EMERGENT PROPERTIES IN A AUTOMATA GAS

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Abstract
Understanding statistical properties of cognitive systems is one of the main goal of complex systems physics. The automata gas is a statistical system whose particles perform information based interactions and use a decision mechanism. The collective nature of such interactions is at the base of the self-organized dynamical states of the system. In some cases it is possible to study the emergence of these states, by using an adiabatic separation between the dynamics time scales and the particle distribution time scale. Then the emergent properties can be related to the solutions of a nonlinear diffusion equation. The considered models have a wide range of application in biology and social sciences to describe the self-organization properties observed in the experiments. Our main interest is to develop a model for the pedestrian dynamics in a urban space. We study the properties of a automata gas in some simple cases and we discuss the applicability of the adiabatic approach.

Key words
Automata gas, cognitive dynamics, self-organized properties

1 Introduction

The theoretical approach to complexity science is related to the definition of various classes of models inspired by the different application fields. Taking the Von Neumann automaton definition[Von Neumann, 1963] as a starting point, there has been recently proposed the "automata gas model"[Turchetti, 2007] as a generalization of a granular flow model, whose elementary particles has an internal cognitive structure able to process information and to take decisions according to an utility function[Domencich, 1975]. This model can be included in the agent based models class[Schweitzer, 2003], that has been developed to simulate biological systems like ants colonies[Detrain, 2006] or social systems like pedestrians in urban spaces[Batty, 2003]. In this paper we present a simple automata gas model to study the existence of emergent properties that can be directly to a cognitive behavior. The automata have a cooperative attitude based on the information on the population collective behavior and move subjected to physical interactions with other automata and the external environment. The physical interactions consider both inelastic collisions due to the finite volume effects, and a "visual interaction" that introduces a repulsive force among the automata: in such a way the automata tend to avoid hard collisions when it is possible.

Conversely the cognitive behavior is based on the existence of an internal cognitive state for each automaton[Gelder, 1998], whose dynamics is given by a stochastic differential equation in the landscape potential defined by the utility function[Domencich, 1975]. The external information modifies the landscape utility potential and determines the automaton propensity towards a certain choice. The cognitive dynamics introduces a nonlinear relation between the utility and the decision probability (in the Bayes-De Finetti interpretation of subjective probability[de Finetti, 1972]), that is consistent with recent observations and experiments in neuroscience[Nadel, 2003]. Our aim is to study phase transitions into self-organized states that are consequence of the cognitive automaton dynamics. Similar problems have been considered by other authors[Camazine, 2001] in biological systems to distinguish the collective behaviors due to physical interactions from the strategies related to evolution and learning processes. A rigorous proof of the results presented in the paper would require to prove a generalized law of large numbers for interacting particles with collisional finite volume effects: this is beyond the purpose of this paper. The model we are interested, is relevant for simulation of pedestrian dynamic problems in urban spaces. Pedestrian dynamics has been considered by physicists to study the appearance of congestion effects in panic stampede and criticalities in crowd dynamics[Helbing, 2005]. The attention was focused mainly on physical interactions among pedestrians to improve the safety[Helbing, 2005]. But recently

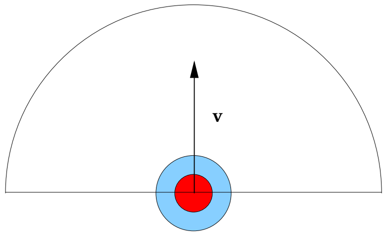

Figure 1. Sketch of automaton physical dimensions: the dark circle is the incompressible body of radius r_b; the light circle is the social space of radius $2r_b$ and the larger semicircle is the frontal visual space of radius $5r_b$.

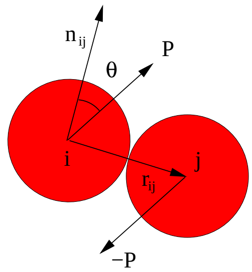

Figure 2. Physical collision between two incompressible automata in the center of mass reference system.

