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ITERATIVE LEARNING CONTROL DESIGN FOR A DISCRETE-TIME SYSTEM UNDER DELAY ALONG THE SAMPLE TRAJECTORY AND INPUT BACKLASH

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Abstract

Actuator components of gantry robots, such as reduction gears or clutches, typically have nonlinear characteristics such as dead zone, hysteresis, or backlash. Iterative learning control (ILC) is widely used to achieve the high accuracy of repetitive operations performed by such robots. These nonlinearities can severely limit the achievable accuracy. However, their impact on ILC is not well understood. This paper considers a discrete-time system under a control delay along the sample trajectory and with input backlash. The method of vector Lyapunov functions for repetitive processes is applied to design an ILC law that ensures the convergence of the learning error. An example is given to demonstrate the effectiveness of the proposed ILC algorithm.

Key words

Cyber-physical systems, machine learning, iterative learning control, input backlash, control delay, repetitive processes, error convergence, vector Lyapunov function, linear matrix inequalities.

1 Introduction

The idea of machine learning appeared long ago. For more detail, the reader is recommended an interesting historical survey [Fradkov and Shepeljavyi, 2022] with special emphasis on the contribution of V.A. Yakubovich and his scientific school to machine learning, pattern recognition, adaptive systems, and robotics.

Back in the 1960s, Ya.Z. Tsypkin [Tsypkin, 1971] defined the concept of learning in control systems as follows:

"By learning we mean the process of developing in a certain system one or another response to external signals through multiple impacts on the system and external adjustments."

The principle of iterative learning control (ILC) arose from attempts to improve the accuracy of repetitive tasks performed by robots and fully matches this concept. The key feature of ILC is that the dynamics repeat over a finite duration, resetting to the starting location once each repetition is complete. In the literature, each repetition is termed a trial (pass or iteration), and the finite duration is known as the trial length. Moreover, in a substantial body of the current literature, the trial length does not change between trials.

Once a trial is complete, all information generated during its execution is available to design the control input for the next trial. The design problem is to use this previous trial information in the best way. The most common approach is to compute the next trial input as that used on the previous trial plus a correction term based on the previous trial information.

Early results on ILC research following the pioneering publications can be found, e.g., in the survey papers [Bristow et al., 2006; Ahn et al., 2007]. Moreover, ILC remains an established area of research in new theoretical results, design methods, experimental validation, and implementation. Recent developments include applications to additive manufacturing [Lim et al., 2017], high-precision multilayer laser deposition systems [Sammons et al., 2019], center-articulated industrial vehicles [Dekker et al., 2019], and marine systems [Sornmo et al., 2016]. Also, there has been very productive use in health care applications, e.g., robotic-assisted stroke rehabilitation [Sakariya et al., 2020; Seel et al., 2016; Freeman et al., 2012; Meadmore et al., 2014] with supporting clinical trials, and heart ventricular support devices [Ketelhut et al., 2019].

The fundamental idea of the research approach in this paper is based on the physical analogy with vector fields. Models of iterative learning control processes belong to the class of repetitive processes, a special case of the so-called 2D systems. These systems have an essential peculiarity as follows. The classical method of Lyapunov functions cannot be applied to analyze their stability (hence, to determine convergence conditions for ILC algorithms) since unlike ordinary differential or difference equations, it is impossible to find the gradient or first difference of the Lyapunov function from the original equations of the system without finding their solutions.

The divergence operator is widely used to study vector fields. This operator maps a vector field onto a scalar one and determines, for each point, how much the incoming and outgoing fields diverge from a small neighborhood of a given point. In other words, it shows how much the incoming and outgoing flows differ from one another. In 2D models, it is easy to find the generalized energy along the repetition trajectory and the generalized energy between repetitions. Treating these quantities as components of a 2D vector field, based on the known properties of divergence, it can be argued that, first, if such a field has a negative divergence at some point, this point will be an energy sink; if the divergence at some point is positive, this point will be an energy source; finally, when the divergence is zero, the field either contains no sources and sinks or they balance each other. According to the aforesaid, it is natural to assume that if the divergence of such a field takes negative values everywhere, this field will correspond to a stable system

Rigorous mathematical derivations confirm this assumption and provide a constructive method for analyzing the stability of repetitive processes [Pakshin et al., 2016]. This method is used below to obtain convergence conditions for ILC algorithms.

As has been mentioned, the principle of iterative learning control historically emerged from the problems of increasing the accuracy of repetitive operations performed by robots and is currently quite widespread in industrial gantry robots. Such robots are part of modern smart manufacturing (SM) systems, which are complex cyber-physical systems (CPS) that differ significantly from a simple combination of autonomous robots performing individual operations. CPS are defined by the integration of cybernetic and physical components such as communication and control networks, sensors, and actuators in a multilayer architecture [Saez et al., 2020; Uzhva and Granichin, 2021]. Flexibility and reconfigurability are key elements of SM that ensure on-the-fly changes to the production process. These concepts give the advantage to improve efficiency and reduce production costs [Qamsane et al., 2019].

