Precise control of complex lagrangian systems

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Abstract - **This paper deals with the problem of decomposition and precise control by complex objects. Decomposition is based on concepts of an object technical controllability and model reference adaptive control. Precise accuracy is attained with the help of adaptive and optimal control. Computer simulation demonstrates good results.**

1. INTRODUCTION

As a complex we assume an object with some interconnected subsystems [1, 2]. In the wake of [3], by the Lagrangian systems are meant those where the controlled plant obeys a mathematical model (MM) of the form of Lagrange equations. A MM of such an object is usually multiconnected nonlinear and nonstationary one. Synthesis of control algorithms for such an object is not a simple problem. All the more it is difficult for precise control.

Qualitatively under precise control we mean the situation when the motion of any subsystem and the system in the whole are coincided with prescribed motions with prescribed accuracies.

Usual method for such an object control is decomposition [1, 2]. In this paper we assume that an object in the whole could be represented as a set of interconnected subsystems. For every subsystem a component of interconnections is selected and compensated on the base of adaptive or optimal control [4-6]. For this goal we use two approaches to the decomposition. The first one is based on the decomposition of an object MM. In this case the object has to satisfy to the property of technical controllability [7]. The second approach is based on the decomposition of a system control MM. In this case special adaptive control algorithms are derived [8].

In this paper we assume that different subsystems could have actuators of two different natures: dc motors [9] or jet propulsion engines [10].

2. PROBLEM STATEMENT

We consider controlled plants with the MM in the form of Lagrange equation

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = Q, \qquad (1)$$

where $q = (q_i)$ and $Q = (Q_i)$ $(i = \overline{1,n})$ are vectors of generalized coordinates and forces; $T = \frac{1}{2}\dot{q}^T A(q)\dot{q}$ is the kinetic energy; $A(q) = \left(a_{ij}(q)\right)$ $(i, j = \overline{1,n})$ is a symmetrical positive definite matrix (T denotes transposition).

As an example of such an object it could be considered a free–flying space robotic module [8].

By performing differentiation in (1), we proceed to the equation [7]

$$A(q)\ddot{q} + \sum_{s=1}^{n}\left[\dot{q}^T D_s(q)\dot{q}\right]e_s = S(q)M, \qquad (2)$$

where $M = (M_i)$ $(i = \overline{1,n})$ is a vector of control actions to the plant from a controller.

As actuators we select jet propulsion engines with MM [10]

$$m^r = m_0^r sign(u^r), \qquad (3)$$

and dc motors with MM [9]

$$\begin{aligned} J\ddot{\varphi} &= \frac{1}{r}m^d - \frac{1}{r^2}m^m, \\ \tau_e \dot{m}^d + m^d &= \frac{k_m}{R}u^{dc} - \frac{k_m k_\omega}{R} r\dot{\varphi}. \end{aligned} \qquad (4)$$

In (3) and (4) $m_0^r = const > 0$; m^r, m^m are moving forces or moments; r is a reduction coefficient; J, R, τ, k_m, k_ω are a motor constructive parameters; u^r and u^{dc} are control algorithms to be discovered.

We assume that during the object operating:

- matrices A(q), D_s(q), S(q) $(s=\overline{1, n})$ are known [8];
- vectors $q = q(t)$, $\dot{q} = \dot{q}(t)$ are measurable;
- for every q_i $(i = \overline{1, n})$ there exists a function $q_i^0(t)$ and an equation

$$\ddot{q}_i + d_i\dot{q}_i + k_i q = k_i q_i^0(t), \qquad (5)$$

where the function $q_i^0(t)$ and the numbers $k_i > 0,\ d_i > 0$ are prescribed in advance.

The problem:

It is necessary to discover control algorithms

$$M = M(t,q,\dot{q})$$

that guarantees the motion (5).

