STOCHASTIC RESONANCE IN CHUA'S CIRCUIT DRIVEN BY ALPHA-STABLE NOISE

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Abstract
In this paper, the stochastic resonance in Chua's circuit driven by alpha-stable noise has been investigated. Spectral power amplification has been analyzed by changing the characteristic exponent and scale parameters of alpha-stable noise. The results provide a visible stochastic resonance effect in Chua's circuit and the fading is observed as the noise becomes more impulsive (i.e., smaller characteristic exponent) The simulations reveal that the average time (mean residence time) spent by the trajectory in an attractor can be varied by changing the characteristic exponent, skewness and scale parameters of alpha-stable noise.

Key words
Chua's circuit, Alpha-stable (α-stable) noise , Stochastic Resonance (SR), Mean residence time, Heavy-tailed distributions.

1 Introduction

In general, noise is considered to have negative effects on the performance of a system and there have been an effort to remove or eliminate these disturbances from system with various approaches such as filtering and feedback compensation. However, throughout last three decades the studies have demonstrated the positive impacts of the noise. In nonlinear dynamical systems, the presence of an optimal amount of noise can provide improvement in the degree of coherence [Gang et al., 1993], amplification of the weak periodic signals [Jung and Hänggi, 1991], enhancement in the signal-to-noise ratio (SNR) [Benzi et al., 1981] or reducement in the probability of decision error [Kay et al., 2006; Peng and Varshney, 2015]. This phenomenon is called as stochastic resonance (SR) and originally introduced to explain the periodic climatic changes of the Earth's ice ages [Benzi et al., 1981]. Stochastic resonance has been observed in many fields of science, for example, physics [Anishchenko et al., 1999; Gammaitoni et al., 1998], electronic circuits [Anishchenko et al., 1993; Anishchenko et al., 1992], chemical reactions [Leonard and Reichl, 1994], biological systems [Moss et al., 2004]. Experiments have demonstrated that stochastic resonance occurs in sensory systems such as human tactile sensation [Collins et al., 1996], human visual perception and human vision [Lugo et al., 2008; Ranjit et al., 2015]. The existence of stochastic resonance has been investigated also at the level of vital animal behaviour such as mechanoreceptors in crayfish [Douglass et al., 1993], bacterium growth system [Chun et al., 2006] and the animal feeding behavior [Russell et al., 1999].

Anishchenko *et al.* investigated the SR phenomena in Chua's circuit driven by Gaussian noise in [Anishchenko et al., 1993; Anishchenko et al., 1994; Anishchenko et al., 1992]. SR in the presence of multiplicative noise and the effect of oscillations in a symmetric double well potential driven by a subthreshold periodic forcing has been demonstrated in [Gammaitoni et al., 1998]. The selection of external periodic signal frequency larger than the characteristic frequency results in the occurrence of stochastic resonance at only external periodic signal not at its harmonics. High frequency SR in Chua's circuit and the effect of SR in a Chua's circuit perturbed by Gaussian noise has been studied experimentally in [Gomes et al., 2003] and [Korneta et al., 2006], respectively.

The most extensively studied noise is the additive zero-mean white Gaussian noise however it can be non-Gaussian noise. The investigation of the double-well potential model driven by the α-stable and Lévy type noise in [Dybiec and Gudowska-Nowak, 2006; Dybiec and Gudowska-Nowak, 2009] have shown the presence of stochastic resonance. The SR effect also occurs in the simple threshold sensor system driven by α-stable noise [Zhi-Rui et al., 2014].

Although traditional SR requires the weak and periodic signal, aperiodic and suprathreshold signals (i.e., its amplitude is above a certain threshold value) have been investigated in [Barbay et al., 2000; Collins et al., 1995] and [McDonnell et al., 2008; Stocks, 2000], respectively.

The quantitative characteristics of SR depend on the physical mechanism of the system, the kind of nonlinear system driven, the character of the input signal and the noise. The calculation or measurement of these characteristics such as the spectral power amplification (SPA) [Jung and Hänggi, 1991], the signal-to-noise ratio (SNR) [Benzi et al., 1981], the mutual information [Bulsara and Zador, 1996], the correlation coefficient [Collins et al., 1995] and the residence-time distributions [Gammaitoni et al., 1998] indicate that at an optimal amount of noise the maximum of these characteristics is achieved.

In this paper, we have investigated the stochastic resonance in Chua's circuit driven by α-stable noise based on the work in [Yılmaz, 2012].

