CYBERNETICS AND PHYSICS, VOL. 1, NO. 2, 2012 , 79–88

RANDOMIZATION IN CONTROLS FOR THE OPTIMIZATION OF A SMALL UAV FLIGHT UNDER UNKNOWN ARBITRARY WIND DISTURBANCES

Konstantin Amelin
Department of Mathematics and Mechanics
Saint Petersburg State University
Russia
konstantinamelin@gmail.com

Abstract

This paper deals with the possibilities of the randomized control for optimizing the trajectory of the UAV horizontal flying under unknown wind disturbances. We suggest a small UAV which is equipped for navigation only with the GPS module. Only the positions data received iteratively at discrete time instants can be used. The user must be able to add test perturbations through the input channel. The assumptions concerning the noise are reduced to a minimum: it can virtually be arbitrary yet independent of test perturbations. The theoretical results are illustrated by simulations. Operability of the new algorithm under irregular noise in observations in comparison with traditional approaches is illustrated by simulation examples. For practical use we designed a small autonomous unmanned planner with an autopilot and additional microcomputer on the board. The effective interoperability process between autopilot and microcomputer by SIP was organized. The connection between microcomputer and ground base or microcomputers of the other UAVs was established by Wi-Fi or Internet connection.

Key words

Randomized control, wind disturbances, small UAV, optimization, unknown arbitrary noise.

Nowadays single or group autonomous unmanned aerial vehicles (UAVs) are applied more often for area investigation or monitoring, searching and tracking of the people, vehicles *etc*. The quality of the task fulfilling depends significantly on the exact positioning and the following by a given trajectory.

The massive inertial navigation gyroscopes, magnetometers, accelerometers, and various measuring sensors are used for a large UAV. They are able to reduce the negative effects of measurement errors during the motion and provide a fairly accurate positioning. The GPS navigation network development simplifies greatly the solving of a positioning task in a combination with an inertial system. Technological advances, miniaturization of actuators, their growth and the availability features expansion allow to begin an effective usage of a small UAV for the investigation or monitoring of an areas. The relatively small size and light weight are, on the one hand, the basis of "cheap" appropriate technical solutions, but, on the other hand, does not allow to use of powerful inertial navigation systems which is limited the use of an inertial system only to maintain the "current" equilibrium. For the positioning of these objects GPS sensor are used mainly. The problems of a positioning accuracy and a compliance with prescribed trajectory are solved usually with filters which predict a possible bias in the next moment. The decision about necessary course corrections are made on the basis of these predictions.

The problem of predicting values of a random process generated by white-noise disturbances passed through the linear filter is most typical for the Kalman filtering. Studying the case of observations with statistical errors goes back to the pioneer work [Kalmal and Bucy, 1961] it seems to be complete by now.

In parallel with statistical statements, minimax problem statements are also developed. Here, uncertainties are considered as boundedness in a certain sense, but in other respects, they may be arbitrary. In these statements with a priori knowledge of the noise level, it is usually sufficient to derive predictions in the form of sets whose sizes are stabilized in time (see, e. g. [Bai, Nagpal and Tempo, 1996], [Garulli, Giarre and Zappa, 2002], [Polyak and Sherbakov, 2002], [Polyak and Topunov, 2008]). In this case, no consideration is given to the possibilities of deriving estimates which converge to the true point of unknown parameters. Practical use of estimates-sets leads to intricate problems of robust stability.

Insufficient variety of input signals complicates the problem of an identification. An opportunity for a control system to produce a special control (trial, test, probing) signal in the input channel can significantly alleviate the problem of unknown parameters reconstructing. For example, if a harmonic signal is sent to the input of a steady-state linear plant, the plant's output will also be a harmonic signal after the transition process (it is assumed that there is no noise). The amplitude of this signal is proportional to the value of the plant transfer function at the point which corresponds to the frequency of the input harmonic signal. By varying the frequency, we can reconstruct the whole plant transfer function. In a similar way, a series of single pulses in the input channel allows for reconstructing the plant pulse function.

Moreover, special randomized test signals in the input channel allow identification of the control plant parameters when we consider a plant model with almost arbitrary additive external noise [Granichin and Fomin, 1986]. The procedure suggested in [Granichin and Fomin, 1986] is valid for any noise and does not require a priori knowledge of its characteristics; noise may be not random or may be white or correlated, with zero-mean or bias; a signal-noise ratio may be high or low. The recovery of unknown parameter values is provided by the properties of randomized test signals which are added together with an intrinsic adaptive control signal from a closed loop. This approach follows from Feldbaum's concept of *dual control* [Feldbaum, 1960]: *control must be not only directing, but also learning*.

Consider a dynamical system

$$y_t = G_\star(z^{-1})u_t + v_t \tag{1}$$

with input u_t and output y_t shown in Fig. 1.

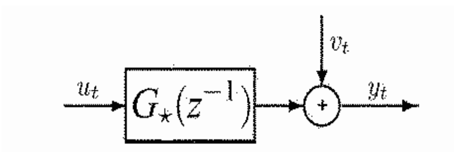

Fig. 1. Dynamical system.

Noise v_t describes all other sources, apart from u_t, which cause variation in y_t. z^{-1} is a delay operator: $z^{-1}u_t = u_{t-1}$. The transfer function $G_\star(z^{-1})$ belongs to a set of transfer functions $G(\theta, z^{-1})$ parameterized by θ, i. e. $G_\star(z^{-1}) = G(\theta_\star, z^{-1})$ for some $\theta_\star$. The structure of the model class $G(\theta, z^{-1})$ is known but $\theta_\star$ itself is unknown. The problem under consideration is to determine, based on a finite set of input and output data collected at time $t = 1, 2, \ldots, N$, a confidence region $\widehat{\Theta}$ for $\theta_\star$ with a specified probability chosen by a user. Moreover, $\widehat{\Theta}$ must be constructed without any a priori knowledge of the noise level, distribution, or correlation.