3. DECOMPOSITION OF AN OBJECT MATHEMATICAL MODEL WITH ACTUATORS TO TWO SUBSYSTEMS

MM of the object (2) could be rewritten in the form

$$\begin{aligned} &A_{11}(q)\ddot{q}^r + A_{12}(q)\ddot{q}^{dc} + N^r(q,\dot{q}) = \\ &\qquad = S_{11}(q)M^r + S_{12}(q)M^{dc}, \\ &A_{21}(q)\ddot{q}^r + A_{22}(q)\ddot{q}^{dc} + N^{dc}(q,\dot{q}) = \\ &\qquad = S_{21}(q)M^r + S_{22}(q)M^{dc}, \end{aligned} \tag{6}$$

where

$$(q^r)^T = \left(q_1,\dots,q_{n_1}\right);\quad (q^{dc})^T = \left(q_{n_1+1},\dots,q_n\right);$$

$$N^r(q,\dot{q}) = \sum_{s=1}^{n_1}[\dot{q}^T D_s(q)\dot{q}]e_s;$$

$$N^{dc}(q,\dot{q}) = \sum_{s=n_1+1}^{n}[\dot{q}^T D_s(q)\dot{q}]e_s;$$

$$(M^r)^T = \left(M_1,\ M_2,\dots,M_{n_1}\right);$$

$$(M^{dc})^T = \left(M_{n_1+1},\ M_{n_1+2},\dots,M_n\right).$$

To the system (6) it is necessary to add the equations for actuators (3) and (4) presented in a matrix form

$$\begin{aligned} &M^r = M_0^r sign(U^r), \\ &A_m\ddot{q}^{dc} = R(M^d - M^m), \\ &\tau\dot{M}^d + M^d = \rho U^m - \beta\dot{q}^{dc} \end{aligned} \tag{7}$$

where

$$(M_0^r)^T = diag(M_{10}^r,\ M_{20}^r,\dots,M_{n_1}^r)$$

$$(sign(U^r))^T = (sign(u_1^r),\dots,sign(u_{n_1}^r)),$$

$$A_m = diag(J_{n_1+1}r_{n_1+1}^2,\dots,\ J_n r_n^2),$$

$$R = diag(r_{n_1+1},\ \dots,\ r_n),$$

$$(M^d)^T = (M_{n_1+1}^d,\dots,M_n^d),$$

$$(M_m)^T = (M_{n_1+1}^m,\dots,M_n^m),$$

$$\tau = diag(\tau_{e,(n_1+1)},\dots,\tau_{e,n}),$$

$$\rho = diag(\frac{k_{m,(n_1+1)}}{R_{n_1+1}},\dots,\frac{k_{m,n}}{R_n}),$$

$$\beta = diag(\frac{k_{m,(n_1+1)}k_{\omega,(n_1+1)}}{R_{n_1+1}},\dots,\frac{k_{m,n}k_{\omega,n}}{R_n}).$$

For control algorithms synthesis it is possible to take the condition

$$\tau = 0 \tag{8}$$

and to simplify the system (6) to the form

$$\begin{aligned} &\ddot{q}^r = A_{11}^{-1}(q)S_{11}(q)M_0^r sign(U^r) + f^r(*), \\ &\ddot{q}^{dc} = [A_{22}(q) + S_{22}(q)R^{-1}A_m]^{-1}S_{22}(q)\rho U^{dc} + \\ &\qquad + f^{dc}(*). \end{aligned} \tag{9}$$

So if we have the MM (6) [8] then the MM of the system together with actuators can be received in the form (9) by adding constant certain matrices $A_m,\ R,\ \tau,\ \rho,\ \beta$.

4. ADAPTIVE PROGRAMMED CONTROL OF AN OBJECT SUBSYSTEM WITH DC MOTOR ACTUATORS

Let us take the second equation of the MM (9) and try to discover the desired control algorithm. For example an algorithm could be taken in the programmed adaptive form

$$\begin{aligned} &U^{dc} = (S_{22}\rho)^{-1}[A_{22}(q) + S_{22}(q)R^{-1}A_m]* \\ &*[K^{dc}(q^{dc,0} - q^{dc}) - D^{dc}\dot{q}^{dc} + S^{dc}]. \end{aligned} \tag{10}$$

Really, in the algorithm (10) every term is known besides matrices $K^{dc},\ D^{dc}$ and a vector S^{dc} which we may take to realize the motion (5).