The long-term non-stop operation of SM systems raises increased demands for the robots included in them. Actuator components of gantry robots (e.g., reduction gears or clutches) typically have nonlinear characteristics such as dead zone, hysteresis, or backlash. These nonlinearities can severely limit the achievable accuracy [Pakshin et al., 2020; Pakshin and Emelyanova, 2023]. Therefore, it is crucial to analyze and design ILC algorithms considering these nonlinearities. The delay is another factor that may have an adverse effect here.

A substantial part of the ILC literature, including experimental validation, involves a linear dynamic model for design. By analogy with other approaches, such a design may encounter implementation difficulties due to nonlinear effects in actuators. This form of nonlinearity is more complicated for analysis. It is often represented as the sum of a linear term and an unknown but bounded term. A typical example is the saturation of actuators. Modifications to control laws to compensate the impact of saturation have seen progress reported, e.g., in [Sebastian et al., 2019; Pakshin et al., 2020]. Backlash is another common problem for actuators, but this case has attracted relatively less attention of researchers in the ILC setting; see[Wei et al., 2017a; He et al., 2019; Zhou et al., 2020] for the currently available results.

Previous research on ILC with backlash includes [He et al., 2019], where the Timoshenko beam system described by a second-order distributed parameter model was considered. In this model, the input backlash is divided into a linear input and an unknown bounded term, estimated by an observer. Also, in [Zhou et al., 2020], a model of a two-link rigid-flexible manipulator with input backlash was considered. The input backlash was analyzed in an identical manner and was combined with an external disturbance. Both of these designs apply only to the specific systems described. In [Wei et al., 2017b], an adaptive ILC scheme was presented for a particular class of nonlinear systems with unknown time-varying delays and control preceded by an unknown nonlinear backlash-like hysteresis.

In contrast to the publications mentioned above, this paper proposes an ILC design approach for discrete-time systems with input backlash and a delay along the sample trajectory. The approach is based on the development of the earlier results [Pakshin and Emelyanova, 2023], where systems with input saturation were studied, to the class of systems under consideration.

2 Problem Statement

Consider a discrete-time stochastic system in repetitive mode described by the state-space model

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + B\psi(u_k(p-d)), \\ y_k(p) &= Cx_k(p),\ 0 \le p \le N-1,\ k \ge 0, \end{aligned} \quad (1)$$

where $x_k(p) \in \mathbb{R}^{n_x}$ is the state vector, $u_k(p) \in \mathbb{R}$ is the scalar control variable, $y_k(p) \in \mathbb{R}$ is the controllable scalar output variable (often called the trial or pass profile), d is the number of control delay samples, and $\psi(u_k(p))$ (further written as $\psi_k(p)$ for brevity) is

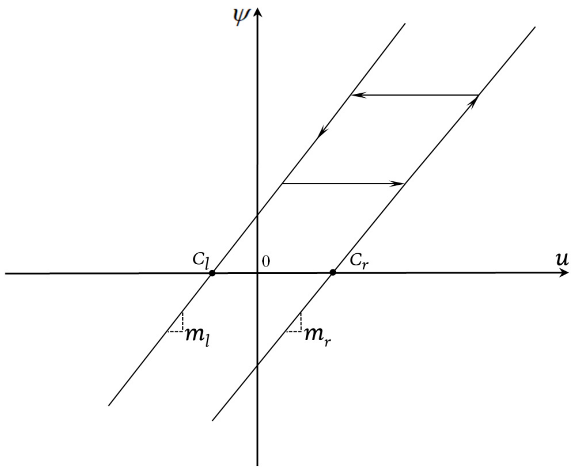

Figure 1. Backlash model.

the backlash-type nonlinear function (Fig. 1). Following [Tao and Kokotović, 1993], in the case under consideration, this nonlinear function is given by

$$\psi(u_k(p)) = \text{back}(u_k(p)) \\ = \begin{cases} m_l(u_k(p) - c_l), & \text{if } u_k(p) \le \underline{u}_k(p), \\ m_r(u_k(p) - c_r), & \text{if } u_k(p) \ge \overline{u}_k(p), \\ \psi_k(p-1), & \text{if } \underline{u}_k(p) < u_k(p) < \overline{u}_k(p), \end{cases} \quad (2)$$

where $m_l,\ m_r,\ c_r$ are positive constants, c_l is a negative constant, and

$$\underline{u}_k(p) = \frac{1}{m_l}\text{back}(u_k(p-1)) + c_l,$$
$$\overline{u}_k(p) = \frac{1}{m_r}\text{back}(u_k(p-1)) + c_r.$$

Attention is focused on the case $m_r = m_l = m$, which arises in applications. Without loss of generality, the boundary conditions have the form $x_k(0) = 0$ and $y_0(p) = f(p)$, where $f(p)$ is a known scalar function of $p,\ 0 \le p \le N-1$. Also, the pair $\{A, B\}$ is assumed to be controllable and $CB \neq 0$.

