

## STUDY OF MULTISTABLE VISUAL PERCEPTION USING THE SYNERGETIC MODEL

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### Abstract

We have studied in the synergetic model of multistable visual perception. Exploring the Necker cube as the essential example of an ambiguous figure, we measured dynamical of two and three coexisting percepts as a function of bias parameter change. The bifurcation analysis allowed us to estimate the different coexistent region of perceptions.

### Key words

Multistable, perception, bifurcation.

### 1 Introduction

The term multistable used in perception, means that an ambiguous stimulus received by the brain can be interpreted in different ways. This phenomenon was intensively studied by psychologists since 1832 [Kuhn and Hawkins, 1963]. Multistable perception can be evoked by visual patterns that are too ambiguous for the human visual system to recognize an only interpretation, such as, Necker cube, ruby vase, etc. Since the most of ambiguous images have only two possible meanings, this kind of perception is often referred to as bistable perception. When a person observes such an image for a long time, the attention intermittently switches between different percepts. This alternation was attributed to neuronal adaptation or saturation [Köhler and Wallach, 1944].

Nowadays, there are several hypotheses about possible mechanisms underlying these switches. Some of them suppose the influence of stochastic processes inherent to neural network activity [Pisarchik et al., 2014]. Other hypotheses [Kelso, 2012] relate the

switches to proper dynamics of the brain neural network.

Generalized multistability means the coexistence of several stable states or attractors for the same set of parameters. The stability of an attractor depends on the velocity at which the system comes back to this state after a small disturbance from this attractor. Theoretically, multistability can be detected by varying initial conditions for all system variables. Experimental detection of multistability is more complicated. The common ways are to change a control parameter forth and back and find a hysteresis or to add noise which induces switches between coexisting states [Pisarchik and Feudel, 2014].

Issues of visual percepts can be attributed to disorder of visual processing. Brain noise plays an important and benefit role in the detection of perceptions and decision making, permit change between different percepts state [Moreno-Bote et al., 2007]. Alternation of perception is a stochastic process according to the Markov chain. Any decision would be impossible without noise which produces probabilistic chooses.

In the recent paper [Pisarchik et al., 2014], the Necker cube has been used as the essential example of an ambiguous figure, where the contrast of inner lines was taken as a control parameter to measure the dynamic hysteresis of two coexisting percepts. The level of brain noise was estimated from the dependence of the hysteresis on the speed of the contrast change. In addition to internal noise, external noise can also induce alternation in perceptions. The results of psychological experiments have been interpreted on the base of a stochastic energy model.

However, there are other models which can be used to reveal mechanics underlying changes between different

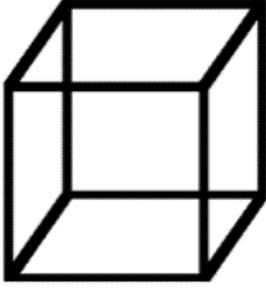


Figure 1. Necker Cube

visual percepts. In this paper, we focus on the synergistic model, first introduced by Haken [Haken, 1979] for studying bistable visual perception. The rest of the paper is organized as follows. In Sec. 2 we introduce the model, in Sec. 3 we present the bifurcation diagram, and in Sec. 4 we summarize our results.

## 2 Model

In 1944, Khler proposed a psychological hypothesis about perception saturation [Köhler and Wallach, 1944]. He suggested and many psychologists supported his opinion, that the observed switches between different percepts of ambiguous images result from fatigue, inhibitions or neuronal saturation. This perceptual behavior can be represented by a mathematical model of human perception of ambiguous patterns [Kohonen, 1989; Haken, 1979; Ditzinger and Haken 1989; Haken, 2004] [Kohonen, 1987; Haken, 1979; Ditzinger and Haken, 1989, 1990], which is in concordance with the experimental result. This mathematical model is a straightforward extension of a general algorithm for pattern ambiguous recognition on the one hand and saturation of attention parameter on the other [Haken, 1987]. In this paper, we focus on the Ditzinger and Haken model [Ditzinger and Haken 1989], for the recognition of ambiguous patterns, which consist of a set of four coupled nonlinear differential equation, two variable states for the saturation attention parameter and two variable state for the perception of ambiguous patterns.