The second equation of the MM (9) together with the algorithm (10) could be represented in the form

$$\begin{aligned} &\ddot{q}^{dc} + D^{dc}\dot{q}^{dc} + K^{dc}q^{dc} = K^{dc}q^{dc,0}(t) + \\ &\qquad + [f^{dc}(t) + S^{dc}]. \end{aligned} \tag{11}$$

Let us take matrices $K^{dc},\ D^{dc}$ as diagonal ones that is

$$\begin{aligned} &K^{dc} = diag(k_{n_1+1},\ k_{n_1+2},\dots,k_n), \\ &D^{dc} = diag(d_{n_1+1},\ d_{n_1+2},\dots,d_n) \end{aligned} \tag{12}$$

where $k_i,\ d_i\ (i = \overline{n_1+1,\ n})$ are coincided with desired numbers in (5).

Then the system (11) is decomposed to $(n-n_1)$ interconnected equations

$$\ddot{q}_i^{dc} + d_i\dot{q}_i^{dc} + k_i q_i^{dc} = k_i q_i^{dc,0}(t) + \\ +[f_i^{dc}(t) + S_i^{dc}] \quad (i = \overline{n_1+1,\ n}). \qquad (13)$$

From the equation (13) we see that if it is valid the equality

$$[f_i^{dc}(t) + S_i^{dc}] \equiv 0 \quad (i = \overline{n_1+1,\ n}) \qquad (14)$$

then the problem for generalized coordinates $q_i \quad (i = \overline{n_1+1,\ n})$ is solved.

5. MODEL REFERENCE ADAPTIVE CONTROL FOR SUBSYSTEM WITH DC MOTOR ACTUATORS

Let us set up the problem to compensate the action of the term $f_i^{dc}(t) \quad (i = \overline{n_1+1,\ n})$ in the equation (13) on the desired movement $q_i^{dc} = q_i^{dc}(t)$ with the help of a purposeful variation of the component S_i^{dc}. For this goal we use the well known principle of model reference adaptive control [5].

Let us take a reference model in the form

$$\ddot{q}_{mi}^{dc} + d_i\dot{q}_{mi}^{dc} + k_i q_{mi}^{dc} = k_i q_i^{dc,0}(t)\ . \qquad (15)$$

From (13) and (15) we receive an equation with respect to the error $\varepsilon_i = q_i^{dc} - q_{mi}^{dc}$ in the form

$$\ddot{\varepsilon}_i + d_i\dot{\varepsilon}_{ri} + k_i\varepsilon_{ri} = [f_i^{dc}(t) + S_i]. \qquad (16)$$

The equation (16) can be rewritten in a matrix form

$$\dot{x}_i = A_i x_i + \rho_i(y_i), \\ \dot{y}_i = \psi_i + \mu_i(t) \qquad (17)$$

where $\varepsilon_i = x_{i1},\ \dot{\varepsilon}_i = x_{i2},\ f_i^{dc}(t) + S_i = y_i$, $\dot{f}_i^{dc}(t) = \mu_i(t),\ \ \dot{S}_i = \psi_i,\ \ x_i^T = (x_{i1}\ \ x_{i2})$,

$$\rho_i^T(y_i) = (0\ \ y_i),\ A_i = \begin{pmatrix} 0 & 1 \\ -k_i & -d_i \end{pmatrix}.$$

Now we can choose an algorithm for S_i purposeful variation from the condition of an asymptotical convergence of the system (17) with respect to the movement

$$x_i \equiv 0,\quad y_i \equiv 0. \qquad (18)$$

For this goal we take Lyapunov's function in the form

$$V_i(x_i, y_i) = \kappa_i x_i^T P_i x_i + y_i^2 \qquad (19)$$

where P_i is a positive definite matrix, $\kappa_i = const > 0$. The derivative of $V_i(x_i, y_i)$ with respect to the time on the strength of the system (17) is determined by the equality

$$\dot{V}_i(x_i, y_i) = \kappa_i x_i^T Q_i x_i + \\ +2y_i[\sigma_i + \mu_i(t) + \psi_i] \qquad (20)$$

where Q_i is the prescribed negative definite matrix, $\sigma_i = (p_{21}^i x_{i1} + p_{22}^i x_{i2}),\ p_{jk}^i$ are elements of the matrix $P_i = (p_{jk}^i) \ \ (j,k = 1,2)$.