The initial condition $x_k(0)$ is the same for all $k = 0,\ 1\ \ldots,$ whereas $u_k(p)$ is known and bounded for $p \in [-d,\ 0]$.

Let $y_{\text{ref}}(p) \in \mathbb{R}$, $0 \le p \le N-1$, denote the supplied reference signal. Then

$$e_k(p) = y_{\text{ref}}(p) - y_k(p) \quad (3)$$

is the error on trial k.

The control design problem is to construct a control input sequence $\{u_k\}$, such that

$$|e_k(p)| \le \kappa\varrho^k + \mu,\ \kappa > 0,\ \mu \ge 0,\ 0 < \varrho < 1, \quad (4)$$
$$\lim_{k\to\infty} |u_k(p)| = |u_\infty(p)| < \infty,\ p \in [0, N-1], \quad (5)$$

where the bounded variable $u_\infty(p)$ is termed the learned control and $|\cdot|$ indicates the chosen norm (the absolute value for scalar functions). If there is no backlash, the design presented in this paper reduces to the linear dynamics case and $\lim_{k\to\infty} |e_k(p)| = 0$ naturally holds.

3 An Auxiliary Model in the Repetitive Process Form

Consider an auxiliary vector $\hat{x}_k$ of dimension d with the components $\check{x}_{ki}(p) = \psi_k(p-i)$, $i = 1, \ldots, d$. It is easy to see that this vector satisfies the equation

$$\check{x}_k(p+1) = A_d\check{x}_k(p) + B_d\psi_k(p), \quad (6)$$

where

$$A_d = \begin{bmatrix} 0\ 0 \ldots 0\ 0 \\ I\ 0 \ldots 0\ 0 \\ 0\ I \ldots 0\ 0 \\ \vdots\ \vdots\ \ddots\ \vdots\ \vdots \\ 0\ 0 \ldots I\ 0 \end{bmatrix},\quad B_d = [I\ 0\ 0 \ldots 0]^T.$$

Therefore, the first equation in (1) can be written as

$$x_k(p+1) = Ax_k(p) + BC_d\check{x}_k(p), \quad (7)$$

where $C_d = [\underbrace{0 \ldots 0}_{d-1}\ I]$.

With $\bar{x}_{k+1}(p) = [x^T_{k+1}(p)\ \check{x}^T_{k+1}(p)]^T$, equations (6), (7) can be reduced to the single equation

$$\bar{x}_k(p+1) = \check{A}\bar{x}_k(p) + \check{B}\psi_k(p),$$
$$y_k(p) = \check{C}\bar{x}_k(p), \quad (8)$$

where

$$\check{A} = \begin{bmatrix} A & BC_d \\ 0 & A_d \end{bmatrix},\quad \check{B} = \begin{bmatrix} 0 \\ B_d \end{bmatrix},\quad \check{C} = [C\ 0].$$

Assume that the state vector is available for control design and the matrix CB is nonsingular. In the case of no delay, the nonsingularity condition allows deriving a simple equation for the learning error as a function of the number of passes. For the extended model, $\check{C}\check{B} = 0$, and additional transformations are needed. First, it is necessary to introduce the auxiliary variable

$$\eta_{k+1}(p+1) = \bar{x}_{k+1}(p) - \bar{x}_k(p) \quad (9)$$

and obtain equations for the increments of the extended state vector. According to (8), this variable satisfies the equation

$$\eta_{k+1}(p+1) = \check{A}\eta_{k+1}(p) + \check{B}\Delta\psi_{k+1}(p-1), \quad (10)$$

where $\Delta\psi_{k+1}(p-1) = \psi_{k+1}(p-1) - \psi_k(p-1)$. Due to the structure of the matrices $\check{A}$ and $\check{B}$,

$$\check{C}\check{A}^d\check{B} = CB. \quad (11)$$

Consider the biased learning error $\bar{e}_k(p) = e_k(p+d)$. In view of (3) and (8)–(10), it obeys the equation

$$\bar{e}_{k+1}(p) = -\check{C}\check{A}^{d+1}\eta_{k+1}(p) + \bar{e}_k(p) - CB\Delta\psi_{k+1}(p-1). \quad (12)$$