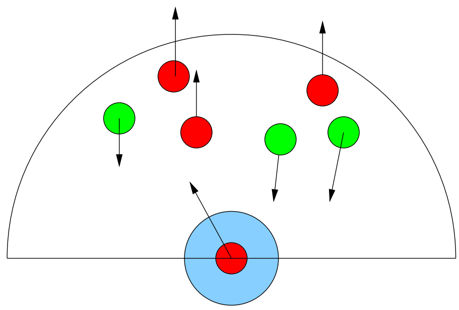

Figure 3. Collision avoiding strategy of automata as a consequence of a local vision mechanism.

the urban planners and sociologists have pointed out the complexity characters of the urban mobility and its relevance for life quality[Fruin, 1971], that cannot be simulated using the classical origin-destination approach[Cascetta, 2001]. Under this point of view, the automata gas model may become a useful instrument for the urban planners both to take into some the cognitive aspects of pedestrian mobility related to the appearance of preferred paths or habits and to produce a mobility "governance"[Giorgini, 2007].

2 The automata gas model: physical interactions

The automata gas model is a generalization of a 2-dimensional granular flow model. The elementary particles (automata) have a finite body dimension r_b, that defines the microscopic spatial scale of the system, a social space space of dimension $2r_b$ and a visual space of dimension $\simeq 5r_b$. We also provide the automata of an inertial mass m. In the figure 1, we plot a sketch of the automaton physical properties; to reproduce the effect of a frontal vision, the visual space is limited to a semicircle centered at the velocity direction. Each automaton i tends to move at a desired velocity $\vec{v}_{0i}$ according to

$$\dot{\vec{v}}_i = -\gamma(\vec{v}_i - \vec{v}_{0i}) \quad (1)$$

where the parameter $1/\gamma$ defines the microscopic relaxation time scale of the system. The automata perform inelastic collisions due to their body incompressibility and the momentum $\vec{P}$ in the center of mass reference system changes according to

$$\vec{P}' = \mathcal{R}(\theta + \phi)\sqrt{1 - \eta \sin^2\theta}\,\vec{P} \quad (2)$$

where $\mathcal{R}(\theta)$ is a rotation matrix of an angle θ in the plane: θ is the collision angle as defined in the figure 2. The parameter η controls the kinetic energy loss which is maximal when $\theta = \pi/2$ (head-on collision), and the angle $\phi \in [0, \theta]$ introduces a sliding effect between the colliding automata. We remark that $\eta = 1$ and $\phi = 0$ define a classical inelastic collision whereas $\eta = 0$ and $\phi = \theta$ give a purely elastic collision.

The visual space introduces a topological long range interaction among the automata in the following way: each automaton checks all the other automata in his visual space to detect if somebody will enter into his social space; in such a case he rotates and reduces the velocity in order to perform a strategy to avoid future collisions. The rotation velocity ω and the deceleration rate α are constant (not depending from the distance of the "dangerous" automata). Therefore the net result is a repulsive force among the automata coming from opposite directions and the automaton tendency to move towards empty regions of the space or to follow the automata moving in the same direction (see fig. 3). The repulsive force turns out to be proportional to the density of counteracting automata in the visual space and to the rotational velocity and deceleration rate values. We remark that the "local vision" has not the features of a Newtonian force since the Action-Reaction principle is not valid for a frontal vision. The long range character local vision allows the formation of of self-organized dynamical states: let us consider two counteracting automata flows along a narrow corridor. The collision dynamics defined by eq. (2) introduces a stochastic perturbation in the velocity equation (1) which has a finite transverse component with respect to the desired velocity (directed along the corridor). If γ is not too big and the local vision mechanism is switched off, both the automata populations tends to distribute disorderly along the corridor according to maximal entropy principle (see fig. 4 top). If we switch on the local vision, it appears a density depend force in

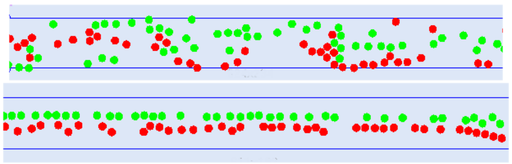

Figure 4. Dynamics of two counteracting flow of automata moving along a corridor. When the automata interact only by means of inelastic collisions a disordered state is observed along the corridor(top); if we switch on the local vision interaction, a two streams ordered flow appear along the corridor (bottom).