The rest of this paper is organised as follows: In Section 2, α-stable distributions have been briefly introduced. In Section 3, stochastic resonance has been investigated for the model of Chua's circuit driven by α-stable noise and in Section 4, the effect of α-stable noise parameters on the mean residence time (MRT) has been analyzed via the numerical studies.

2 Alpha-stable distributions

Although there is no analytical expression for α-stable density functions, the characteristic function of a random variable X which has a stable distribution can be described as [Nikias and Shao, 1995; Samoradnitsky and Taqqu, 1994]

$$\varphi(w) = \begin{cases} exp\left\{-\sigma^{\alpha}|w|^{\alpha}[1 - i\beta sign(w)\tan(\frac{\pi\alpha}{2})] + i\mu w\right\} & \text{for } \alpha \neq 1 \\ exp\left\{-\sigma^{\alpha}|w|[1 + i\beta sign(w)\frac{2}{\pi}log(|w|)] + i\mu w\right\} & \text{for } \alpha = 1 \end{cases} \tag{1}$$

where *sign(w)* is signum function and $0<\alpha\leq 2$, $\beta \in [-1, 1]$, $\sigma \in \mathbb{R}_{+}$, and $\mu \in \mathbb{R}$

A stable distribution is characterized by four parameters: α, β, μ, σ and denoted by S_{α} (σ,β,μ). The characteristic exponent denoted by α measures the tail thickness of the distribution (smaller α implies heavier tails i.e., more impulsive behavior). The skewness parameter β measures the symmetry of the distribution where $\beta = 0$ refers to symmetric distribution, $\beta > 0$ right-skewed distribution and $\beta < 0$ left-skewed distribution. μ is a location parameter and the scaling parameter σ determines the spread of distribution around its location parameter μ. In fact the Gaussian, Cauchy, and Levy distributions are special cases of the alpha-stable distributions with (α=2), ($\alpha = 1, \beta = 0$), and ($\alpha = 1/2, \beta = 1$), respectively.

The numerical approximation of α-stable density functions have been evaluated [Nolan, 1997] by the Fourier transform of the characteristic function of α-stable distributions given in Eq. (1) as

$$f\left(y; \alpha, \beta, \gamma, \mu\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-jwy}\varphi(w)dw. \tag{2}$$

It is shown as in Fig. (1) that the distribution is highly impulsive with small characteristic exponent α. Fig. (2) shows the dependence of density on skewness parameter β. When the skewness parameter β is positive the distribution is skewed to the right, when β is negative it is skewed to the left shown as in Fig. (2).

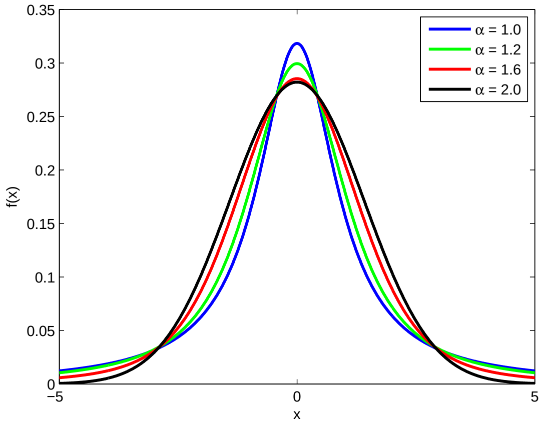

Figure 1. Symmetric α-stable densities for various α, $\sigma = 1.0$, $\mu = 0$.

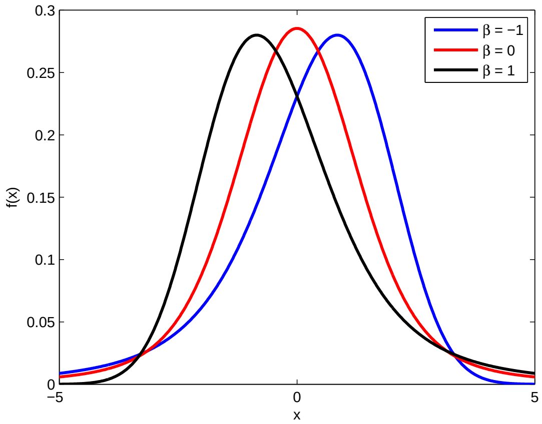

Figure 2. α-stable densities for various β, $\alpha = 1.6$, $\sigma = 1.0$, $\mu = 0$.

