# VIBRATION SUPPRESSION OF HELICOPTER BLADES BY PENDULUM ABSORBERS (FIRST ELASTIC MODE OF THE BLADE)

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**Abstract**

The pendulum absorbers have been used for suppressing the vibrations in helicopter blades. The aim of this study is to clarify the mechanism of the vibration suppression. But, most of the previous studies analyzed its characteristics based on the linear theory and explained focusing on the anti-resonance point. Since the pendulum may vibrate with large amplitude, it is expected that the nonlinearity have essential influence on its vibration characteristics. Therefore, we investigated the vibration suppression of a pendulum absorber considering its nonlinearity. In our first report, we proposed a 2-degree-of-freedom (2DOF) model composed of a rigid blade and a pendulum absorber. The blade is excited by giving a sinusoidal deflection at its end. In the second report, we proposed a 3DOF model by adding the fuselage, where the blade is also considered rigid. The blade is excited by a distributed force which changes sinusoidally. In this paper, a 3DOF model which is composed of a mass corresponding to a rotor mount on the fuselage, a flexible blade, and a pendulum is discussed. A periodic distributed force is given on the blade. This study clarified the mechanism of the pendulum absorbers for suppressing the vibrations of helicopters based on the nonlinearity theory.

**Key words**

Absorber , Pendulum , Helicopter Blade , Nonlinear Vibration

## 1 Introduction

Recently, the use of helicopters has spread rapidly for the prevention of disasters and the emergency medical care because of its superior mobility. However, due to the large vibration of a helicopter, there are possibilities to exert harmful influences to passengers and emergency patients. In addition, the vibration induces fatigue in the structure. Especially, vibration of the blade of helicopter is extremely important because it is the principal cause of helicopter vibration. Periodic external force working to the blade is induced by the aerodynamic force, and it excites the vibration of the blade at integral multiple frequencies of the rotational

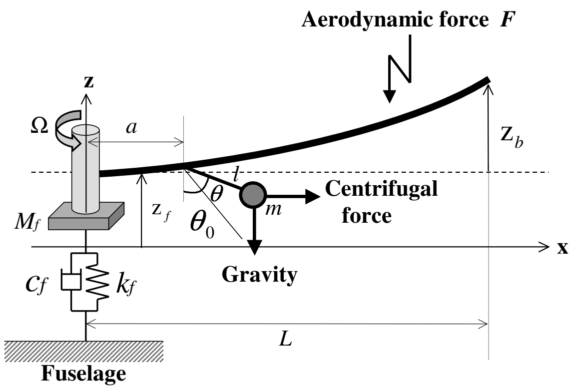



Figure 1. Model for a blade, a pendulum absorber and a rotor mount.

speed [Saito, et al., 1993; Kato and Imanaga, 1985].

Nowadays, pendulum absorber is applied to helicopters as one of the vibration suppression devices of helicopters. Though the pendulum absorbers have designed to use anti-resonance point based on the linear theory, there is no detailed study of its characteristics [Taylor and Teare,1975; Amerand Neff,1974; Viswanathan and McClure,1983]. Our previous paper [Nagasaka, et al.,2007] modeled a rigid blade and a pendulum absorber as 2DOF oscillatory system with the forced displacement. And the vibration suppression mechanism, the effect of parameters of a pendulum, and the limit of its effectiveness are clarified by numerical simulation, theoretical analysis and experiments considering nonlinearity.

This paper treats the first elastic mode of the blade, and consider 3DOF vibration system with a blade fixed to the shaft, a pendulum absorber, and a rotor mount including drive-train elastically-supported to fuselage(See Fig1). It is assumed that external force that has integral multiple frequencies of rotational speed of a blade and amplitude that proportional to the square of a blade velocity works to the system. A pendulum absorber suppress vibration of a blade, and as a result, vibration of a rotor mount is suppressed and the vibration transmitted to fuselage decreases.

Organization of the paper is as follows. Second section explains the modeling of the study and deals with the energy of blade. Third section presents the simulation results. Finally fourth section shows the conclusions for the study.