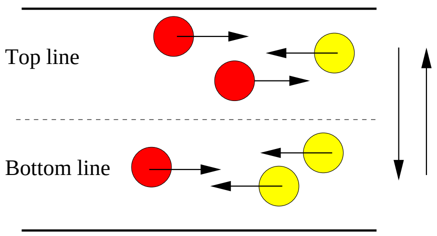

Figure 5. Transverse dynamics along the corridor according to the detailed balance equations; the arrows denote the transition probabilities between the top and bottom lines.

the transverse dynamics that increases the displacement probability of an automaton when he meet a cluster of counteracting automata. As a result a self-organized state is created, with two ordered streams moving along corridor (see fig. 4 bottom). To get a qualitative explanation of the observed phenomenon, one can consider a detailed balance equation to describe the transverse dynamics along the corridor. Let us divide the corridor in two lines and let $p^{\pm}_{t,b}$ the probabilities that an automaton moves in the positive $(+)$ or negative $(-)$ direction along the top t or the bottom b lines. The detailed balance equations read (cfr. fig. 5)

$$\begin{aligned} \frac{dp^+_t}{dt} &= -p^+_{tb}p^+_t + p^+_{bt}p^+_b \\ \frac{dp^-_t}{dt} &= -p^-_{tb}p^-_t + p^-_{bt}p^-_b \end{aligned} \tag{3}$$

with the constraint $p^{\pm}_t + p^{\pm}_b = 1$. $p^{\pm}_{tb,bt}$ are the transition probabilities that depend nonlinearly on the automata distribution when the vision mechanism is considered due to the presence of multiple collisions. In a perturbative approach, we set

$$\begin{aligned} p^+_{tb} &= Dp^-_t\left(1 + c(p^-_t - p^-_b)\right) + D'p^+_t \\ p^+_{bt} &= Dp^-_b\left(1 + c(p^-_b - p^-_t)\right) + D'p^+_b \end{aligned} \tag{4}$$

and analogous formulas for p^-_{tb} and p^-_{bt}. The coefficients D and D' describe the effects of binary interactions (we have an obvious inequality $D' < D$ since the interactions among the automata moving in the same direction is weak), whereas the coefficient c weights the collective interactions due to local vision that depend on the counteracting automata distribution in both the lines. After some algebraic manipulations, it is possible to explicitly compute the equilibrium states of eq. (3), from the equation

$$\left(2cDp^+_t(1 - p^+_t) - D'\right)(p^+_t - p^-_t) = 0 \tag{5}$$

$p^+_t = p^-_t = 1/2$ is the uniform distribution along the corridor (ergodic solution), but we have another equilibrium solution at

$$p^+_t = p^-_b = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{2D'}{cD}}\right) \qquad \text{if} \quad \frac{D'}{cD} \le \frac{1}{2} \tag{6}$$

Therefore it exists a critical value $c_* = 2D'/D$ for the collective effects weight over which a self-organized stable equilibrium appears corresponding to a non-uniform transverse distribution of automata along the corridor; at the same time the ergodic solution becomes unstable. This picture is consistent with the simulations show in fig. 4. In order to apply the detailed balance equation a physical separation between the two lines is needed to avoid instabilities due to collisions; under this point of view one has to take into account the finite volume effects and the continuous limit is not trivial. We finally remark that the self-organization due to the local vision cannot be considered the result of a cognitive process since it is essentially related to a collective effect and not to an internal cognitive process of automata; indeed the same phenomenon is observed in various granular flow models.

3 The automata gas model: cognitive behavior

The mathematization of cognitive interactions is still an open problem, and under a certain point of view the game theory is simulation of decision mechanism[Parsons, 2002]. Therefore one can consider a class of models based on simple assumptions that are supported by experimental observations. Being inspired by recent experimental results in neuroscience research[Nadel, 2003], a dynamical model for the cognitive behavior should take into account the following remarks:

- the decisions are always mediated by the brain activity (existence of a cognitive space);
- there is a nonlinear subjective relation between the "utility" of a decision and the probability of taking that decision: overestimate of small advantages and underestimate of the large advantages with respect to a linear relation;
- there is an aversion to risk.

Out dynamical model to introduce an automaton cognitive behavior, can be formulated according the following assumptions:

- the brain activity can be represented by a dynamical system defined on a n-dimensional cognitive space;
- the utility associated to a decision introduces a landscape potential in the cognitive space whose shape depends on the external information;
- the cognitive state is stochastically perturbed;
- let E_j $j = 1, ..., n+1$ the events associated to the possible decisions, there exists a probability function $\mathcal{P}(E_j; X)$ which is a function of the cognitive states.