The standard approach to obtaining confidence regions is to use *an asymptotic theory* of system identification (see, e. g. [Ljung, 1999]). Although these results have been used successfully in many applications, asymptotic estimates are only reliable when the data volume N tends to infinity. When the number of points of data measurements is finite the asymptotic theory may cause erroneous results even for large data sets. Another way—*Set Membership Identification*—assumes that the boundaries of all uncertain system components are a priori known. As a result, the guaranteed region of parameters is defined as a set of values that do not violate a priori boundaries [Bai, Nagpal and Tempo, 1996], [Garulli, Giarre and Zappa, 2002], [Polyak and Sherbakov, 2002].

In [Granichin, 1988], [Granichin, 1989], [Granichin, 1992], [Granichin, 2004], for the case of arbitrary noise (e. g., *unknown but bounded noise*), the randomization was used to develop an identification algorithm which allowed for obtaining an asymptotically confidence region of an indefinitely small size. These results were extended to the case of time-varying parameters in [Vakhitov, Granichin and Vlasov, 2010], [Amelin and Granichin, 2011]. The information about the maximum possible amplitude of the noise has only been used in the formulas for estimating the rate of convergence, i. e. this knowledge is not required for operability of an identification algorithm.

In [Granichin, 2012], [Amelin and Granichin, 2012] it was presented a procedure which gives rigorously guaranteed nonasymptotic confidence regions for unknown parameters of a linear dynamical control plant which is disturbed by arbitrary noise. The procedure consists of simple input design steps followed by an algorithm named LSCR (Leave-out Sign-dominant Correlation Regions) which is mostly promoted by M. Campi and E. Weyer [Campi and Weyer, 2010]. But the LSCR method is difficult to use directly when we consider identification problems in the context of adaptive control under arbitrary noise. In particular, the practical application of the LSCR method to systems with feedback, which was considered in [Campi and Weyer, 2010] (Remark 3 on p. 2711), is only possible for an a priori chosen stationary control law. If the control plant is not stabilized and input and output variables increase infinitely, a linear model is usually not valid from the practical point of view, and the regions obtained by the algorithms from [Campi and Weyer, 2010], which were proved theoretically, may not be directly relevant to the original problem statement. If the closed-loop regulator changes in time depending on current observations, it implies that one of the main conditions of the applicability of the LSCR method from [Campi and Weyer, 2010] is violated.

The main contribution of this paper is extending the results of [Amelin and Granichin, 2011], [Amelin and Granichin, 2012] to a more practice problems of a small UAV fly path optimization.

The paper is organized as follows: At the beginning we summarize the result of [Amelin and Granichin, 2012] about randomized control strategies. Then, in Section III, we formulate a linear filtering problem under arbitrary noise in observations. Section IV provides the simple model of horizontal UAV flying under noise discreet position measurement and the rules to form control inputs (control synthesis) which allow to achieve the better performance of flying path. Section V presents a simulation study example. The technical characteristics of UAV which is built to approve the main theoretical result is given in Section VI. At the end, we make conclusions.

1 Control Actions with Randomized Test Signals

The procedure discussed further is intended to identify the unknown parameters of a dynamic scalar linear control plant which is described by an autoregressive moving average model. It is based on reparameterization of a plant mathematical model. Instead of the plant natural parameters — dynamic coefficients — it is convenient to use some other parameters which are in one-to-one correspondence with them. Such reparameterization is a result of rewriting the plant's equation in a moving average model form which makes it possible to use the LSCR procedure for building the confidence region even in the cases if an adaptive algorithm is used in the feedback channel.

We assume that a control plant has scalar inputs and outputs and it is described by Equation (1) in discrete time with $G_\star(z^{-1}) = B_\star(z^{-1})/A_\star(z^{-1})$, where

$$A_\star(\lambda) = 1 + a_\star^1\lambda + \cdots + a_\star^{n_a}\lambda^{n_a},$$

$$B_\star(\lambda) = b_\star^l\lambda^l + b_\star^{l+1}\lambda^{l+1} + \cdots + b_\star^{n_b}\lambda^{n_b},$$

natural numbers n_a, n_b are the output and input (control) model orders; l is a delay in a control, $1 \le l \le n_b$; $a_\star^1, \ldots, a_\star^{n_a}, b_\star^l, \ldots, b_\star^{n_b}$ are plant parameters, a part of which is unknown.

It is required to define with a given probability an area of reliability for unknown coefficients of the plant (1) by the observations of outputs $\{y_t\}$ on a finite interval of time $t = 1, 2, \ldots, N$, and known inputs (controls) $\{u_t\}$ which can be chosen.

Let $s \le n_a + n_b - l + 1$ be a positive integer number. (It is usually equal to the amount of unknown parameters of the plant (1)). And let $N = s \cdot N_\Delta$ with some N_Δ.

Let us choose a sequence of independent random variables symmetrically distributed around zero (a randomized test perturbation) $\Delta_0, \Delta_1, \ldots, \Delta_{N_\Delta - 1}$ and add them to the input channel once per every s time moments (at the beginning of each time interval) in order to "enrich" the variety of observations.