## 2 Modeling

In our previous research [Nagasaka, et al.,2007], we assumed that a helicopter blade is a rigid body connected to a rotor by a hinge. We analyzed this model and made experiments with the corresponding setup. In this paper, a model with elastic blades are studied numerically. For simplicity, only the first elastic mode is considered.

Figure1 shows the theoretical model of 3DOF system composed of a rotor blade, a pendulum absorber and a mount. The blade is a thin beam with a rectangular cross section. One end is fixed to the vertical rotating shaft and the other end is free. The length of the blade is $L$ and its mass is $M$. A pendulum with the mass $m$ and the length $l$ is mounted on the blade at the position $a$ from the rotating shaft. This pendulum can swing in a vertical plane defined by the shaft and the blade. It is assumed that the blade is flexible in this vertical plane. In the following analysis, only the first mode flexural oscillation is considered and its high mode and the vibrations in the other directions, torsional and translational vibrations, are neglected. The mount with mass $M_f$, which composed with the shaft and the driving system, is supported on the main body by the spring with the spring constant $k_f$ and the damper with the damping coefficient $c_f$. It is assumed that this mount can move in the vertical direction. The pendulum suppress the vibration of this mount. Therefore, the vibration transferred to the main body is decreased. The stationary coordinate system O$-xyz$ is defined in the stationary condition. The origin is located at the cross point of the shaft and the blade, the x-axis is taken in horizontal direction which coincides with the blade, and the z-direction is taken in the vertical direction. The blade rotates around the z-axis with the angular velocity $\Omega$. It is assumed that an external force with the angular frequency $\omega$ （=n$\Omega$, n：integer） and with the magnitude proportional to the square of $\Omega$ works to all parts of the blade.

Let the vertical deflections of the free end and the end fixed to the shaft be $z_b$ and $z_f$, and the angle of the pendulum in the equilibrium condition and the angle in oscillation be $\theta_0$ and $\theta$, respectively.

### 2.1 Energy of blade

It is assumed that the shape of the first mode of the beam during the rotation can be approximated by the shape of the first mode of a non-rotating beam. Then, the vertical deflection $h(s)$ of the beam of length $L$ at the position of distance $s$ from the origin is represented by

$$\begin{aligned} h(s) = (\sin\lambda + \sinh\lambda)(\cos\lambda\frac{s}{L} - \cosh\lambda\frac{s}{L}) \\ -(\cos\lambda + \cosh\lambda)(\sin\lambda\frac{s}{L} - \sinh\lambda\frac{s}{L}) \end{aligned} \tag{1}$$

Where $\lambda$ is the characteristic root corresponding to the first mode. Then the vertical deflection of the beam at the position $s$, $w(s,t)$, is represented by

$$w(s,t) = \frac{h(s)}{h(L)} z_b \tag{2}$$

The point at the distance $s$ shifts towards the origin due to the deflection $w(s,t)$ and this distance $u$ is given as follows.

$$\begin{aligned} u &= \int_0^s \frac{1}{2}\left(\frac{\partial w}{\partial s}\right)^2 ds \\ &= \frac{1}{2h^2(L)}\int_0^s \left(\frac{dh(s)}{ds}\right)^2 ds z_b^2 = H(s) z_b^2 \end{aligned} \tag{3}$$

where

$$H(s) = \frac{1}{2h^2(L)}\int_0^s \left(\frac{dh(s)}{ds}\right)^2 ds \tag{4}$$

The coordinates $(x, y, z)$ representing the position of the point on the beam at the distance $s$ are represented by

$$\left.\begin{aligned} x &= (s-u)\cos\Omega t \\ y &= (s-u)\sin\Omega t \\ z &= z_f + \frac{h(s)}{h(L)} z_b \end{aligned}\right\} \tag{5}$$