In a commonly used ILC law, the input for the next trial as the sum of the previous trial input plus a correction term that uses previous trial data. This approach is considered below: the control law has the structure

$$\psi_{k+1}(p) = \mathrm{back}(\psi_k(p) + \delta u_{k+1}(p)), \quad (13)$$

where $\delta u_{k+1}(p)$ is the control update given by

$$\delta u_{k+1}(p) = K_1 H\eta_{k+1}(p+1) + K_2\bar{e}_k(p+1), \quad (14)$$

where K_1 and K_2 are matrices of compatible dimensions to be designed and $H = [\underbrace{I}_{n_x}\ 0]$. With $\varphi_k(p) = \Delta\psi_{k+1}(p-1) - \delta u_{k+1}(p-1)$, the system model in increments can be written as

$$\begin{aligned}\eta_{k+1}(p+1) &= (\check{A} + \check{B}K_1H)\eta_{k+1}(p) \\ &\quad + \check{B}K_2\bar{e}_k(p) + \check{B}\varphi_k(p), \\ \bar{e}_{k+1}(p) &= -\check{C}\check{A}^d(\check{A} + \check{B}K_1H)\eta_{k+1}(p) \\ &\quad + (I - CBK_2)\bar{e}_k(p) - CB\varphi_k(p).\end{aligned} \quad (15)$$

It follows from (2) and Fig. 1 that $\Delta\psi_{k+1}(p-1) = \mathrm{back}(u_{k+1}(p-1)) - \mathrm{back}(u_k(p-1))$ satisfies the constraints

$$m\delta u_{k+1}(p-1) - m\Delta c \le \Delta\psi_{k+1}(p-1) \le m\delta u_{k+1}(p-1) + m\Delta c,$$

where $\Delta c = c_r - c_l$. Also, $\varphi_k(p)$ satisfies the constraints

$$m_1\delta u_{k+1}(p-1) - m\Delta c \le \varphi_k(p) \le m_1\delta u_{k+1}(p-1) + m\Delta c,$$

or

$$m^2(\Delta c)^2 - [\varphi_k(p) - m_1\delta u_{k+1}(p-1)]^2 \ge 0, \quad (16)$$

where $m_1 = m-1$. If $m_r = m_l = m$, then $\mathrm{back}(mu) = m\mathrm{back}(u)$; without loss of generality, the case $m = 1$ will be considered below. In this case, the quadratic constraint (16) takes the form

$$\Delta c^2 - \varphi_k(p)^2 \ge 0. \quad (17)$$

The model (15) represents a discrete repetitive process, a particular class of 2D systems [Rogers et al., 2007]. In the presence of backlash, the ILC dynamics are nonlinear, and a stability theory has been recently developed for nonlinear repetitive processes. One approach is based on vector Lyapunov functions [Pakshin et al., 2016]. This theory is used for ILC design below, starting from the convergence conditions presented in the next section.

4 The Convergence Theorem

Consider the vector Lyapunov function

$$V(\eta_{k+1}(p), \bar{e}_k(p)) = \begin{bmatrix} V_1(\eta_{k+1}(p)) \\ V_2(\bar{e}_k(p)) \end{bmatrix} \quad (18)$$

on the trajectories of system (15), where $V_1(\eta_{k+1}(p)) > 0,\ \eta \ne 0,\ V_2(\bar{e}_k(p)) > 0,\ \bar{e}_k(p) \ne 0$, $V_1(0) = 0$, and $V_2(0) = 0$. The discrete counterpart of the divergence operator along the trajectories of this system is defined as

$$\mathcal{D}V(\eta_{k+1}(p), \bar{e}_k(p)) = V_1(\eta_{k+1}(p+1)) - V_1(\eta_{k+1}(p)) + V_2(\bar{e}_{k+1}(p)) - V_2(\bar{e}_k(p)). \quad (19)$$

Theorem 1. *If there exist a vector Lyapunov function* (18)*, positive numbers* $c_1, c_2,$ *and* c_3*, and a nonnegative number* γ *such that*

$$c_1|\eta_{k+1}(p)|^2 \leqslant V_1(\eta_{k+1}(p)) \leqslant c_2|\eta_{k+1}(p)|^2, \quad (20)$$

$$c_1|\bar{e}_k(p)|^2 \leqslant V_2(\bar{e}_k(p)) \leqslant c_2|\bar{e}_k(p)|^2, \quad (21)$$

$$\mathcal{D}_dV(\eta_{k+1}(p), \bar{e}_k(p)) \leqslant \gamma - c_3(|\eta_{k+1}(p)|^2 + |\bar{e}_k(p)|^2), \quad (22)$$

then the convergence conditions (4)*,* (5) *hold.*

Proof. In the case $\gamma = 0$, the proof coincides with that of Theorem 1 in [Pakshin et al., 2016]. Let $\gamma \ne 0$; calculating the divergence along the trajectories of (15) and following to the outline of the proof from [Pakshin et al., 2016] give

$$|\bar{e}_k(p)|^2 \leqslant \frac{1}{c_1}\left[\lambda^k \sum_{q=0}^{p} \lambda^{p-q} V_2(\bar{e}_0(q)) + \gamma \sum_{n=0}^{k-1}\left(\sum_{q=0}^{p}\lambda^{p-q}\right)\lambda^{k-1-n}\right], \quad (23)$$

where $0 < \lambda < 1$. Since the value $||\bar{e}_0(q)||^2$ is bounded for all $0 \leqslant q \leqslant N-1$, there exists $\bar{\mu} > 0$ such that $||\bar{e}_0(q)||^2 \leqslant \bar{\mu}$. In view of (21), it follows that

$$\sum_{q=0}^{p} \lambda^{p-q} V_2(\bar{e}_0(q)) \le c_2\bar{\mu}\sum_{q=0}^{\infty}\lambda^{p-q} = \frac{c_2\bar{\mu}}{1-\lambda}. \quad (24)$$