In this paper we suppose that the sign of the coordinate y_i is known. Then we choose the desired algorithm in the form

$$\psi_i = -\sigma_i - \bar{k}_i sign(y_i) \qquad (21)$$

where $\bar{k}_i > 0$ and

$$\bar{k}_i > |\mu_i(t)|\ . \qquad (22)$$

Then we have inequalities

$$V_i(x_i, y_i) > 0, \qquad \dot{V}_{mi}(x_{mi}, y_{mi}) < 0 \qquad (23)$$

which ensure the solution of the problem.

6. DECOMPOSITION OF THE OBJECT SUBSYSTEM WITH JET PROPULSION ENGINE ACTUATORS

Let us take the first equation of the MM (9)

$$\ddot{q}^r = A_{11}^{-1}(q)S_{11}(q)M_0^r sign(U^r) + f^r(*), \quad (24)$$

and try to discover the desired control algorithm too. The equation (24) can be rewritten in the form

$$\ddot{q}^r = R^r(q,t)M_0^r sign(U^r)\,. \qquad (25)$$

We assume that there exists a working domain $G^W(q, \dot{q}, t)$ where an object with MM (25) possesses by the technical controllability [7]. At this case the MM (25) is decomposed to the system of nonstationary equations in the form

$$\ddot{q}_i^r = \rho_i^r(t)sign(u_i^r),\quad \rho_i^r(t) \ge \rho_{i0}^r, \\ \rho_{i0}^r = const > 0\ \ (i = \overline{1, n_1})\,. \qquad (26)$$

We suppose that for $f^r(*) = f^r(t)$ there is the possibility to know variation ranges that is to know numbers $f_{i0}^r \quad (i = \overline{1,\ n_1})$ in inequalities

$$\left| f_i^r(t) \right| \le f_{i0}^r, \quad f_{i0}^r = const. \qquad (27)$$

7. PRECISE CONTROL OF AN OBJECT SUBSYSTEM WITH JET PROPULSION ENGINE ACTUATORS

Let us introduce notations

$$x_i^r = q_i^{r,0}(t) - q_i^r(t),$$
$$\dot{x}_i^r = \dot{q}_i^{r,0}(t) - \dot{q}_i^r(t) \ \ (i = \overline{1, n_1}) \qquad (28)$$

and rewrite (26) in the form

$$\ddot{x}_i^r = -(\rho_i^r(t))^* sign(u_i^r) \quad (i = \overline{1, n_1}). \qquad (29)$$

We assume that at (29)

$$(\rho_i^r(t))^* \ge \rho_{i0}^r,$$
$$\rho_{i0}^r = const > 0 \ \ (i = \overline{1, n_1}). \qquad (30)$$

7.1. Optimal control for a special condition

At first we will suppose a special not real condition for the system (29) in the form

$$(\rho_i^r(t))^* \equiv \rho_{i0}^r \quad (i = \overline{1, n_1}). \qquad (31)$$

Then a very known algorithm [6]

$$u_i^r = x_i^r + \frac{\left(\dot{x}_i^r\right)^2}{2(\rho_{i0}^r)} sign(\dot{x}_i^r) \ \ (i = \overline{1, n_1}) \qquad (32)$$

guarantees to the movement of the system (29) (with conditions (30) and (31)) the optimal asymptotical convergence to the movement

$$x_i^r \equiv 0, \ \dot{x}_i^r \equiv 0 \ \ (i = \overline{1, n_1}). \qquad (33)$$