The kinetic energy of the beam is given as follows:

$$\begin{aligned} T_b &= \frac{1}{2}\rho A \int_0^L (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) ds \\ &= \frac{1}{2}\rho A \left( \frac{\Omega^2 L^3}{3} + L\dot{z_f}^2 \right. \\ &\quad \left. -2H_1\Omega^2 z_b^2 + H_2\dot{z_b}^2 + 2H_4\dot{z_b}\dot{z_f} \right) \end{aligned} \tag{6}$$

where the accuracies of the quantities $z_b$ and $\dot{z_b}$ are $O(\epsilon)$ and the quantities smaller than and equal to $O(\epsilon^3)$ are neglected. The potential energy of the blade is given by

$$\begin{aligned} U_b &= \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 w}{\partial s^2}\right)^2 ds \\ &\quad +\rho Ag \int_0^L \left\{ z_f + \frac{h(s)}{h(L)} z_b \right\} ds \\ &= \frac{1}{2} EIH_3 z_b^2 + \rho Ag(Lz_f + H_4 z_b) \end{aligned} \tag{7}$$

where E is Young's modulus and I is the cross-sectional area of moment of inertia and

$$\left.\begin{aligned} H_1 &= \int_0^L sH(s)ds \\ H_2 &= \frac{1}{h^2(L)}\int_0^L h^2(s)ds \\ H_3 &= \frac{1}{h^2(L)}\int_0^L \left(\frac{d^2h(s)}{ds^2}\right)^2 ds \\ H_4 &= \frac{1}{h(L)}\int_0^L h(s)ds \\ H_5 &= \frac{h(a)}{h(L)} \end{aligned}\right\} \tag{8}$$

### 2.2 Energy of pendulum

Let the coordinate of the pendulum mass be $(x_p, y_p, z_p)$. They are represented by

$$\left.\begin{aligned} x_p &= \{a + l\sin(\theta+\theta_0)\}\cos\Omega t \\ y_p &= \{a + l\sin(\theta+\theta_0)\}\sin\Omega t \\ z_p &= z_f + \frac{h(a)}{h(L)} z_b - l\cos(\theta+\theta_0) \end{aligned}\right\} \tag{9}$$

The kinetic energy of the pendulum is given as follows.

$$\begin{aligned} T_p &= \frac{1}{2}m(\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2) \\ &= \frac{1}{2}m\Big[\Omega^2\{a + l\sin(\theta+\theta_0)\}^2 + l^2\dot{\theta}^2 \\ &\quad +\dot{z_f}^2 + H_5^2\dot{z_b}^2 + 2H_5\dot{z_b}\dot{z_f} \\ &\quad +2l\dot{\theta}(\dot{z_f} + H_5\dot{z_b})\sin(\theta+\theta_0)\Big] \end{aligned} \tag{10}$$

The potential energy of the pendulum is given by

$$U_p = mgz_p = mgz_f + H_5 z_b - l\cos(\theta+\theta_0) \tag{11}$$

### 2.3 Energy of rotor mount

Suppose that the mount moves in the vertical direction. Let mass of the mount be $M_f$ and the position be $z_f$. The kinetic energy of the mount is represented by

$$T_f = \frac{1}{2}M_f\dot{z_f}^2 \tag{12}$$

The potential energy of the mount is represented by

$$U_f = \frac{1}{2}k_f z_f^2 + M_f g z_f \tag{13}$$

### 2.4 Nonconservative force

The nonconcervative force working to the blade is the viscous damping force and the aerodynamic force. Assume that the viscous damping forces proportional to the velocity $\dot{z_f}$ of the body, the velocity $\dot{z_b}$ of the free end of the blade and the angular velocity $\dot{\theta}$ of the pendulum. Let the coefficients of these viscous damping force be $c_f$, $c_b$ and $c_p$, respectively. The dissipating function of the system is

$$D = \frac{1}{2}c_b\dot{z_b}^2 + \frac{1}{2}c_p\dot{\theta}^2 + \frac{1}{2}c_f\dot{z_f}^2 \tag{14}$$

Let the external force distribute on the whole blade and works periodically. Let the magnitude of the force per unit length be $F$ and its frequency be $\omega$. Since the magnitude $F$ is proportional to the square of the blade velocity, it can be represented by

$$F = F_L(\Omega s)^2 \tag{15}$$

where $F_L$ is the coefficient.