Considering (24), inequality (23) implies

$$|\bar{e}_k(p)|^2 \leqslant \alpha\lambda^k + \beta, \quad (25)$$

where

$$\alpha = \frac{c_2\bar{\mu}}{c_1(1-\lambda)},\ \beta = \frac{\gamma}{c_1(1-\lambda)^2},\ 1 \leqslant p \leqslant N.$$

By definition, $\bar{e}_k(p)$ is the biased learning error. Hence, condition (25) leads to (4) with the parameters $\kappa = \sqrt{\alpha}$, $\varrho = \sqrt{\lambda}$, and $\mu = \sqrt{\beta}$.

It suffices to show the boundedness conditions (5) for this ILC law. First, note that by the convergence condition (4) and (3), $|Cx_\infty(p)| = \lim_{k\to\infty} |Cx_k(p)|$ is bounded for all p. It follows from (1) that

$$Cx_k(p+d+1) = CAx_k(p+d) + CB\psi_k(p)$$

and

$$\psi_k(p) = (CB)^{-1}(Cx_k(p+d+1) - CA^p x_k(p+d)). \quad (26)$$

Hence,

$$|\psi_\infty(0)| \le |(CB)^{-1}|(|Cx_\infty(d+1)| + |CAx_\infty(d)|) < \infty$$

because $|Cx_\infty(p)| < \infty$ for all p, and

$$|\psi_\infty(1)| \le |(CB)^{-1}|(|Cx_\infty(d+2)| + |CA^2 x_\infty(d)| + |CAB||\psi_\infty(0)|) < \infty$$

because $|Cx_\infty(p)| < \infty$ for all p, and according to the previous inequality, $|\Psi_\infty(0)| < \infty$.

Continuing this procedure gives

$$|\Psi_\infty(p-1)| \le |(CB)^{-1}|(|Cx_\infty(p+d)| + |\sum_{q=0}^{p-2} |CA^{p-1-q}B||\Psi_\infty(q)| + |CA^pB||x_\infty(d)|) < \infty,\ p \in [1, N],$$

because $|Cx_\infty(p)| < \infty$ for all p, and according to the previous steps, $|\Psi_\infty(q)| < \infty,\ q = 0, 1, \ldots, p-2$.

Finally, by the definition of the inverse backlash function [Tao and Kokotović, 1993], the boundedness condition (5) is valid.

5 Iterative Learning Control Design

Theorem 1 yields various sufficient convergence conditions depending on the particular choice of the entries in the vector Lyapunov function (18). Choosing these entries as quadratic forms allows reducing the original design problem to linear matrix inequalities (LMIs). If these LMIs are solvable, then the update law is easily obtained from them. However, it is a non-trivial task to determine the level of conservativeness of this solution in advance, and the solvability domain may be overly bounded. This difficulty can be overcome, e.g., as follows. Suppose that an update law obtained by using approximation the original nonlinear model with a linear model ensures convergence. Then if the original nonlinear system with this update law satisfies the conditions of Theorem 1, then this law will ensure the convergence of the learning error in the original system. The approaches based on these ideas are described in detail below.

5.1 The direct method

Consider a vector Lyapunov function (18) along the trajectories of (15) with entries as the quadratic forms

$$V_1(\eta_{k+1}(p)) = \eta_{k+1}^\top(p) P_1 \eta_{k+1}(p),$$
$$V_2(\bar{e}_k(p)) = \bar{e}_k^\top(p) P_2 \bar{e}_k(p),$$

where $P_1 \succ 0$ and $P_2 \succ 0$, $P = \mathrm{diag}[P_1\ P_2]$. Let $\xi_k(p) = [\eta_{k+1}(p)^\top\ \bar{e}_k(p)]^\top$, simply written below as $\xi,\ \eta,\ \bar{e}$ for brevity. Computing (19) along the trajectories of (15) gives

$$\mathcal{D}V(\eta,\bar{e}) = [(\bar{A}+\bar{B}K\bar{H})\xi + \bar{B}\varphi]^\top P[(\bar{A} + \bar{B}_iK\bar{H})\xi + \bar{B}\varphi] - \xi^\top P\xi, \quad (27)$$

where

$$K = [K_1\ K_2],\ \bar{H} = \begin{bmatrix} H & 0 \\ 0 & 1 \end{bmatrix},$$
$$\bar{A} = \begin{bmatrix} \check{A} & 0 \\ -\check{C}\check{A}^{d+1} & I \end{bmatrix}, \bar{B} = \begin{bmatrix} \check{B} \\ -CB \end{bmatrix}.$$

Since $V_1(\eta) \succ 0$ and $V_2(\bar{e}) \succ 0$, conditions (20) and (21) of Theorem 1 are valid.