To be more precise, we will build controls $\{u_t\}_{t=0}^{N-l}$ by the rule

$$u_{sn+i-l} = \begin{cases} \Delta_n + \bar{u}_{sn-l}, & i = 0, \\ \bar{u}_{sn+i-l}, & i = 1, 2, \ldots, s-1, \end{cases}$$

$n = 0, \ldots, N_\Delta - 1$, where "intrinsic" controls $\{\bar{u}_t\}$ are determined by an adjustable feedback law

$$\bar{u}_t = \mathscr{U}_t(y_t, y_{t-1}, \ldots, \bar{u}_{t-1}, \ldots), t \ge 0,\ \bar{u}_{-k} = 0, k > 0.$$

The type and characteristics of a feedback depend on specific practical problems. In particular, it is possible to use a trivial law of "intrinsic" feedback: $\bar{u}_t = 0$, $t = 0, 1, \ldots, N - l$.

Main assumption

A1. The user can choose Δ_n and this choice does not affect to the external noise $v_{sn}, \ldots, v_{s(n+1)-1}$. (In the mathematical sense Δ_n does not depend on $\{v_t\}_{t=1}^{s(n+1)-1}$.)

Note, that no assumptions are made about the noise v_t and the upper limits of noise amplitudes. If the noise is random there are no assumptions about the zero-mean or any autocorrelation properties.

For time instant sn, $n = 0, \ldots, N_\Delta - 1$, we can denote $\bar{v}_{sn} = v_{sn} + (1 - A_\star(z^{-1}))y_{sn} + (B_\star(z^{-1}) - b_\star^l z^{-l})u_{sn}$ and rewrite Equation (1) in the following form:

$$y_{sn} = \Delta_n\theta_\star^1 + \theta_\star^1\bar{u}_{sn-l} + \bar{v}_{sn},$$

where $\theta_\star^1 = b_\star^l$. This equation shows a direct relation between observation y_{sn} and test signal Δ_n which does not depend on the "new" noise $\bar{v}_{sn}$.

Similarly, we rewrite Equation (1) for the rest of times $sn + k - 1$, $k = 2, \ldots, s$, sequentially excluding the variables $y_{sn+k-1}, \ldots, y_{sn}$ from the left-hand side of the equation, using the same equation (1) for early time instants:

$$y_{sn+k-1} = \Delta_n\theta_\star^k + \sum_{i=0}^{k-1}\theta_\star^{k-i}\bar{u}_{sn-l+i} + \bar{v}_{sn+k-1}, \quad (2)$$

where $\theta_\star^{k-i}$, $i = 0, \ldots, k-1$ are the corresponding coefficients of the remaining right-hand side terms with $\bar{u}_{sn-l+i}$.

In [Granichin and Fomin, 1986] and [**?**], the authors suggest forming new parameters as s-vector $\theta_\star$ of coefficients $\theta_\star^k$ obtained in (2). They also give conditions for the invertibility of such reparameterization procedure.

The next formula follows immediately from the above definition $\theta_\star = \mathbb{A}^{-1}\mathbb{B}$, where $s \times s$ matrix $\mathbb{A}$ and s-vector $\mathbb{B}$ are

$$\mathbb{A} = \begin{pmatrix} 1 & 0 & \ldots & 0 & 0 \\ a_\star^1 & 1 & \ldots & 0 & 0 \\ a_\star^2 & a_\star^1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \ldots & a_\star^{n_a} & \ldots & a_\star^1 \; 1 \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} b_\star^l \\ \vdots \\ b_\star^{n_b} \\ \vdots \\ 0 \end{pmatrix}.$$

Consider the conditions of the existence of a corresponding inverse function.

Assumption

A2. Let s be a positive integer such that a set of the plant's unknown parameters are uniquely determined by some function $\tau(\theta)$ from the above-defined vector $\theta_\star$.

By Lemma 5.5.1 on p. 224 from [Fomin, 1985] Assumption **A2** holds for $s = n_a + n_b - l + 1$ when the plant's orders n_a, n_b are known and the following assumption is satisfied:

A3. The polynomials $z^{n_a} A_\star(z^{-1})$ and $z^{n_b} B_\star(z^{-1})$ are mutually prime.

In [Fomin, 1985] there is also the algorithm for the inverse function $\tau(\theta)$.

In practice only part of plant parameters are unknown usually. Sometimes, unknown parameters correspond to the low degrees of z^{-1} which are smaller than some $\bar{n}_a$ and $\bar{n}_b$ respectively. In this case we can choose $s = \bar{n}_a + \bar{n}_b - l + 1$ which is significantly less than $n_a + n_b - l + 1$. Moreover the "new" noise $\bar{v}_{sn+k-1}$ in (2) can be divided into two parts: nonmeasurable $\tilde{v}_{sn+k-1}$ and measurable ψ_{sn+k-1}. The latter is determined by observable inputs and outputs with known coefficients (see the example below).

Example. Consider the second-order plant

$$y_t + a_\star^1 y_{t-1} + y_{t-2} = b_\star^1 u_{t-1} + 1.6 u_{t-2} + v_t, \qquad (3)$$

$t = 1, 2, \dots, N$, with unknown coefficients $a_\star^1$ and $b_\star^1 \neq 0$.