3 The Observation of Stochastic Resonance in Chua's Circuit driven by α-stable noise

When the GCC [Suykens et al., 1997] is driven by the stochastic input then the governing stochastic differential equation can be written as

$$\begin{aligned} \dot{x} &= a\left[y - h(x)\right] \\ \dot{y} &= x - y + z \\ \dot{z} &= -by + E(t) + \xi(t) \end{aligned} \tag{3}$$

where a and b denote bifurcation parameters, the external forcing signal $E(t) = Asinw_o t$ and $\xi(t)$ indicates α-stable noise and the piecewise-linear characteristics $h(x)$ is given as

$$h(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x+1| - |x-1|) \quad (4)$$

The parameters of Chua's circuit have been chosen such that two single-scroll symmetrical attractors are present in the absence of external forcing and the noise.

The characteristic frequency of Chua's circuit, f_{ch}, is located at 0.225 Hz which corresponds to the maximum peak in the power spectrum of autonomous Chua's circuit. Without noise and external forcing there is no possibility to jump from one attractor to the other. However the jumps between the attractors occur if the external periodic forcing signal is suprathreshold. A minimum threshold amplitude value of periodic signal A_{th} is determined as a function its frequency. These threshold values required to induce jumps are plotted in Fig. (3) as a function of the ratio of input frequency to the characteristic frequency.

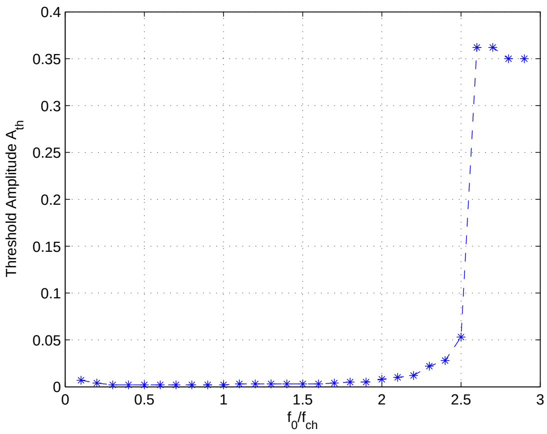

Figure 3. Threshold amplitude value of the external periodic signal as a function of frequency ratio f_o/f_{ch}.

Fig. (3) shows the threshold value of the amplitude of external periodic forcing is low when it is close to the characteristic frequency and it increases for larger frequencies than the characteristic frequency.

In this study, the frequency of the external periodic signal is set at $f_0 = 0.6075$ Hz where $f_0/f_{ch} = 2.7$ where the corresponding threshold amplitude is $A_{th} = 0.362$. Since we want to induce jumps between the attractors by the addition of noise, the amplitude of the external forcing is set as $A = 0.35$ i.e., this value is close to but below the minimum threshold value to ensure that the input signal is sub-threshold. Under these conditions, the addition of small amount of noise enables to induce jumps between the attractors. To study the amplification of the input signal as a function of the noise intensity for observing the effect of SR and analyze the response of model by varying alpha-stable parameters we define a spectral amplification parameter η

$$\eta = \frac{S_{out}(w_i)}{S_{in}(w_i)} \quad (5)$$

where $w_i = 2\pi f_i$ is the input signal frequency (rad/s), $S_{in}(w_i)$ is the average power of input signal and the power spectral density of output state x at the input frequency is defined as $S_{out}(w_i) = \int_{w_i-\triangle w}^{w_i+\triangle w} S(w)dw$.

The amplification of the sub-threshold input periodic signal as a function of the noise intensity for various α parameters with $\beta = 0$ is shown as in Fig. (4). The simulation results show a visible stochastic resonance (SR) effect in Chua's circuit driven by α-stable noise.

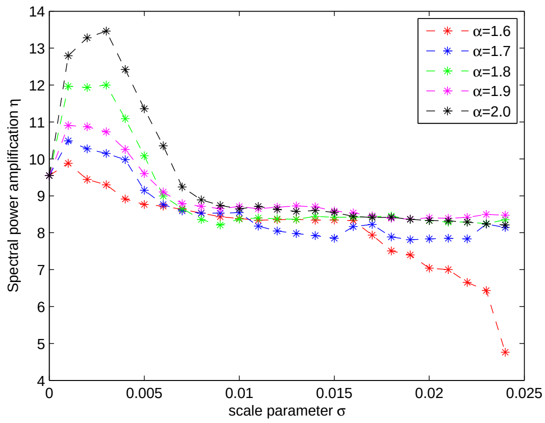

Figure 4. Spectral power amplification for various α parameters.