The first assumption essentially states that the brain is a huge dynamical systems, that can be macroscopically described, when we limit his activity to simple choice operations. The existence of an utility function in the decision mechanism has been proposed in various contexts[Domencich, 1975]; our point of view is to consider the utility function as a potential $V(X; I)$ in the cognitive space, whose minima could be related to the utility of the different choices and depend on the external information I. Moreover we attribute the same utility function to all the automata in a given population. The third assumption is quite obvious since one expects that any decision can be influenced by several unpredictable uncorrelated factors. Finally the last assumption allows to introduce a subjective rationality since choice probabilities is different for automata with a different cognitive state X or a different information level. The cognitive dynamics for the i-automaton is defined by the stochastic differential equation

$$dX_i = -\frac{\partial V}{\partial X}(X_i; I)dt + \sqrt{2T_i}dw_i(t) \qquad (7)$$

where T_i is the individual "social temperature" (i.e. a measure of the individual influence of external random factors on the cognitive state) and $w_i(t)$ are independent Wiener processes. The information I is obtained from the external environment and it introduces a global coupling among the automata since the automata dynamics changes the environment itself. If we neglect I dependence the distribution function $\varrho_i(X, t)$ of the cognitive states satisfies a Fokker-Planck equation

$$\frac{\partial \varrho_i}{\partial t} = \frac{\partial}{\partial X}\frac{\partial V}{\partial X}\varrho_i + T_i \frac{\partial^2 \varrho_i}{\partial X^2} \qquad (8)$$

In this paper we discuss the case $I = I(\rho)$ where $\rho(x, v, t)$ is the phase space density of the automata population and the events E_j are related to possible choices of the desired velocity. For a fixed cognitive state and if one neglects the finite volume effects, the automata distribution $\rho(x, v, t)$ could be related to the solution of a self-consistent Vlasov problem, where the collective interactions are introduced by the local vision mechanism. In the sequel we analyze the case in which the distribution ρ relaxes toward an equilibrium distribution and the relaxation time is much faster than the relaxation time of the each cognitive distribution $\varrho_i(X, t)$; then we can applied an adiabatic approximation to describe the ρ evolution due to the I changes, by assuming that ρ is always in a stationary state for the cognitive dynamics. In such a case, ρ can be considered a function of the total cognitive states distribution

$$\varrho(X, t) = \frac{1}{N}\sum_{i=1}^{N} \varrho_i(X, t) \qquad (9)$$

where the sum run over a fixed automata population. and we have $I = I(\varrho(X, t))$ in the equation (7) and the cognitive dynamics separates from the spatial dynamics. Under the hypothesis that one can proves a strong law of large numbers[Oelschlager, 1989]

$$\lim_{N\to\infty}\left|\frac{1}{N}\sum_{i=1}^{N}\delta(X - X_i(t)) - \frac{1}{N}\sum_{i=1}^{N}\varrho_i(X)\right| \to 0 \qquad (10)$$

it is possible to justified the coupled Fokker-Planck equations for the distribution ϱ_i

$$\frac{\partial \varrho_i}{\partial t} = \frac{\partial}{\partial X}\frac{\partial V}{\partial X}(x, I(\varrho))\varrho_i + T_i \frac{\partial^2 \varrho_i}{\partial X^2} \qquad i = 1, ..., N \qquad (11)$$

If $\lim_{|x|\to\infty} V(x) \simeq |x|^\alpha$ with $\alpha > 0$, from the system (11) we get the self-consistent stationary solution

$$\varrho(x) = \frac{1}{N}\sum_{i=1}^{N} A_i \exp -\frac{V(x, I(\varrho))}{T_i} \qquad (12)$$

where A_i are normalization constants.