Denote

$$\tau_\star = \begin{pmatrix} a_\star^1 \\ b_\star^1 \end{pmatrix}.$$

Let $s = 2$ and vector $\theta_\star$ of the "new" parameters be

$$\theta_\star = \begin{pmatrix} b_\star^1 \\ 1.6 - a_\star^1 b_\star^1 \end{pmatrix} \in \mathbb{R}^2.$$

In this case the inverse function $\tau(\theta)$ is

$$\tau(\theta) = \begin{pmatrix} \frac{1.6 - \theta^2}{\theta^1} \\ \theta^1 \end{pmatrix}.$$

Equations (2) have the following forms:

$$y_{2n} = \Delta_n \theta_\star^1 + \theta_\star^1 \bar{u}_{2n-1} + \psi_{2n} + \tilde{v}_{2n},$$

$$y_{2n+1} = \Delta_n \theta_\star^2 + \theta_\star^2 \bar{u}_{2n-1} + \theta_\star^1 \bar{u}_{2n} + \psi_{2n+1} + \tilde{v}_{2n+1},$$

where $\psi_{2n+k} = 1.6 \bar{u}_{2n-2+k} - y_{2n-2+k}$, $k = 0, 1$, $\tilde{v}_{2n} = v_{2n} - a_\star^1 y_{2n-1}$, $\tilde{v}_{2n+1} = v_{2n+1} + a_\star^1 (a_\star^1 y_{2n-1} + y_{2n-2} - 1.6 \bar{u}_{2n-2} - v_{2n})$.

Procedure for Constructing Confidence Regions

1. Using observational data we can write predictors as a function of θ

$$\hat{y}_{sn+k-1}(\theta) = \Delta_n \theta^k + \sum_{i=0}^{k-1} \theta^{k-i} \bar{u}_{sn+k-l-i}, \qquad (4)$$

$$n = 0, \dots, N_\Delta - 1,\ k = 1, \dots, s.$$

2. We can calculate the prediction error

$$\varepsilon_t(\theta) = y_t - \hat{y}_t(\theta),\ t = 1, \dots, N.$$

3. According to the observed data we form a set of functions of θ

$$f_{sn+k-1}(\theta) = \Delta_n \varepsilon_{sn+k-1}(\theta),\ n = 0, \dots, N_\Delta - 1,$$

$$k = 1, \dots, s.$$

4. Choose a positive integer $M > 2s$ and construct M different binary stochastic strings (of zeros and ones) $(h_{i,1}, \dots, h_{i,N})$, $i = 0, 1, \dots, M-1$, as follows: $h_{0,j} = 0$, $j = 1, \dots, N$, all the other elements $h_{i,j}$ take the values of zero or one with the equal probability $\frac{1}{2}$.
   We calculate

$$g_i^k(\theta) = \sum_{n=0}^{N_\Delta - 1} h_{i,ns+k} \cdot f_{ns+k-1}(\theta),\ i = 0, \dots, M-1,$$

$$k = 1, \dots, s.$$

5. Choose q from the interval $[1; M/2s]$. For $k = 1, \dots, s$, construct a region $\widehat{\Theta}^k$ such that at least q of the $g_i^k(\theta)$ functions are strictly higher than 0 and at least q of them are strictly lower than 0.
   We define the confidence set by the formula

$$\widehat{\Theta} = \bigcap_{k=1}^{s} \widehat{\Theta}^k. \qquad (5)$$

Remarks. 1. The procedure described above is similar to the one suggested in [Campi and Weyer, 2010] but it has two significant differences from it. First, we consider a confidence set $\widehat{\Theta}$ in state space $\mathbb{R}^s$ instead of $\mathbb{R}^{n_a+n_b}$. The confidence regions $\widehat{\Theta}^k$, $k = 1, \dots, s$, are the subsets of $\mathbb{R}^k$ instead of $\widehat{\Theta}^k \subset \mathbb{R}^{n_a+n_b}$. Second, randomized trial perturbations are included through the input channel only once per every s time instants instead of permanent perturbations in [Campi and Weyer, 2010].

2. If we can divide the "new" noise $\bar{v}_{sn+k-1}$ in (2) into two parts — $\tilde{v}_{sn+k-1}$ and ψ_{sn+k-1} — where the first part is nonmeasurable whereas the second is determined by observable inputs and outputs with known coefficients then in the above-described procedure we can use stronger predictors instead of (4)

$$\hat{y}_{sn+k-1}(\theta) = \Delta_n \theta^k + \sum_{i=0}^{k-1} \theta^{k-i} \bar{u}_{sn+k-l-i} + \psi_{sn+k-1}.$$

The probability that $\theta_\star$ belongs to each of $\widehat{\Theta}^k$, $k = 1, 2, \ldots, s$, is given in the following theorem.

Theorem 1: Let condition **A1** be satisfied. Consider $k \in \{1, 2, \ldots, s\}$ and assume that $Prob(g_i^k(\theta_\star) = 0) = 0$. **Then**,

$$Prob\{\theta_\star \in \widehat{\Theta}^k\} = 1 - 2q/M, \qquad (6)$$

where M, q and $\widehat{\Theta}^k$ are from steps 4 and 5 of the above-described procedure.

Proof: See [Amelin and Granichin, 2012].

The next corollary follows directly from Theorem 1.

Corollary 2: Under the conditions of Theorem 1

$$Prob\{\theta_\star \in \widehat{\Theta}\} \geq 1 - 2sq/M, \qquad (7)$$

where $\widehat{\Theta}$ is taken from (5).

Note, that the value of the probability in (6) is accurate but not the lower limit as it was pointed out in [Campi and Weyer, 2010]. Inequality in (7) is obtained because the events $\{\theta_\star \notin \widehat{\Theta}^k\}$, $k = 1, \ldots, s$ may overlap.

From the above it is easy to derive.

Theorem 3: Let conditions **A1–A2** be satisfied and assume that $Prob(g_i^k(\theta_\star) = 0) = 0$. **Then**, the set $\tau(\widehat{\Theta})$ is the confidence set for unknown parameters of the plant (1) with a confidence level of no less than $1 - 2sq/M$.