This model simulates the recognition of ambiguous patterns, such as different orientations of the Necker cube face in Fig. 1. The model is given by the following equations

$$\begin{aligned}
 \dot{\xi}_1(t) &= \xi_1 \left[ \lambda_1 - A\xi_1^2 - B\xi_2^2 + \right. \\
 &\quad \left. 4(B-A)\alpha\xi_2^2 \left( 1 - \frac{2\xi_2^4}{(\xi_1^2 + \xi_2^2)^2} \right) \right], \\
 \dot{\xi}_2(t) &= \xi_2 \left[ \lambda_2 - B\xi_1^2 - A\xi_2^2 - \right. \\
 &\quad \left. 4(B-A)\alpha\xi_1^2 \left( 1 - \frac{2\xi_1^4}{(\xi_1^2 + \xi_2^2)^2} \right) \right], \\
 \dot{\lambda}_1(t) &= \gamma(1 - \lambda_1 - \xi_1^2) + F(t), \\
 \dot{\lambda}_2(t) &= \gamma(1 - \lambda_2 - \xi_2^2) + F(t).
 \end{aligned} \tag{1}$$

where  $\xi_1$  and  $\xi_2$  represent variables for the perception or recognitions of ambiguous patterns for two different cube orientations (Fig. 1), and  $\lambda_1$  and  $\lambda_2$  are variables associated with corresponding saturation attention. In this model, the attention parameters are subjected to a damping mechanism mimicking the effect of saturation of attention and synaptic connections with the constant parameters  $A = 1.5$ ,  $B = 2$ ,  $\gamma = 0.1$ ,  $\alpha$  is a bias parameter referred to the perception preference, and  $F(t)$  is the perturbation factor, which can be either harmonic or stochastic modulation.

Figure 2 illustrates the variables for the perception and saturation attention under the influence of bias parameter  $\alpha$  for  $F(t) = 0$ . When  $\alpha = 0$  there are no preference of any percepts in the ambiguous patterns because the pulse widths of the perception and saturation attention variables do not display any difference, as seen in Fig. 2(a). We refer these processes to as perceptions without bias. For  $\alpha = 0.064$  we can see in Fig. 2(b) the preference of one percepts because the pulse widths of both the perception variable  $\xi_2$  and the saturation attention variable  $\lambda_2$  increase with respect to another perception variable  $\xi_1$  and the variable  $\lambda_1$ , respectively. This means that one of the two possible percepts has a preference over the other. This process was described by Ditzinger and Haken [Kohonen, 1989; Haken, 1979] as perceptions with different bias. An additional increase in parameter  $\alpha = 0.128$  induces a high preference of one percepts over the other. It is clearly seen in Fig. 2(c) that the pulse widths of both the perception variable  $\xi_2$  and the saturation attention  $\lambda_2$  are longer than the perception variable  $\xi_1$  and the variable  $\lambda_1$ , respectively. Here, we focus on the percept  $\xi_2$  of the ambiguous image.