Under the constraints (17), condition (22) will be true if

$$\mathcal{D}V(\eta,\bar{e}) + \tau((\Delta c)^2 - \varphi^2) \le \gamma - \xi^\top[Q + (K\bar{H})^\top RK\bar{H}]\xi \quad (28)$$

holds for all φ and ξ, where $Q \succ 0$ and $R \succ 0$ are matrices of compatible dimensions and $\tau > 0$. (For details, see [Tarbouriech et al., 2011; Yakubovich et al., 2004].) Inequality (28) holds if $\gamma = \tau(\Delta c)^2$ and

$$\begin{bmatrix} (\bar{A}+\bar{B}K\bar{H})^\top P(\bar{A}+\bar{B}K\bar{H}) - P + M & (\bar{A}+\bar{B}K\bar{H})^\top P\bar{B} \\ \bar{B}^T P(\bar{A}+\bar{B}K\bar{H}) & \bar{B}^\top P\bar{B} - \tau \end{bmatrix} \preceq 0,$$

where $M = Q + (K\bar{H})^\top RK\bar{H}$. Rewriting this last inequality as

$$\begin{bmatrix} -P & 0 \\ 0 & -\tau \end{bmatrix} + \begin{bmatrix} (\bar{A}+\bar{B}_iK\bar{H})^\top & I & (K\bar{H})^\top \\ \bar{B}^\top & 0 & 0 \end{bmatrix} \times \begin{bmatrix} P & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} (\bar{A}+\bar{B}K\bar{H}) & \bar{B} \\ I & 0 \\ K\bar{H} & 0 \end{bmatrix} \preceq 0$$

and applying Schur's complement lemma gives

$$\begin{bmatrix} -X & 0 & (\bar{A}+\bar{B}Y\bar{H})^\top & X & (YH)^\top \\ 0 & -\tau & \bar{B}^\top & 0 & 0 \\ (\bar{A}+\bar{B}Y\bar{H}) & \bar{B} & -X & 0 & 0 \\ X & 0 & 0 & -Q^{-1} & 0 \\ YH & 0 & 0 & 0 & -R^{-1} \end{bmatrix} \preceq 0, \quad (29)$$

where $X = P^{-1}$, $Y = KW$ and W is the solution of

$$HX = WH. \quad (30)$$

If the system of linear matrix inequalities (LMIs) and linear matrix equation (LME) (5.1), (30) is solvable with respect to the variables X, Y, and W, then the ILC law (13) with $\delta u_{k+1}(p)$ given by (14) and

$$K = [K_1 \; K_2] = YW^{-1}$$

will ensure the convergence condition (4). The matrices Q and R play here the same role as weight matrices in the LQR theory.

5.2 The method based on an auxiliary Riccati inequality

In this method, first, the ILC law will be obtained for the linear incremental model, and then for this law the convergence conditions based on the extended model, which include backlash will be checked.

This approach has the following motivation: as a rule, it is necessary to correct the weight matrices repeatedly during control design in order to obtain a satisfactory rate of convergence of the learning error. For such a correction in the linear model, a wider freedom of choice is provided, due to which a decrease in conservatism can be expected. Consider the discrete Riccati inequality

$$\bar{A}^\top \bar{P}\bar{A} - (1-\sigma)\bar{P} - \bar{A}^\top \bar{P}\bar{B}[\bar{B}^\top \bar{P}\bar{B} + R]^{-1}\bar{B}^\top \bar{P}\bar{A} + Q \preceq 0 \quad (31)$$

with respect to a positive definite matrix $\bar{P} = \mathrm{diag}[P_1 \; P_2]$, where P_1 and P_2 have the same dimensions as in previous subsection and $0 < \sigma < 1$. According to Schur's complement lemma, if the LMI

$$\begin{bmatrix} (1-\sigma)\bar{X} & X\bar{A}^\top & \bar{X} \\ \bar{A}\bar{X} & \bar{X}+\bar{B}R^{-1}\bar{B}^\top & 0 \\ \bar{X} & 0 & Q^{-1} \end{bmatrix} \succeq 0, \; X \succ 0, \quad (32)$$

are solvable with respect to $X = \mathrm{diag}[X_1 \; X_2] \succ 0$, where X_1 and X_2 have the same dimensions as P_1 and P_2, then $P = X^{-1}$. Let

$$L = [\underbrace{L_1}_{n_x} \; \underbrace{L_2}_{d} \; \underbrace{L_3}_{1}] = -[\bar{B}^\top \bar{P}\bar{B}_i + R]^{-1}\bar{B}^\top \bar{P}\bar{A} \quad (33)$$