Example

We return to control plant (3) with $N = 960$ and unknown parameters $a_\star$ and $b_\star$.

Define the functions $f_t(\theta)$:

$$f_{2n}(\theta) = \Delta_n(y_{2n} - \Delta_n\theta^1 - \theta^1\bar{u}_{2n-1} - \psi_{2n}),\ n = 0, \ldots, N_\Delta - 1,$$

$$f_{2n+1}(\theta) = \Delta_n(y_{2sn+1} - \Delta_n\theta^2 - \theta^2\bar{u}_{2n-1} - \theta^1\bar{u}_{2n} - \psi_{2n+1}).$$

Let us choose $M = 480$ and $q = 6$ and calculate the empirical correlations

$$g_i^k(\theta) = \sum_{n=0}^{499} h_{i,2n+k} \cdot f_{2n+k-1}(\theta),\ i = 1, \ldots, 479,\ k = 1, 2.$$

For $k = 1, 2$ we construct the regions $\widehat{\Theta}^k$ which only include the values of θ for which no less than 6 of the functions $g_i^k(\theta)$, $i = 1, \ldots, 479$, are strictly higher than zero and no less than 6 of them are strictly lower than zero.

By virtue of Theorem 3, the vector of true parameters with a probability of more than $95\% = (1 - 2 \cdot 2 \cdot 6/480) \cdot 100\%$ belongs to the confidence set $\tau(\widehat{\Theta}) = \tau(\widehat{\Theta}^1 \bigcap \widehat{\Theta}^2)$.

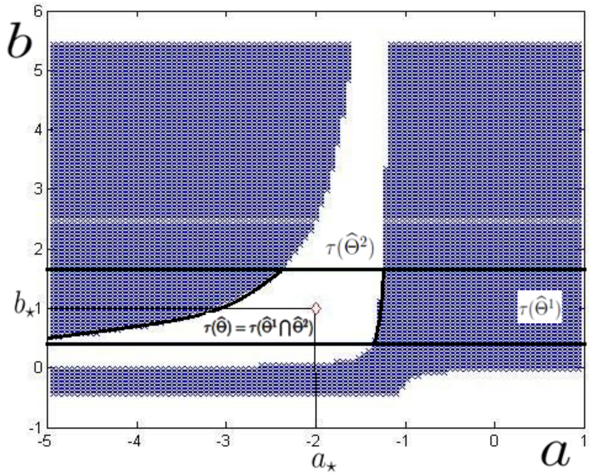

Fig. 2. Confidence set $\tau(\widehat{\Theta})$.

Fig. 2 shows the regions $\tau(\widehat{\Theta})$, $\tau(\widehat{\Theta}^1)$, and $\tau(\widehat{\Theta}^2)$ obtained from the simulation with true values $a_\star = -2$ and $b_\star = 1$, and characteristics of noise and stabilizing feedback like those in Section II of [Amelin and Granichin, 2012]: $\{v_t\}$ is i.i.d. sequence of normally distributed noise with mean 0.5 and dispersion 0.1 (bias noise).

2 Prediction of a Random Process Observed Under Arbitrary Bounded Disturbances

We will confine ourselves to considering the following problem statement: a scalar signal is observed that satisfies the equation

$$y_t = \varphi_t^{\mathrm{T}} \theta_t + v_t, \qquad (8)$$

which is a mixture of the transformed vector process $\{\theta_t\}$, $\theta_t \in \mathbb{R}^s$, and the observation noise $\{v_t\}$. Here, $\{\varphi_t\}$ are s-dimension vectors which are known at the time instant t. The vector process $\{\theta_t\}$ is generated by a linear filter

$$\theta_{t+1} = \mathrm{A}\theta_t + w_{t+1}, \qquad (9)$$

in which A is the known matrix:

$$\| \mathrm{A} \| = \sqrt{\lambda_{\max}(\mathrm{A}\mathrm{A}^{\mathrm{T}})} \leq 1,$$

and $\{w_t\}$ is a realization of a sequence of zero-mean independent random vectors. (Hereinafter, $||\cdot||$ is used to mean a norm, $\lambda_{\max}(\mathrm{A})$ and $\lambda_{\min}(\mathrm{A})$ are the maximum and minimum eigenvalues of the matrix A).

The main conditions at which the main results will be formulated are the following:

B1. The inputs $\{\varphi_t\}_{t\geq 1}$ represent a sequence of independent identically distributed random vectors with bounded *known* mathematical expectations: $\| E\varphi_t \| \leq M_\varphi < \infty$, $\forall t$ vectors φ_t do not depend on the random values $w_1,\dots,w_t$ and on $v_1,\dots,v_t\}$, if $v_1,\dots,v_t$ are random. The random vectors

$$\Delta_t = \varphi_t - E\varphi_t$$

have symmetric distribution functions $\mathrm{P}(\cdot)$ with the covariance matrices that satisfy the conditions

$$E\Delta_t\Delta_t^{\mathrm{T}} = \mathrm{B} > 0, \quad \| \mathrm{B} \| \leq \sigma_\Delta^2 < \infty$$

and the bounded fourth statistical moment:

$$E \| \Delta_t \|^4 = M_4^4 < \infty.$$

Hereinafter, $\mathrm{B} > 0$ means that B is a positive definite matrix.