From the principle of virtual work, we can obtain the force $Q_b$ working to the blade and the force $Q_f$ working to the mount as follows.

$$\left.\begin{aligned} Q_b &= \int_0^L F\frac{h(s)}{h(L)}ds\cos\omega t \\ &= \Omega^2 F_L H_6 \cos\omega t \\ Q_f &= \int_0^L F ds\cos\omega t = \frac{1}{3}L^3\Omega^2 F_L\cos\omega t \end{aligned}\right\} \quad (16)$$

where

$$H_6 = \frac{1}{h(L)}\int_0^L s^2 h(s)ds \quad (17)$$

### 2.5 Lagrange equation

The lagrangean $L$ is given by

$$\begin{aligned} L &= (T_b - U_b) + (T_p - U_p) + (T_f - U_f) \\ &= \frac{1}{2}\rho A\Bigg(\frac{\Omega^2 L^3}{3} + L\dot{z_f}^2 \\ &\quad -2H_1\Omega^2 z_b^2 + H_2\dot{z_b}^2 + 2H_4\dot{z_b}\dot{z_f}\Bigg) \\ &\quad -\left\{\frac{1}{2}EIH_3 z_b^2 + \rho Ag(Lz_f + H_4 z_b)\right\} \\ &\quad +\frac{1}{2}m\Big[\Omega^2\{a + l\sin(\theta+\theta_0)\}^2 \\ &\quad +l^2\dot{\theta}^2 + \dot{z_f}^2 + H_5^2\dot{z_b}^2 + 2H_5\dot{z_b}\dot{z_f} \\ &\quad +2l\dot{\theta}(\dot{z_f} + H_5\dot{z_b})\sin(\theta+\theta_0)\Big] \\ &\quad -mg\{z_f + H_5 z_b - l\cos(\theta+\theta_0)\} \\ &\quad +\frac{1}{2}M_f\dot{z_f}^2 - \frac{1}{2}k_f z_f^2 - M_f g z_f \end{aligned} \quad (18)$$

Using Lagrange's equations, we can derive the equations of motion for the blade, the pendulum and the mount as follows.

$$\left.\begin{gathered} \Omega^2 F_L H_6\cos\omega t = \\ \rho A(H_2\ddot{z_b} + H_4\ddot{z_f} + 2\Omega^2 H_1 z_b) \\ +m\{H_5^2\ddot{z_b} + H_5\ddot{z_f} \\ +lH_5\ddot{\theta}\sin(\theta+\theta_0) + lH_5\dot{\theta}^2\cos(\theta+\theta_0)\} \\ +c_b\dot{z_b} + EIH_3 z_b + \rho AgH_4 + mgH_5 \end{gathered}\right\} \quad (19)$$

$$\left.\begin{gathered} 0 = ml\{l\ddot{\theta} + \ddot{z_f}\sin(\theta+\theta_0) \\ +H_5\ddot{z_b}\sin(\theta+\theta_0)\} + c_p\dot{\theta} \\ -m\Omega^2 l(a + l\sin(\theta+\theta_0))\cos(\theta+\theta_0) \\ +mgl\sin(\theta+\theta_0) \end{gathered}\right\} \quad (20)$$

$$\left.\begin{gathered} \frac{1}{3}L^3\Omega^2 F_L\cos\omega t = (\rho AL + m + M_f)\ddot{z_f} \\ +(\rho AH_4 + mH_5)\ddot{z_b} \\ +m\ell\{\ddot{\theta}\sin(\theta+\theta_0) + \dot{\theta}^2\cos(\theta+\theta_0)\} \\ +c_f\dot{z_f} + k_f z_f + (\rho AL + M_f + m)g \end{gathered}\right\} \quad (21)$$

## 3 Simulation results

### 3.1 Vibration suppression with a tuned pendulum

Figure 2 shows natural frequencies of a blade, a pendulum and a rotor mount when the pendulum synchronized to the frequency of the external force with the frequency $\omega = 4\Omega$ [Nagasaka, et al.,2007]. Figures 3 to 5 show their amplitude calculated by integrating Eqs.(19) - (21) numerically.

The parameter values used for the numerical calculations are shown in table 1.

The results of numerical simulations proved that the pendulum behaves as a absorber and can reduce the blade vibration and the rotor mount oscillation at a blade resonance point. Additionally, a pendulum reduces rotor mount resonance. However, unlike the rigid blade, vibration is not suppressed enough in the large rotational speed range.

