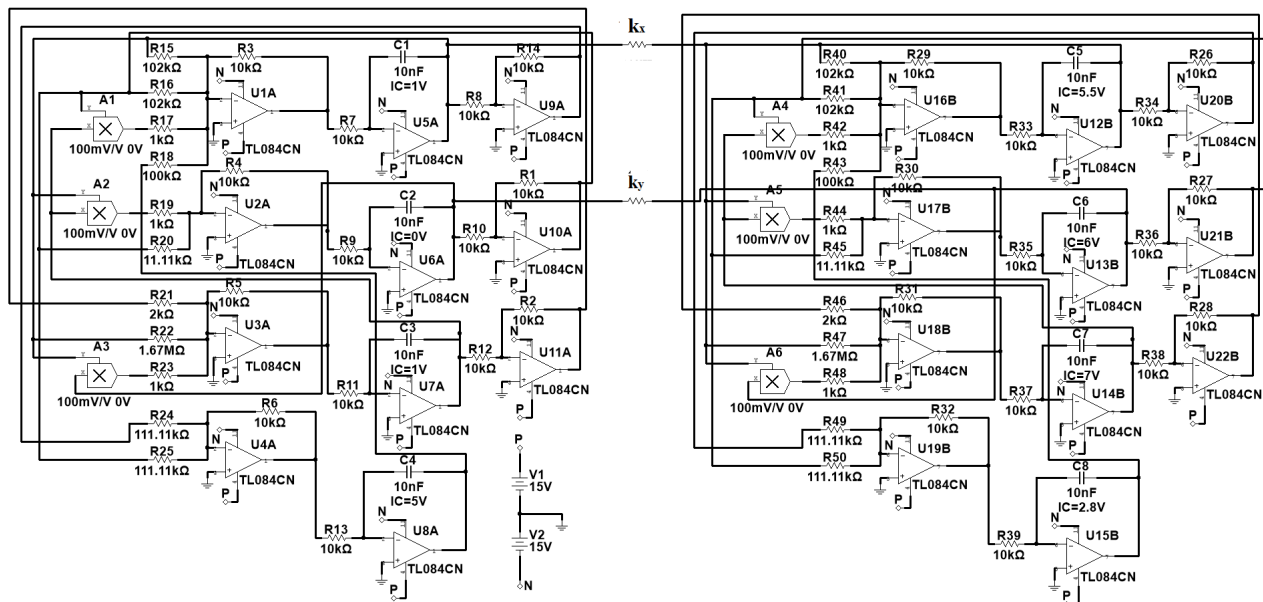
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Figure 4. Analog circuit for the cyclic coupled hyperchaotic systems

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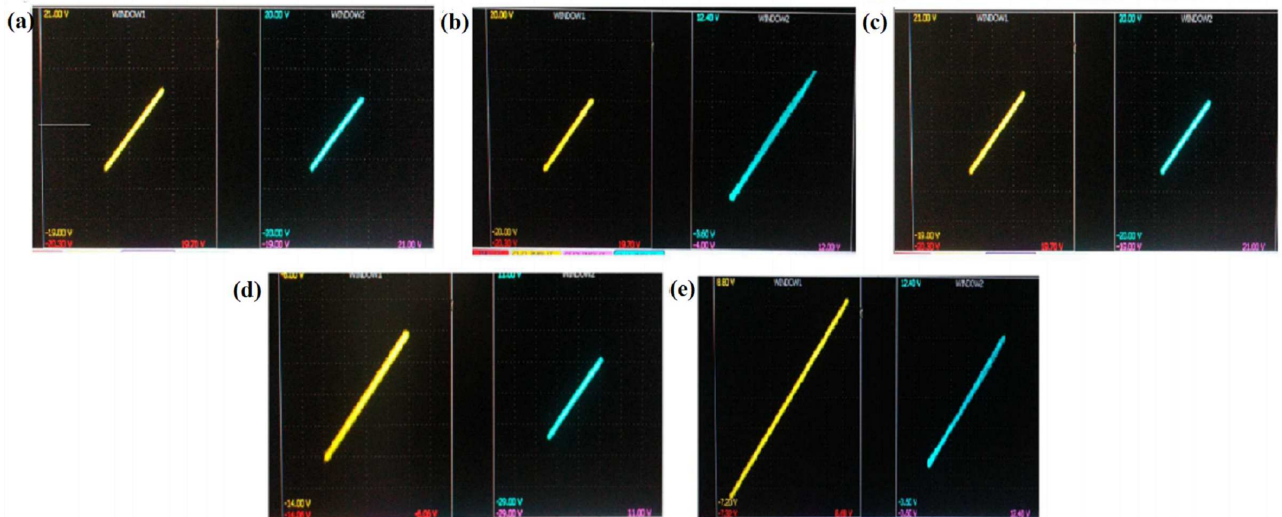
Figure 5. Comfirmation of derived synchronization criterion for the following pair of cyclic coupled variables: (a)$x_1 \rightarrow x_2, y_1 \leftarrow y_2$ (b)$x_1 \rightarrow x_2, z_1 \leftarrow z_2$, (c)$x_1 \rightarrow x_2, w_1 \leftarrow w_2$ (d)$y_1 \rightarrow y_2, w_1 \leftarrow w_2$ and(e) $z_1 \rightarrow z_2, w_1 \leftarrow w_2$ through analog circuit

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