B2. $\forall\, t$ the random vectors w_{t+1} are independent and zero-mean ($\mathrm{E}\{w_{t+1}\} = 0$) that satisfy the condition

$$E w_t w_t^{\mathrm{T}} \leq \mathrm{Q}_w \leq \sigma_w^2 \mathrm{I} < \infty.$$

B3. The sequence of the observation noises $\{v_t\}_{t\geq 1}$ represents either the values of a deterministic unknown bounded function $|v_t| \leq C_v$, $t = 1,2,\dots$, or $\forall\, t$ it is a realization of random vectors which are independent together with Δ_t and bounded in quadratic mean

$$Ev_t^2 \leq C_v^2 < \infty.$$

The problem of *filtering with one-step prediction* consists in designing the estimate $\widehat{\theta}_{t+1}$ of the value of the process $\{\theta_t\}$ at the time instant $t+1$ based on the observations y_k, φ_k, $k \leq t$. The quality of filtering is determined by the mean value of the squared prediction errors

$$E \| \hat{\theta}_{t+1} - \theta_{t+1} \|^2 .$$

It is usually supposed that the vectors $\{\varphi_t\}$ in the observation model are specified by a deterministic sequence. Here, we assume that the sequence of the vectors $\{\varphi_t\}$ is random and satisfies condition **B1**.

Under this assumption Procedure (8) of the measurement of the process θ_t is, in fact, randomized since the sought signal is "weighed" with a randomly chosen set of coefficients φ_t whose values are known at a current moment.

Consider the following algorithm for the next estimate generation

$$\widehat{\theta}_{t+1} = \mathrm{A}\widehat{\theta}_t - \alpha\mathrm{A}\Gamma\Delta_t(\varphi_t^{\mathrm{T}}\widehat{\theta}_t - y_t),\ \Delta_t = \varphi_t - E\varphi_t, \quad (10)$$

where $t = 0,1,\dots,$ $\alpha > 0$ is the step-size value and Γ is a positive definite symmetrical matrix.

The initial data $\widehat{\theta}_0$ are assumed to be given by an arbitrary nonrandom vector from $\mathbb{R}^s$. Algorithm (16) is called randomized because the current measurement is carried out with a randomized input (a set of weights), and the current estimate is changed in a randomized direction $\mathrm{A}\Gamma\Delta_t$.

Substituting (8) and (9) into (16), for the prediction error we get

$$\widehat{\theta}_{t+1} - \theta_{t+1} = \mathrm{A}(\mathrm{I} - \alpha\Gamma\Delta_t\Delta_t^{\mathrm{T}})(\widehat{\theta}_t - \theta_t) -$$

$$-\alpha\mathrm{A}\Gamma\Delta_t(E\varphi_t^{\mathrm{T}}(\widehat{\theta}_t - \theta_t) - v_t) - w_{t+1}.$$

Let us denote $D_t := \|\widehat{\theta}_{t+1} - \theta_{t+1}\|^2$ and $\mathscr{F}_{t-1}$ is a σ-algebra of $\{w_1,\dots,w_{t-1},v_1,\dots,v_{t-1},\Delta_1,\dots,\Delta_{t-1}\}$ Under the assumptions made above, in view of independence of the random vectors Δ_t and w_{t+1}, averaging conditionally with respect to the prehistory of all random processes up to the instant of time t except w_t and using the assumption **B2**, we deduce that

$$E\{D_t|\mathscr{F}_{t-1},\Delta_t\} = \|\mathrm{A}(\mathrm{I} - \alpha\Gamma\Delta_t\Delta_t^{\mathrm{T}})(\widehat{\theta}_t - \theta_t) -$$

$$-\alpha\mathrm{A}\Gamma\Delta_t(E\{\varphi_t^{\mathrm{T}}(\widehat{\theta}_t - \theta^n) - v_t|\mathscr{F}_{t-1},\Delta_t\})\|^2 + s\sigma_w^2.$$

Because of symmetry of the distribution for $\mathrm{P}(\cdot)$ we have

$$(\widehat{\theta}_t - \theta_t)^{\mathrm{T}} E\{(\mathrm{I} - \alpha\Delta_t\Delta_t^{\mathrm{T}}\Gamma)\mathrm{A}^{\mathrm{T}}\alpha\mathrm{A}\Gamma\Delta_t|\mathscr{F}_{t-1}\}\times$$

$$\times(E\{\varphi_t^{\mathrm{T}}(\widehat{\theta}_t - \theta_t) - v_t|\mathscr{F}_{t-1}\}) = 0.$$

Hence, taking the conditional expectation over $\mathscr{F}_{t-1}$ by virtue the assumption **B1** it may be concluded that

$$E\{D_t|\mathscr{F}_{t-1}\} \leq (1 - 2\alpha\lambda_{\min}(\mathrm{B}\Gamma) + \alpha^2\|\Gamma\|^2 M_4^4)\|\mathrm{A}\|^2 D_{t-1}$$

$$+\alpha^2 E\{(\varphi_t^{\mathrm{T}}(\widehat{\theta}_t - \theta_t) - v_t)^2|\mathscr{F}_{t-1}\}\|\mathrm{A}\Gamma\|^2\mathrm{Tr}[\mathrm{B}] + s\sigma_w^2.$$

Further, after taking unconditional mathematical expectation from the both parts of the last formula, using the assumption **B3** and satisfying the inequality below for any $\rho > 0$

$$2E\varphi_t^{\mathrm{T}}(\widehat{\theta}_t - \theta_t)v_t \le \rho M_\varphi v_t^2 + \frac{M_\varphi}{\rho} D_{t-1},$$

for the mean value of the prediction error, we derive the estimate

$$ED_t \le \psi(\alpha,\rho)ED_{t-1} +$$

$$+\alpha^2(1+M_\varphi\rho)\|\mathrm{A}\Gamma\|^2\mathrm{Tr}[\mathrm{B}]C_v^2 + s\sigma_w^2,$$

where

$$\psi(\alpha,\rho) = (1 - 2\alpha\lambda_{\min}(\mathrm{B}\Gamma) +$$

$$+\alpha^2\|\Gamma\|^2(M_4^4 + (M_\varphi + \frac{1}{\rho})M_\varphi \mathrm{Tr}[\mathrm{B}]))\|\mathrm{A}\|^2. \quad (11)$$

From the last inequality it follows a direct conclusion of the next theorem.