(34)

$$F = [\underbrace{F_1}_{n_x} \; \underbrace{0}_{d} \; \underbrace{F_3}_{1}] = L\Theta, \quad (35)$$

where

$$\Theta = \begin{bmatrix} \Theta_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Theta_3 \end{bmatrix}$$

is a matrix with blocks of compatible dimensions that satisfies the LMI

$$\begin{bmatrix} M - M\Theta - \Theta M - Q & \Theta\sqrt{M} \\ \sqrt{M}\Theta & -I \end{bmatrix} \preceq 0 \quad (36)$$

and $M = \bar{A}^\top \bar{P}\bar{B}[\bar{B}^\top \bar{P}\bar{B} + R]^{-1}\bar{B}^\top \bar{P}\bar{A}$.

Theorem 2. *Assume that for some weight matrices $Q \succ 0$ and $R \succ 0$ and scalar $0 < \sigma < 1$, the linear matrix inequalities* (32), (36) *and*

$$\begin{bmatrix} (\bar{A}+\bar{B}K\bar{H})^\top S(\bar{A}+\bar{B}_iK\bar{H}) - S & (\bar{A}+\bar{B}K\bar{H})^\top S\bar{B} \\ \bar{B}^\top S(\bar{A}+\bar{B}K\bar{H}) & \bar{B}^\top S\bar{B} - \tau \end{bmatrix} \prec 0, \quad (37)$$

where

$$K = [F_1\Theta_1 \; F_3\Theta_3], \quad (38)$$

are solvable with respect to X, Θ, $\tau > 0$, and $S = \mathrm{diag}[S_1 \; S_2] \succ 0$ with blocks of the same dimensions as P_1 and P_2. Then the ILC law (13) with $\delta u_{k+1}(p)$ given by (14) and $K = [K_1 \; K_2]$ given by (38) *ensures the convergence condition* (4) *and the boundedness condition* (5).

Now the idea outlined at the beginning of this section can be refined as follows. Consider system (1) without backlash. In this case, the ILC law has form

$$u_{k+1}(p) = u_k(p) + \delta u_{k+1}(p).$$

Let $\delta u_{k+1}(p)$ be obtained as described in Theorem 2; by corollary of Theorem 2 from [Pakshin and Emelianova, 2020], for this linear case, conditions (4), (5) hold with $\gamma = 0$ and, hence, the condition of this Theorem seems less conservative compared to the conditions of the previous section.

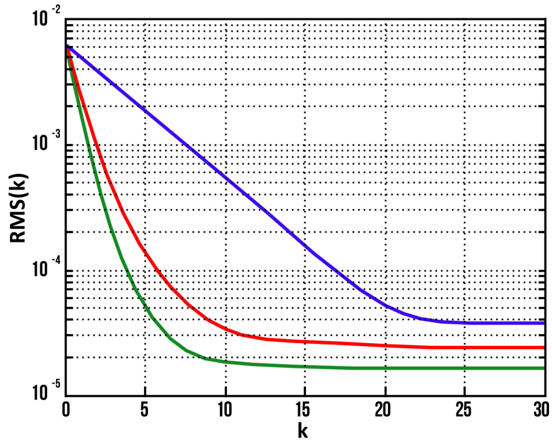

Figure 2. The RMS progression for $c = 0.003$: $\sigma = 0.85$ (green line), $\sigma = 0.8$ (red line), and $\sigma = 0.4$ (blue line).

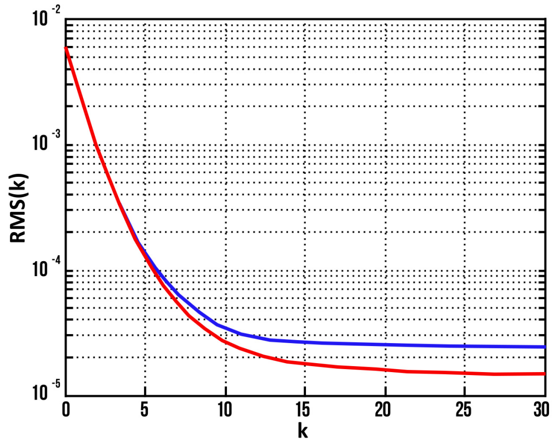

Figure 3. The RMS progression for $c = 0.005$ (blue line) and $c = 0.003$ (red line).

Proof. Inequality (37) implies that, for all ξ and φ including those satisfying (17),

$$[(\bar{A} + \bar{B}K\bar{H})\xi + \bar{B}\varphi]^\top S[(\bar{A} + \bar{B}K\bar{H})\xi + \bar{B}\varphi] - \xi^\top S\xi - \tau\varphi^2 < 0. \quad (39)$$

Since the left-hand side is a quadratic form in ξ and φ and $S > 0$, all the conditions of Theorem 1 are satisfied and the convergence condition (4) holds for $\gamma = \tau(\Delta c)^2$. The boundedness condition (5) is proved similarly to the previous section.