7.2. Suboptimal control for the common case

But the assumption (31) is not true. Really we have the condition (30). Then we will try to use the algorithm (32) for the common case and to prove that it will guarantee a suboptimal motion of the system

$$\ddot{x}_i^r = -(\rho_i^r(t))^* sign(u_i^r),$$
$$u_i^r = x_i^r + \frac{\left(\dot{x}_i^r\right)^2}{2(\rho_{i0}^r)^*} sign(\dot{x}_i^r) \quad (i = \overline{1, n_1}) \qquad (34)$$

together with the asymptotical convergence to the movement (33). For this goal we need to consider a motion of the system (34) on the phase plane $\left\{ x^r{}_{i1}, x^r{}_{i2} \right\}$ with the coordinates $x_i^r = x_{i1}$; $\dot{x}_i^r = x_{i2}$.

Let us consider for the system (34) a Lyapunov's function in the form [11]

$$V_i(x_{i1}, x_{i2}) = \frac{1}{2}\left[S_i(x_{i1}, x_{i2})\right]^2, \qquad (35)$$

where

$$S_i(x_{i1}, x_{i2}) = x_{i1} +$$
$$+ \frac{\left(x_{i2}\right)^2}{2(\rho_{i0}^r)^*} sign(x_{i2}) \quad (i = \overline{1, n_1}). \qquad (36)$$

The Lyapunov's function (35) is definite and positive at any point of the phase plane $\{x_{i1}, x_{i2}\}$, except for the points of the switching line $S_i(x_{i1}, x_{i2}) = 0$ $(i = \overline{1, n_1})$, where it assumes zero values.

For any point of the phase plane $\{x_{i1}, x_{i2}\}$, except for the points of the switching line $S_i(x_{i1}, x_{i2}) = 0$ $(i = \overline{1, n_1})$, we obtain the derivative of Lyapunov's function (35) along the trajectories of system (34):

$$\frac{d}{dt} V_i(x_{i1}, x_{i2}) = S_i(x_{i1}, x_{i2}) \frac{dS_i(x_{i1}, x_{i2})}{dt} \qquad (37)$$
$$(i = \overline{1, n_1}).$$

In [7] are proved inequalities

$$\frac{dS_i(x_{i1}, x_{i2})}{dt} < 0 \quad for \quad S_i(x_{i1}, x_{i2}) > 0,$$
$$\frac{dS_i(x_{i1}, x_{i2})}{dt} > 0 \quad for \quad S_i(x_{i1}, x_{i2}) < 0. \qquad (38)$$

Therefore, for any point of the phase plane $\{x_{i1}, x_{i2}\}$, except for the points of the switching line, inequalities (38) are valid. These inequalities are conditions for occurrence of sliding mode on the switching line as soon as the representing point gets on it [11].

After as the representing point gets on the switching line it moves to the point (33) in the sliding mode. So the movement of the system (34) asymptotically converges to the point (33) in spite of the fact that the switching line equality (31) is not true.

9. SIMULATION RESULTS

In simulation results we demonstrate the model reference adaptive control for the subsystem with actuators as dc motors and for the subsystem with relay actuators. We assume that the decomposition problem is solved and now it is necessary to show that algorithms

(21) and (32) really provide precise control of $q_i(t)$ with respect to $q_i^0(t)$.

Let $k_i = 1$ and $d_i = 1.4$ in (13), $Q_i = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ in (20). Then $P_i = \begin{pmatrix} 2.83 & 1 \\ 1 & 1.43 \end{pmatrix}$ in (19) and $\sigma_i = (1.43\dot{\varepsilon}_i + \varepsilon_i)$ in (21).

In fig.1.a we see $q_{mi}^{dc}(t)$, $q_i^{dc}(t)$ with the disturbance $f_i^{dc}(t)$ but without adaptation that is $S_i \equiv 0$ in (13). The difference between $q_{mi}^{dc}(t)$ and $q_i^{dc}(t)$ is essential.