Theorem 4: Assume that the sequences $\{y_t\}$,$\{\varphi_t\}$,$\{v_t\}$,$\{\theta_t\}$ and $\{w_t\}$ are related by Equations (8) and (9), $\alpha > 0$, Γ is a positive definite matrix, and $\hat{\theta}^0$ is an arbitrary nonrandom vector from $\mathbb{R}^s$.

If the assumptions **B1–B3** are satisfied ***then*** for the prediction errors of the estimates $\{\widehat{\theta}_t\}$ generated by algorithm (16) for any $\rho > 0$ and a sufficiently small α such that $\psi(\alpha,\rho) < 1$, the following inequality are satisfied

$$E\parallel \widehat{\theta}_{t+1} - \theta_{t+1} \parallel^2 \le \psi(\alpha,\rho)^t E \parallel \hat{\theta}^0 - \theta^0 \parallel^2 +$$

$$+\frac{r\sigma_w^2 + \alpha^2(1+M_\varphi\rho)\parallel \mathrm{A}\Gamma \parallel^2 \mathrm{T}r[\mathrm{B}]C_v^2}{1-\psi(\alpha,\rho)}, \quad (12)$$

$t = 0,1,\dots.$

Note, that the result of Theorem 4 is an accurate in the sense that in typical cases inequality (12) transforms into the equation when inequalities are replaced with equalities in the Theorem 4 conditions.

The first term of Inequality (12) right side shows a contribution of uncertainty about the initial data and tends to zero exponentially with time.

It is interesting to analyze the second term. Boundedness of disturbances, which is usually assumed in minimax filtering problems, leads to results whose accuracy is proportional to specific sizes of an uncertainty set. Inequality (12) shows the unexpected novel feature of the randomized algorithm (to be more exact, the algorithm with a randomized measurement process when the current estimate is changed in the chosen random direction). If the upper bound σ_w^2 of variance of uncontrollable part of the process under study is sufficiently small then it is possible to obtain small prediction errors as compared with the level of an observation noise C_v.

3 Randomized Control for Small UAV Under Unknown Arbitrary Wind Disturbances

Consider a simplified model of the UAV flying. We assume that it moves in a horizontal plane with a velocity of a and it is pushed by a wind with mean velocity b to the direction θ. Data from GPS receiver comes into a system trough the time interval δ, i. e. at time instant $T_t = T_0 + \delta t$ system gets a pair of numbers $(\bar{x}_t, \bar{y}_t)$ which are measuring of the current position of (x_t, y_t) with some error (err_{x_t}, err_{y_t}).

To control the UAV, i. e. to synthesize a sequence of control actions on the actuators $\{u_t\}$, it would be better to estimate the unknown parameter $\theta_\star \in \mathbb{R}$ (the wind direction) based on current measurements $\{(\bar{x}_t, \bar{y}_t)\}$.

More precisely, let (A,B) be the goal point. At any time instant T_t UAV is in the point (x_t, y_t). We assume that the elevators, ailerons, and the direction can change the UAV motion course to the angle u and keeping it constant throughout the time from T_t till T_{t+1}. At this time interval the UAV motion in the direction of u interferes with a wind with the constant speed average b and an angle θ_{t+1}. Assuming a constant wind direction $\theta \equiv \theta_t$, a series of successive measurements allows to get the accurate assessment of the wind direction. The optimal control is determined by the equation:

$$a\sin u - b\sin\theta = 0, \quad (13)$$

when UAV moves directly from the point (x_t, y_t) to (A,B) (see Fig. 3). It follows easy from (13) that

$$u = \arcsin\left(\frac{b}{a}\sin\theta\right). \quad (14)$$

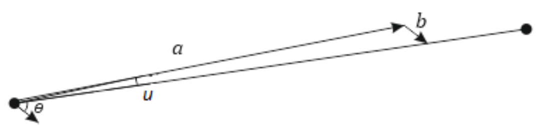

Figure 3. Goal direction, course, wind shift.

From the practical point of view the case $a \gg b$ is very interesting. For example, in the case of a glider discussed in [Amelin, 2010] cruising speed is $a \approx 20$ m/s, and the range of wind velocities at which the glider is commonly used is from 0 till 7 m/s (usually 2-3 m/s). In this case we have

$$\sin u \approx u,$$

and the optimum angle of the course is

$$u = \frac{b}{a}\sin\theta = \mathscr{U}(\theta). \tag{15}$$

In the case of changes wind directions the random nature of the wind angle variation is often assumed:

$$\theta_{t+1} = \theta_t + w_{t+1}, \tag{16}$$

where $\{w_t\}$ are independent, zero-mean and identically distributed random variables:

$$E\{w_t\} = 0,\ E\{w_t^2\} = \sigma_w^2 < \infty. \tag{17}$$

$$E\{w_i, w_j\} = 0,\ \ i \neq j.$$

In order to optimize moving to the goal point *it is required* at the time T_t to design an estimation (prediction) algorithm for θ_{t+1} based on observations $\{(\bar{x}_i, \bar{y}_i)\}_{i=0}^{t}$ and previously chosen control actions u_i, $i = 0, \ldots, t-1$, which minimizes the mean square deviation

$$E\left\{\left(\theta_{t+1} - \widehat{\theta}_{t+1}\right)^2\right\} \to \min.$$

The estimation algorithm design. Using the above theoretic results we consider the randomized approach to synthesis of control actions $\{u_t\}$ and compare it with the algorithm without randomization based on the traditional Kalman filter [Levy, 1997].