Table 1. Parameters for numerical simulations

| parameters | value |
|---|---|
| $M$ | 0.108 [kg] |
| $\rho$ | 2698.0 [kg/m $^3$] |
| $E$ | 7.03 $\times 10^{10}$ [N/m$^2$] |
| $L$ | 0.4 [m] |
| $\lambda$ | 1.875 |
| $c_b$ | 0.0523 [N·s/m] |
| $A$ | 1.0$\times 10^{-4}$ [m$^2$] |
| $\ell$ | 0.007 [m] |
| $a$ | 0.1 [m] |
| $m$ | 0.036 [kg] |
| $c_p$ | 3.1$\times 10^{-5}$ [N·s/rad] |
| $M_f$ | 0.054 [kg] |
| $k_f$ | 1.0 [N/m] |
| $c_f$ | 6.0$\times 10^{-4}$ [N·s/m] |
| $F_L$ | 0.05 [N] |

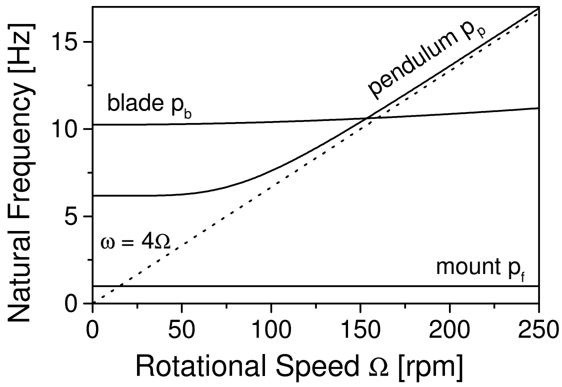



Figure 2. Natural frequencies of a blade, a pendulum absorber and a rotor mount

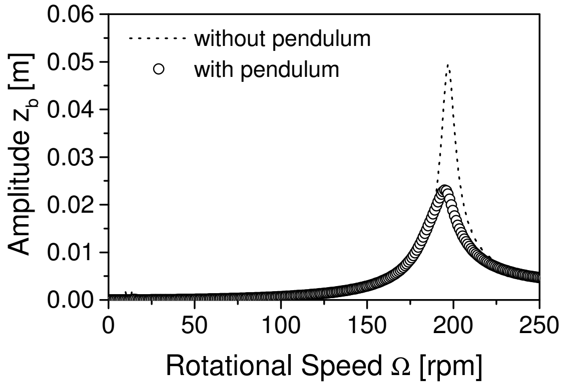



Figure 3. Amplitude of a blade

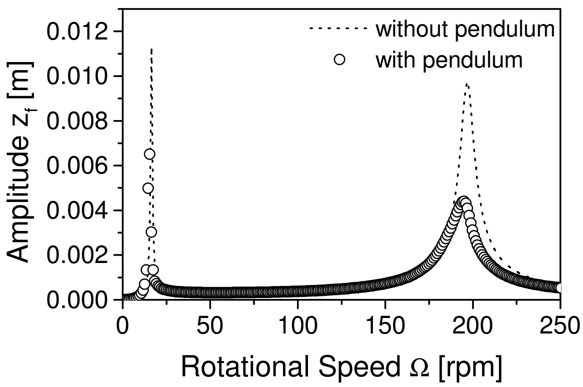



Figure 4. Amplitude of a rotor mount

### 3.2 Effect of a pendulum location $a$

Figure 6 shows the effect of a pendulum location $a$ to vibration suppression capability of a tuned pendulum absorber under constant rotational speed. As the result, it becomes clear that the effect of vibration suppression capability is lost in some specific range. This is due to the location of the node of the first mode is this range.

Generally, in helicopters, it is preferable to install pendulum absorbers as close as possible to the shaft due to the aerodynamic requirement.