For seek of simplicity, we specify the model for the "atomic decision" (i.e. the choice between two possibilities): the utility landscape potential is a double well potential as sketched in fig. 6. Let n_A and n_C the fraction of automata population that chooses A or C, we specify the utility potential according to

$$V(X, I) = kX^2\left(\frac{X^2}{4} + \frac{X}{3}(X_A - X_C) - \frac{X_A X_C}{2}\right) \qquad (13)$$

where $X_{A,C} = X_0 + cn_{A,C}$. The potential (13) simulates a cooperative behavior among the automata, c being the cooperation parameter. When $X < 0$ the choice A turns out more useful than the other choice C and viceversa, as a consequence in a "rational population" we have

$$n_A(t) = \int_{-\infty}^{0} \varrho(X, t)\, dX \qquad n_C(t) = \int_{0}^{\infty} \varrho(X, t)\, dX \qquad (14)$$

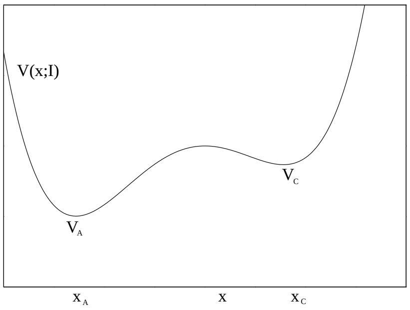

Figure 6. Utility landscape potential in the case of an atomic decision: the minima A and C gives the utility of the considered choices.

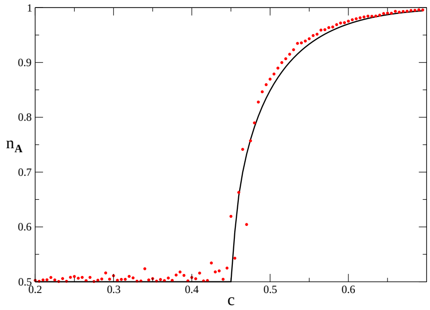

Figure 7. Cooperative solution n_A of the cognitive dynamics (7) using the utility potential (13), with the following parameters $k = 1, X_0 = 1/2$; the individual "social temperatures" are uniformly distributed in the interval $T_0 \pm \Delta T$ where $T_0 = .05$ and $\Delta T = .025$. The dots are the MonteCarlo results on a population of 10^4 automata, whereas the continuous curve refers to the direct solution of eq. (17).

i.e. the probability of choosing A (resp. C) is 1, if this choice is evaluated more useful. Using the constraints $n_A + n_C = 1$, in the stationary state we have the self-consistent equations for n_A

$$n_A = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{0} A_i \exp\left(-\frac{V(x; n_A, n_C)}{T_i}\right) dx \tag{15}$$

$n_A = n_C = 1/2$ is the trivial solution, but other solutions are possible. An approximate computation can be performed if the individual social temperatures T_i are small with respect the utility potential. Let us evaluated the individual transition probability between the choices A and C by the Kramer's theory[Hanggi 1990]

$$P^{(i)}_{AC} = \frac{\omega_A \omega_B}{2\pi} e^{V_A/T_i} \qquad P^{(i)}_{CA} = \frac{\omega_C \omega_B}{2\pi} e^{V_C/T_i} \tag{16}$$

where $V_{A,C} = V(X_{A,C}, I)$ and $\omega_{A,B,C}$ are the eigenvalue moduli of the potential critical points. Then the equilibrium solutions satisfies

$$n_A = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{1 + \frac{\omega_A}{\omega_C}\exp\left(\frac{V_A - V_C}{T_i}\right)} \tag{17}$$

where

$$\frac{\omega_A}{\omega_C} = \sqrt{\frac{X_0 + c n_A}{X_0 + c n_C}} \qquad V_{A,C} = V(-X_0 - c n_{A,C}) \tag{18}$$

The equation can be solved by using a fixed point principle and a bifurcation phenomenon is observed at $c = c_*$, when the following condition holds

$$\frac{1}{N}\sum_{i=1}^{N} \frac{\partial}{\partial n_A}\bigg|_{n_A=1/2} \frac{\omega_A}{\omega_C}\exp\left(\frac{V_A - V_C}{T_i}\right) = 4 \tag{19}$$

After some algebraic manipulations one gets an explicit equation for the critical value c_*

$$4x_0 + 3c = \frac{4}{3}\frac{kc}{\overline{T}}\left(x_0 + \frac{c}{2}\right)^4 \tag{20}$$

where $\overline{T} =< 1/T >^{-1}$ is the harmonic mean of the cognitive temperatures. When the cooperation parameter satisfies $c > c_*$, the trivial solution $n_A = 1/2$ becomes unstable and a self-organized cooperative solution appears, in which the majority of population coordinates the decisions to choose A or C. Of course there is a strict relation between the self-organization phenomenon and the "information" (14) inserted in the utility function. Indeed a key point to model a cognitive behavior remains the definition of the "interesting information" for a certain decision and her influence on an utility function.