Fig.(4) results that the Gaussian case where $\alpha = 2.0$ provides the best amplification. When the distribution of the driving noise is more heavy-tailed (i.e., α gets smaller) the amplification η decreases. SR occurrence is also observed at smaller scale parameter σ values as the noise becomes more impulsive as shown in Fig. (4). Similar fading effect for the threshold systems have been observed in [Kosko and Mitaim, 2001; Kosko and Mitaim, 2003].

4 The Control of Residence Times in Chua's Circuit by Alpha-Stable Noise

The residence time is defined as the time duration that the trajectory spends in one state (i.e., a single scroll attractor in our case) before jumping to another attractor and the determination of the residence time has the quantitive characteristics of the stochastic resonance observed in chaotic dynamical systems [Anishchenko et al., 1993; Anishchenko et al., 1992]. We have determined the mean residence time in an attractor under the various noise parameters to investigate the effect of noise parameters on the average lifetime of a trajectory in an attractor. The mean residence time for $\alpha = 1.8$ has been analyzed for various skewness parameter β and scale parameter σ of noise. When the skewness parameter is chosen as $\beta = 1$ (i.e., the distribution is right-skewed) the trajectory spends more time in the corresponding right-scroll attractor. Also, the MRT decreases exponentially with the increase of scale parameter σ. The mean residence times for various characteristic exponent parameters α in symmetric case is shown in Fig. (6). When the characteristic exponent α increases the mean residence time in the corresponding attractor also increases.

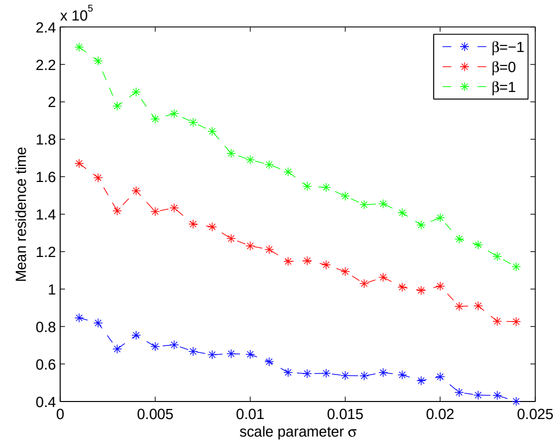

Figure 5. Mean residence times vs. scale parameter σ for $\alpha = 1.8$.

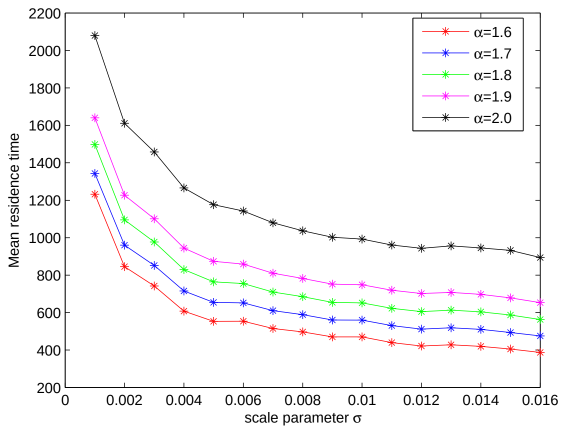

Figure 6. Mean residence times vs. scale parameter σ for various α parameters.

5 Conclusion

In this paper the stochastic resonance in Chua's circuit driven by α-stable noise has been investigated. We first determine a threshold value for the amplitude of input periodic signal as a function of its frequency to ensure that it cannot induce jumps between the attractors in the absence of noise and then we have used spectral power amplification as a measure of SR. The simulation results show a visible stochastic resonance (SR) effect in Chua's circuit driven by α-stable noise. The best amplification has been observed for the Gaussian case. The optimum noise scale parameter σ decreases as the distribution becomes more impulsive. Furthermore the effect of noise parameters on the mean residence times have been analyzed. When scale parameter σ increases then mean residence time decreases exponentially. On the other hand, the decrease in characteristic exponent α (hence the increase of impulsiveness) cause the decrease in the mean residence time in a single scroll attractor. Also the mean residence time can be increased by changing the skewness parameter. The mean residence times in a single scroll attractor have been controlled by varying the parameters of alpha-stable noise.

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