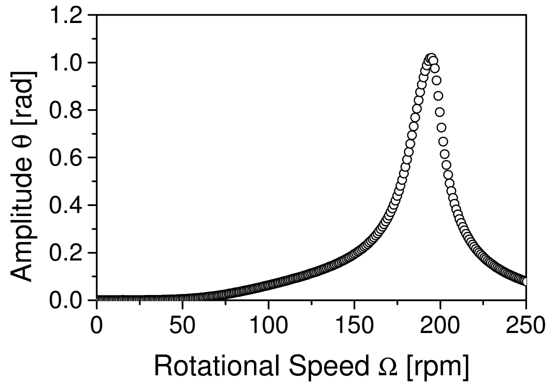



Figure 5. Amplitude of a pendulum

Figure 6 indicates that the optimum location for the pendulum is approximately $a = 0.1$.

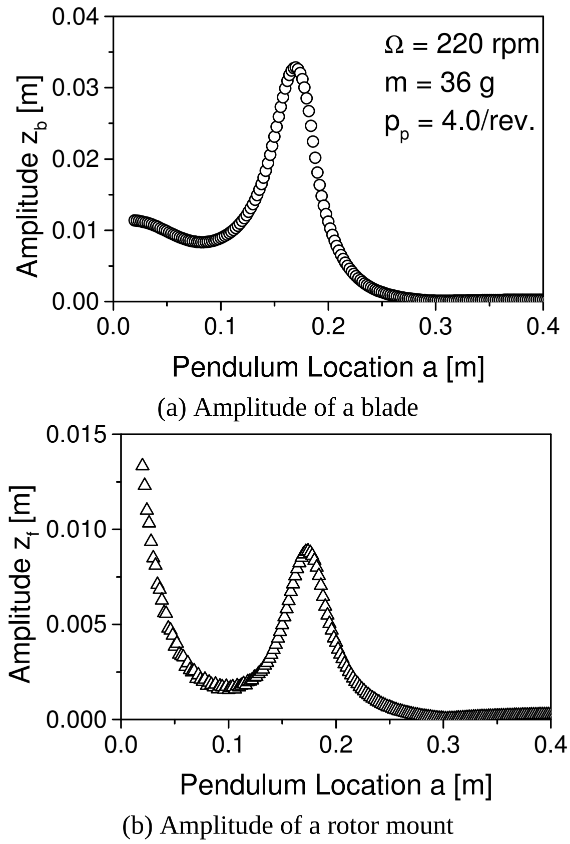
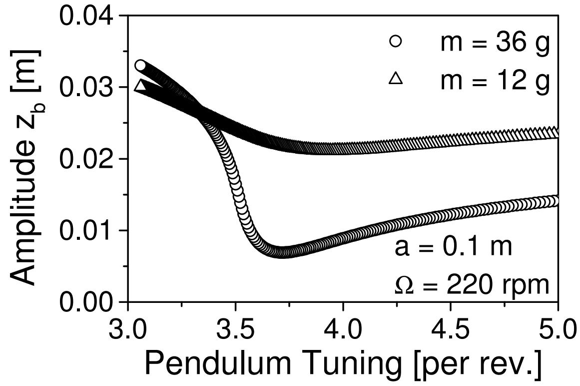


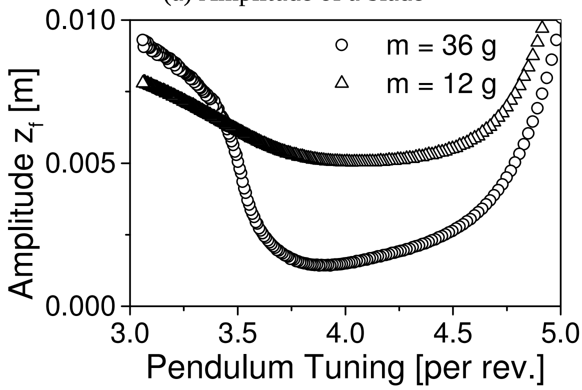

Figure 6. Effect of pendulum location $a$

### 3.3 Effect of a pendulum tuning

As described in the previous paper, the pendulum natural frequency is given by $\Omega\sqrt{1+a/l}$. Therefore a pendulum tuning is determined by coefficient $\sqrt{1+a/l}$. Here, we adopt the pendulum location $a = 0.1$. Then, the natural frequency of the pendulum is tuned by adjusting the pendulum length $l$.