4 Numerical Simulations and Applications

In order to check the validity of the adiabatic assumption and to study the role of the different parameters in the automata gas model, we have performed numerical simulations using models of increasing complexity. First of all we have considered the cognitive dynamics (7) in the case of a double well utility potential (13). We computed the cooperative stationary solution n_A using a MonteCarlo simulation on a population of 10^4 automata by varying the parameter c. The results are reported in fig. 7, where we compared the numerical simulation with the self-consistent solutions (17). The results confirm as the Kramer's theory is effective when the social temperatures are sufficiently smal. Moreover the the transitions probabilities (16) allow to compute an approximate dynamics for the cooperative population fraction $n_A(t)$; the results are shown in fig. 8 where the comparison with MonteCarlo simulations confirms the accuracy of Kramer's approximation. In the previous simulations all the automata have the global information (14) on the decision of other automata. This is a strong requirement for application, then we consider the case in which each automaton has information on the behavior of a limited number of other automata ("friends") randomly chosen in the population, but fixed during the simulation. This is equiv-

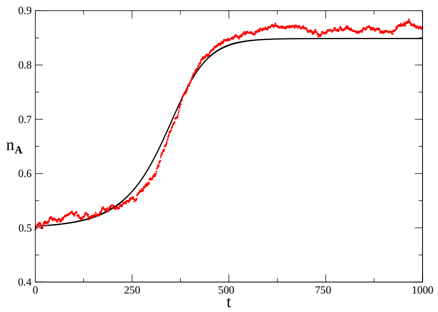

Figure 8. Relaxation process towards a cooperative solution: comparison between the MonteCarlo simulations and the balance equations associated to the transition probabilities (16). The parameters are the same as in fig. 7 with $c = .5$.

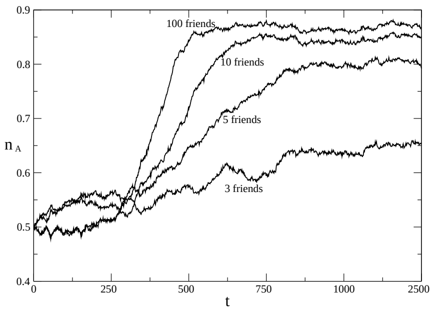

Figure 9. MonteCarlo simulations of the cooperative behavior of 10^4 automata each of one gets information from a limited number of randomly chosen friends (fixed during the simulations). The parameters are the same as in fig. 7 with $c = .5$. The curves refer to the evolution of the cooperative fraction in the case of $3, 5, 10, 100$ friends for each automaton, randomly chosen in the population.

alent to introduce a random communication network among the automata with a constant connectivity. The results are plotted in fig. 9 where we vary the connectivity (i.e. the friends number). The simulations shows that the cooperative behavior is robust with respect to the "friends" number up to a very small number $\simeq 5$. This result is consistent with the study on flocking phenomena where the coordination birds behavior could be explained by the existence of a topological interaction with a small number of neighborhoods[Ballerini et al., 2008].

To introduce the physical dynamics effects, we consider an automata gas model moving from an origin to a destination through a space containing two doors (cfr. fig. 10). The automata perform the physical dynamics described in the second section: finite volume effects and local vision mechanism. The physical dynamics is defined by the following parameters (cfr. eqs. (1,2)): $r_b = m = 1, \gamma = .35, |\vec{v}_{0i}| = 1, \phi = \pi/10$ and $\eta = .1$. The social space and the visual radius are respectively $2r_b$ and $5r_b$, and the visual mechanism uses a rotation velocity $\omega = .8\pi$ and a friction coefficient $\alpha = .5$. The automata perform the cognitive dynamics (7) based on a double well utility potential using the information on