6 Example

As an example, consider the one-axis model of the multi-axis gantry robot described in [Hladowski et al., 2010]. According to the frequency response tests in [Hladowski et al., 2010], an adequate dynamic model for control law design has the 3rd order continuous-time transfer function

$$G(s) = \frac{15.8869(s + 850.3)}{s(s^2 + 707.6s + 3.377 \cdot 10^5)}. \quad (40)$$

The reference trajectory is the same as in [Hladowski et al., 2010] with a trial length of 2 s. For discrete-time design, a sampling period of 0.01 s was used in combination with $m = 1$ and $c_r = -c_l = c$ for the backlash nonlinearity (Fig. 1). The discrete-time control signal was computed with that sampling period and a delay of 1 step. The signal was converted into a control signal through zero-order extrapolation. The resulting discrete-time state-space model has the form (1), where the parameters were obtained from (40) using standard MATLAB functions. For $Q = \mathrm{diag}[1\ 1\ 5 \cdot 10^{-4}\ 10^3\ 1]$, $R = 4.5 \cdot 10^5$, and $\sigma = 0.8$, Theorem 2 gives

$$K_1 = [-2.8\ \ -3.9\ \ -1857.6],\ K_2 = 202.4.$$

The performance of this ILC law was measured by the root-mean-square error for each trial:

$$\mathrm{RMS}(k) = \sqrt{\frac{1}{N}\sum_{p=0}^{N} ||e_k(p)||^2}. \quad (41)$$

According to the figure, the steady state of the RMS value is not equal to zero and it depends on the gains in the update law. For $\sigma = 0.85$,

$$K_1 = [-3.7\ \ -5.2\ \ -2471.5],\ K_2 = 296.6;$$

for $\sigma = 0.4$,

$$K_1 = [-6.0\ \ -8.3\ \ -3971.0],\ K_2 = 130.2.$$

The rates of RMS convergence and the steady-state values are different for the three cases above (Fig.2).

With an increase in the dead zone, the steady-state value of the RMS increases as well (Fig. 3). Figure 4 shows the RMS progression for different delays. The decrease in the steady-state value of RMS with an increase in the delay is explained by higher gains in the update law. In the case of the two-step delay,

$$K_1 = [-6.6\ \ -9.1\ \ -4362.7],\ K_2 = 296.7.$$

In the absence of backlash, as the number of trials increases, the RMS is reduced to the numerical error level (Fig. 5).

7 Conclusion

A new iterative learning control design method has been proposed. Unlike the known ones, it takes into account the influence of input backlash and delay along the sample trajectory. An illustrative example has confirmed the theoretical conclusion that backlash limits the achievable tracking accuracy of the reference learning

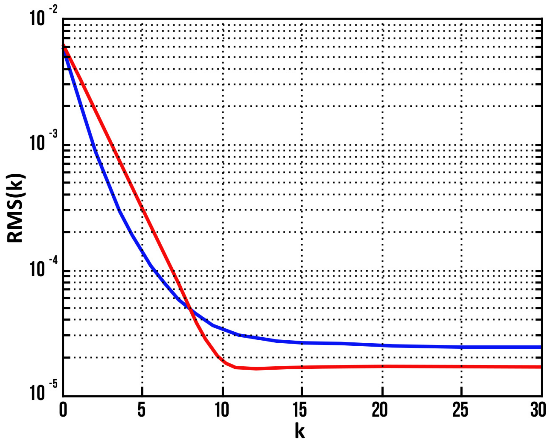

Figure 4. The RMS progression for $c = 0.003$: one-step delay (blue line) and two-step delay (red line).

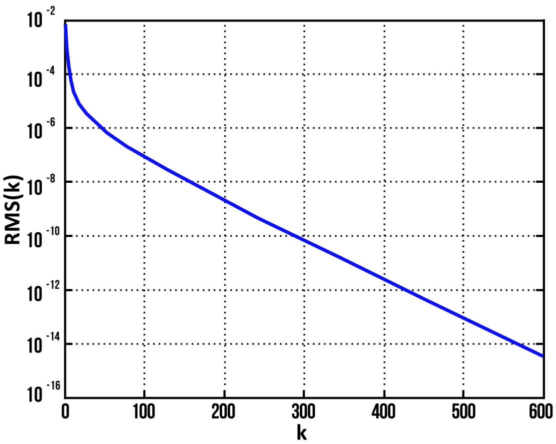

Figure 5. The RMS progression in the absence of backlash.

trajectory. The delay does not affect the achievable accuracy and do not plays significant role in the convergence of the ILC law, as so as in the case of linear systems, see [Tao et al., 2019] and references therein. On the other hand, for the same rate of convergence in a system with delay, higher gains are required in the update law. Increasing these gains can result in lower accuracy in the presence of a saturation-type non-linearity at the input.

8 Acknowledgement

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