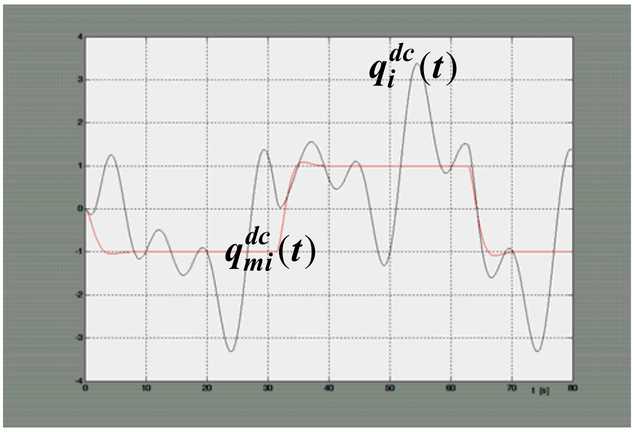

Fig. 1.a.

In fig.1.b we see the same coordinates but with adaptation that is $S_i \neq 0$. The difference between $q_{mi}^{dc}(t)$ and $q_i^{dc}(t)$ is practically null.

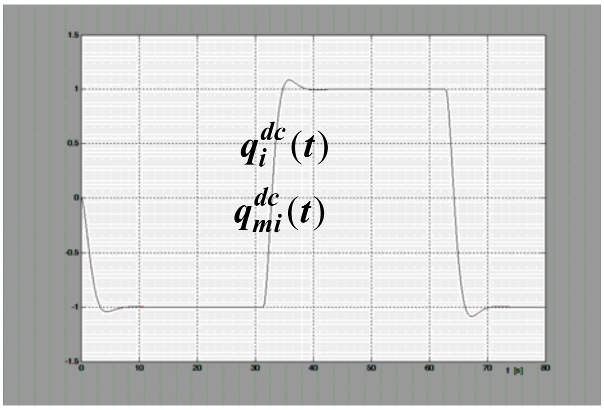

Fig. 1.b.

In fig.2.a we see $q_i^{r,0}(t)$, $q_i^r(t)$ but the condition (30) is not true that is $(\rho_i^r(t))^* < \rho_{i0}^r$. The difference between $q_i^{r,0}(t)$ and $q_i^r(t)$ is essential.

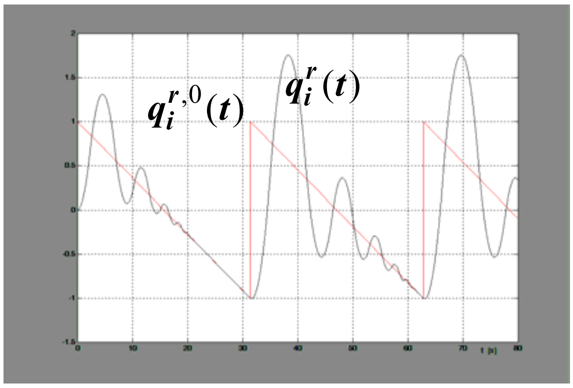

Fig. 2.a.

In fig.2.b we see the same coordinates but with the condition (30) that is $(\rho_i^r(t))^* > \rho_{i0}^r$. The coordinate $q_i^r(t)$ follows a step signal $q_i^{r,0}(t)$ in the suboptimal manner and after that the difference between $q_i^{r,0}(t)$ and $q_i^r(t)$ is practically null.

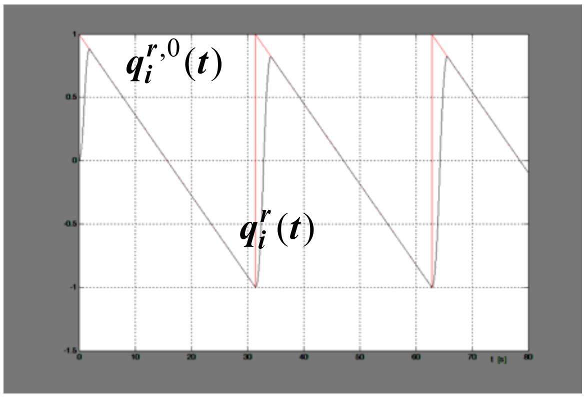

Fig. 2.b.

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