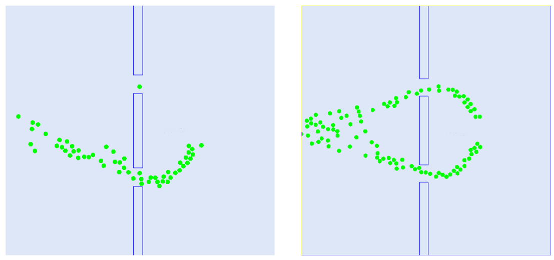

Figure 10. Automata gas model simulation: the automata have to reach the destination on the right choosing between two doors. The cognitive dynamics is based on the double well utility potential (13) ($k = 1$ and $X_0 = 1/2$), which depends on the absolute flux and the crowding nearby the doors. The parameters are $c = .75$ and $c_1 = .15$, whereas the social temperatures are distributed on the interval $T_i \in [.7 \pm .1]$. The left picture refers to a moderate incoming flux and the automata cooperate choosing the bottom door. The right picture refers to larger flux and the automata distribute equally between the two doors.

the total flux $\Phi = \sum v_i$ and the crowding O nearby the doors. The utility of the two choices is computed by using the relations

$$X_{A,C} = X_0 + \frac{c\Phi_{A,C}}{c_1 O^2_{A,C} + 1} \tag{21}$$

where the parameter c measures the cooperation level and c_1 models the automata aversion to enter in a crowded door. The parameters are fixed during the simulations and we vary the flux from the source. The evolution is periodic: i.e. the automata that enter the area have the same cognitive state than the automata that reach the destination area. The simulations show a cooperative behavior if the incoming flux overcome a well defined threshold and almost all the automata choose the same door to reach the destination (see fig. 10 (left)). However if the flux is further increased the occupation level near the door decreases the choice utility, and the automata distribute again between the two doors (see fig. 10 (right)). We remark that each automaton has information only on the chosen door so that there is no a simultaneous comparative evaluations of utilities of the two possible choices.

In the same situation we have also simulated two competing populations that have to cross the doors to reach their destination. The automata use the information of the net fluxes at the doors. By using the same parameters as in fig. 10, we get a coordinate self-organized solution that allows both the populations to reach their destinations (see fig. 11 left). This solution turns out to be robust even if we increase the incoming fluxes. However both the cooperation parameter c and the social temperature spread are critical for the appearance of the cooperative behavior. Indeed if we use a larger spread for the automata social temperatures ($T_i \in [.7 \pm .2]$), the self-organized solution becomes unstable due to collisional effects and the au-

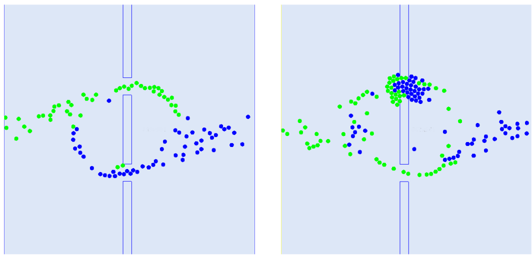

Figure 11. Automata gas model simulation: two competing population move into opposite direction to reach their destination choosing between two doors. The cognitive dynamics is based on the double well utility potential (13) which depends on the net flux at the doors. The parameters are the same as in fig. 10. The left picture show the appearance of a coordinate self-organized solution the populations choose different doors. The right picture refers to population whose social temperatures are spread in a larger interval $T_i \in [.7 \pm .2]$; so that the automata do not coordinate and tend to distribute equally between the two doors, but a critical congested state is soon formed at one door, due to physical collisions and finite volume effects.

tomata distribute between the two doors. But this is not a stationary solution at the incoming flux levels used in the simulations, and a critical congested state is soon observed at one door (see fig. 11 (right)).

5 Conclusion

The automata gas is a very powerful model to simulate complex systems in which a cognitive behavior has a relevant role in determining the global dynamical states. In this space we have discussed the main features of the model and we have considered some simple examples. The assumption of an adiabatic separation between the time-scales of the cognitive and physical dynamics, together with the possibility to prove a generalized law of large numbers for interacting particles, allow to justify a mean field approach, that points out the appearance of self-organized cognitive states. The numerical simulations confirm that our approach can be correctly applied in interesting situations. The application to pedestrian dynamics modeling in urban space, seems to be very promising even if a validation procedure through experimental observations is still a difficult problems.

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