## 3 Bifurcation diagram

In a bistable image, such as the Necker cube, two different percepts are associated with two stable steady states (attractors). A change in a control parameter leads to the deformation of the basins of attraction of these attractors and finally to a change in their stability,

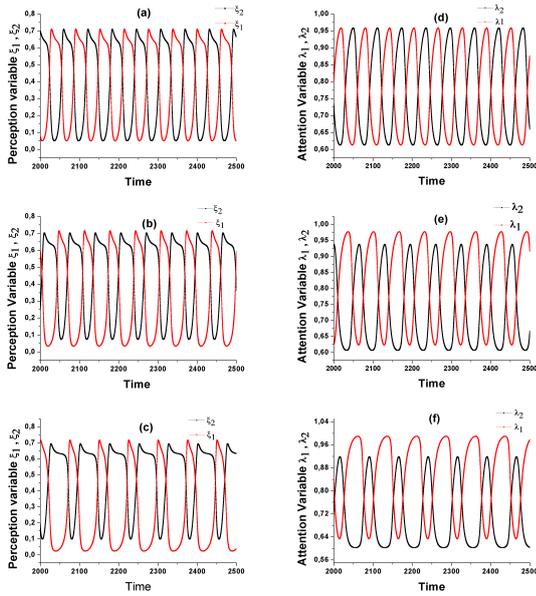


Figure 2. Time series of perception and saturation attention variables for  $F(t) = 0$  and bias parameter (a,d)  $\alpha = 0$ , (b,e)  $\alpha = 0.064$ , and (c,f)  $\alpha = 0.128$ .

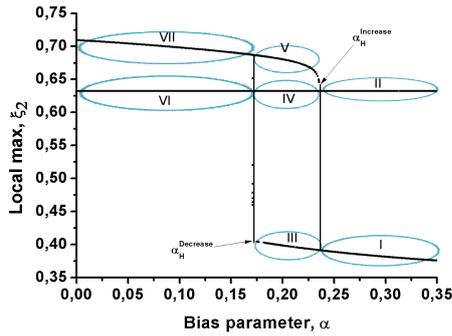


Figure 3. Bifurcation diagram of local maxima of percept  $\xi_2$  as a function of bias parameter  $\alpha$ .

which occurs at a critical point. It should be noted that the parameter variation is practically necessary while dealing with a huge amount of data, in particular, for biological systems. Here, we carry out the bifurcation analysis using the bias  $\alpha$  as a control parameter. This allows one to collect as many data as possible to perform bifurcation analyses. In the Fig. 3 for  $F(t) = 0$ , we can see the bifurcation diagram of local max of percepts  $\xi_2$  as functions of bias parameter  $\alpha$ .

By increasing bias parameter  $\alpha$  top line Fig. 3, one can find a forward Andronov-Hopf bifurcation  $\alpha_H^{Increase}$ , where one of the percepts changes its stability from a periodic to a steady-state behavior, as seen in the time series in Fig. 4(i,a). When  $\alpha$  is decreased, the lower branch in Fig. 3 arrives to a backward Andronov-Hopf bifurcation  $\alpha_H^{Decrease}$ , where another percept variable changes from a fixed point to a

periodic orbit (Fig. 4(e,m)). Whereas one percept is stable for  $\alpha < \alpha_H^{Increase}$ , another percept is stable for  $\alpha > \alpha_H^{Decrease}$ . By using the continuation technique, one more percept is found (middle branch in Fig. 3) with a fixed point around  $\xi_2 = 0.63$  within whole interval of the bias parameter  $0 < \alpha < 0.35$  (Fig. 4(c,g,k)).

In Fig. 3 one can distinguish two regions of bistability. In the first region (for  $\alpha < \alpha_H^{Decrease}$ ), we observe the coexistence of a periodic orbit (upper branch) (Region VII marked by blue oval circles) and a fixed point (lower branch) (Region VI marked by blue oval circles). In the second region (for  $\alpha > \alpha_H^{Increase}$ ), two fixed point attractors coexist, one in the upper branch (Region I marked by blue oval circles) and another in the lower branch (Region II marked by blue oval circles). In particular, a change the bias parameter  $\alpha$  in the interval  $\alpha_H^{Decrease} < \alpha < \alpha_H^{Increase}$  induces multistability. One can see the coexistence of three attractors: the periodic orbit and two fixed points, located, respectively, in the upper branch (Region V marked by blue oval circles), middle branch (Region IV marked by blue oval circles), and lower branch (Region III marked by blue oval circles).