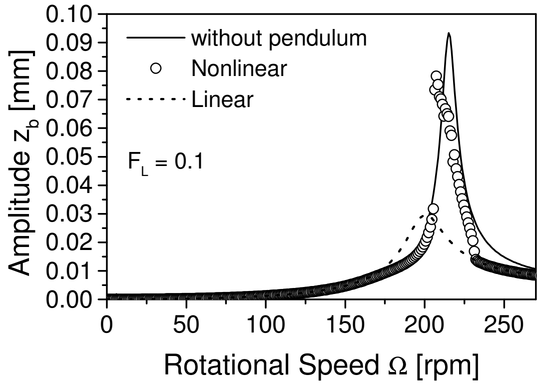



(a) Amplitude of a blade

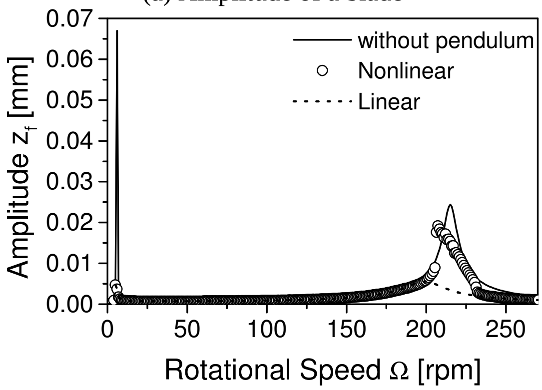



(b) Amplitude of a rotor mount

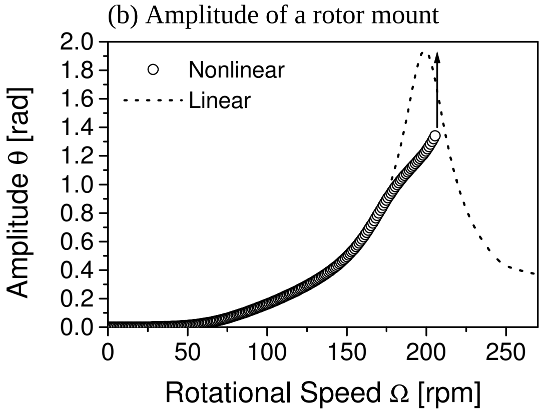
Figure 7. Effect of pendulum tuning




(a) Amplitude of a blade




(b) Amplitude of a rotor mount




(c) Amplitude of a pendulum

Figure 8. Comparison of linear and nonlinear analysis

In order to indicates pendulum tuning, we introduce the number of oscillations per one revolution of a blade. Generally pendulum tuning is determined so as to synchronize to the frequency of the external force according to the linear theory. Therefore, in this case, 4/rev. is the optimum value.

Figure 7 shows the effect of pendulum tuning. This result shows that most effective pendulum tuning is the range from 3.7 to 3.8/rev. The range of optimum tuning varies due to rotational speed $\Omega$, so it is necessary to determine a pendulum tuning for each rotational speed, respectively.

### 3.4 Effect of a pendulum nonlinearity

When the external force is small, as shown in figures 3 to 5, amplitude of a pendulum is smaller than 1.0 rad, and there is not obvious nonlinearity. In case the external force is larger, amplitude of a pendulum exceeds 1.0 rad, and the resonance curve of a blade and a fuselage become soft-spring type because of nonlinearity of a pendulum, see figures 8. Additionally, proximity at 200 rpm, amplitude of a pendulum exceeds $\pi/2$, and a pendulum begin to rotate. In the result the effect of vibration suppression capability is lost. Also, in figures 8, the results of linear simulation is indicated. In linear simulation, a pendulum never rotates even though the amplitude of a pendulum exceeds $\pi/2$. This result differ from actual phenomenon. Hence, nonlinear simulation is important.

## 4 Conclusion

The objective of this study is to test the usage/application of the pendulum absorber for the suppressing of vibration in the helicopter blades. This study treat a blade as the first elastic mode and its main discussion on 3DOF model with a blade under external force, a pendulum, and a rotor mount elastically-supported to fuselage. According to the result of numerical simulations, it became significant that a pendulum absorber can suppress vibration of both the blade and the rotor mount when the natural frequency of pendulum is synchronized to the frequency of the external force. Moreover the effect of the pendulum location to vibration suppression is also explained in the study.

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