In Fig. 4 we plot the time series of the percepts  $\xi_2$  of the systems Eq. (1)–(4) for different values of the bias parameter  $\alpha$  and different initial conditions. In Table I we show the parameters and initial conditions used for Fig. 4.

Table I

Fig.4	$\alpha$	$\xi_1$	$\xi_2$	$\lambda_1$	$\lambda_2$
(a),(b)	0.3	0.558	0.382	0.689	0.854
(c),(d)	0.3	0	0.632	1	0.6
(e),(f)	0.2	0.540	0.399	0.708	0.841
(g),(h)	0.2	0	0.632	1	0.6
(i),(j)	0.2	0.125	0.677	0.817	0.735
(k),(l)	0.1	0	0.632	1	0.6
(m),(n)	0.1	0.083	0.697	0.800	0.737

## 4 Conclusion

Perception multistability has been studied using the synergetic model. Our results have confirmed the hypothesis that the alternation between competing percepts is associated with activation of different states of neural activity driven by a bias parameter. The states with weak and strong stability may contribute to brain pathologies. We believe that the testing methodology proposed in this work can help in understanding pathological brain states.

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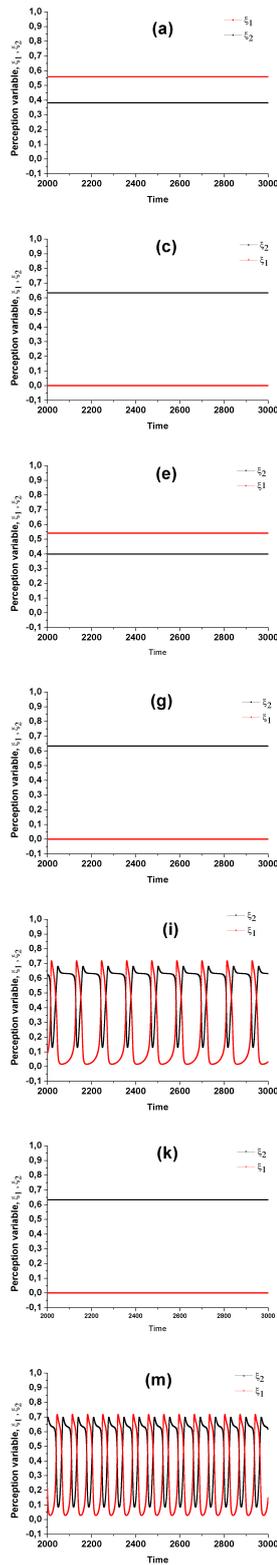


Figure 4. Time series in Region I of bistable section with (a) two fixed points  $\xi_1, \xi_2$ , (b)  $\lambda_1, \lambda_1$  for  $\alpha = 0.3$ ; Region II of bistable section with two fixed points (c)  $\xi_1, \xi_2$ , (d)  $\lambda_1, \lambda_1$  for  $\alpha = 0.3$ ; multistable section **Region III** fixed point (e)  $\xi_1, \xi_2$ , (f)  $\lambda_1, \lambda_1$  for  $\alpha = 0.2$ ; Region IV of multistable section, point (g)  $\xi_1, \xi_2$ , (h)  $\lambda_1, \lambda_1$  for  $\alpha = 0.2$ ; Region V of multistable section, periodic orbit (i)  $\xi_1, \xi_2$ , (j)  $\lambda_1, \lambda_1$  for  $\alpha = 0.2$ ; Region VI of bistable section, fixed point (k)  $\xi_1, \xi_2$ , (l)  $\lambda_1, \lambda_1$  for  $\alpha = 0.1$ ; Region VII of bistable section, periodic orbit (m)  $\xi_1, \xi_2$ , (n)  $\lambda_1, \lambda_1$  for  $\alpha = 0.1$ .

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