For the evaluation of emerging of on-board navigation system errors a small inertial system together with the data of GPS receiver is used.

At the beginning of each iteration the course u_t is chosen and zero value for the gyro is set in this direction. When UAV "is demolished" from the chosen direction the inertial system "adjusts" the control mechanisms on the interval $[T_t, T_{t+1}]$ so that to compensate this effect. But anyway at the end of the interval $[T_t, T_{t+1}]$ a discrepancy value ε sets when the new GPS data comes and a new goal direction is recalculated. As a result the gyro "adjusted" again, and the magnitude of this angle is taken as the "residual" to clarify the current estimate $\hat{\theta}_{t+1}$ (see Fig. 4).

It is necessary to note that, if the UAV course "adjusted" on the whole interval $\{T_t, T_{t+1}\}$ depending on

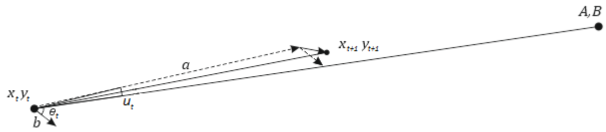

Figure 4. "Residual" in the chosen course.

the deviation from the desired trajectory of a gyroscope then it is not entirely correct to say that θ_t is the angle of the wind direction. More precisely, θ_t is the angle correcting exposure determined by the wind.

The new observation data $(\bar{x}_t, \bar{y}_t)$ form GPS system allows to calculate the "residual"

$$\varepsilon_t = a\sin u_t - b\sin\theta_t + v_t, \tag{18}$$

with some noise v_t which is determined an inaccuracy of observations $\bar{x}_{t+1}, \bar{y}_{t+1}, \bar{x}_t, \bar{y}_t$

If we assume that $a \gg b$ and admissible u_t is small enough: $\sin u_t \approx u_t$, then we obtain the observation model

$$\varepsilon_k = a u_t - b\sin\theta_t + v_t, \tag{19}$$

which "suggests" the following randomized algorithm:

1. To choose a sequence (called as earlier randomized trial perturbation) Δ_n of independent, identically distributed (i.i.d.) random variables which are equal to $\pm\beta$ with same probabilities $\frac{1}{2}$.
2. To form control action u_t by the rule:

$$\begin{cases} u_t = \bar{u}_{t-1} + \Delta_t, \\ \widehat{\theta}_{t+1} = \widehat{\theta}_t - \alpha\Delta_t\varepsilon_t, \\ \bar{u}_t = \mathscr{U}(\widehat{\theta}_{t+1}), \end{cases} \tag{20}$$

where $\bar{u}_0 = 0$, $\widehat{\theta}_0 = 0$, $\alpha, \beta > 0$ are constants, $\mathscr{U}(\cdot)$ is determined by (15).

The prediction algorithm for $\widehat{\theta}_{t+1}$ coincides with the randomized algorithm (16) with $\mathrm{A} = \Gamma = 1$, and for the u_t syntheses, in fact, encouraged to use the strategy (2) with $s = l = N_\Delta = 1$ and $\mathscr{U}_t(\cdot) = \mathscr{U}(\widehat{\theta}_{t+1})$ from (15).

4 Simulation

The efficiency of the algorithm (20) was tested by simulations in comparison with the estimates which are delivered by Kalman filter (see [Kalmal and Bucy, 1961])

$$\begin{cases} u_t = \bar{u}_{t-1}, \\ \widehat{\theta}_{t+1} = \widehat{\theta}_t - K_t\varepsilon_t, \\ \bar{u}_t = \mathscr{U}(\widehat{\theta}_{t+1}), \end{cases} \tag{21}$$

$$K_t = \frac{\gamma_{t-1}}{\frac{1000}{3} + \gamma_{t-1}\varphi_{t-1}^2},$$

$$\gamma_t = \gamma_{t-1} - \frac{\varphi_{t-1}^2 \gamma_{t-1}^2}{\frac{16}{3} + \gamma_{t-1}\varphi_{t-1}^2} + \frac{64}{3},\ \gamma_0 = 0,$$

and by LMS algorithm (see [Ljung, 1999])

$$\begin{cases} u_t = \bar{u}_{t-1}, \\ \widehat{\theta}_{t+1} = \widehat{\theta}_t - \alpha\varepsilon_t, \\ \bar{u}_t = \mathscr{U}(\widehat{\theta}_{t+1}). \end{cases} \quad (22)$$

Measurements in the computer simulation were performed with an interval of time $\delta = 2$ c as an average delay time GPS module data update, the process θ_t is observed in the time interval from 1 till 250; $\sigma_w = 8/\sqrt{3}$. Flight plan is $x_0 = 0, y_0 = 0, A = 5000, B = 5000$. The initial wind angle is $\theta_0 = 3^0$ with respect to the initial flight course. The selected coefficients are $\alpha = 0.1$, $\beta = 0.01$.

Figures 5–7 show the comparative behavior of path trajectories corresponding to the three above mentioned algorithms in typical cases for three different kinds of noises

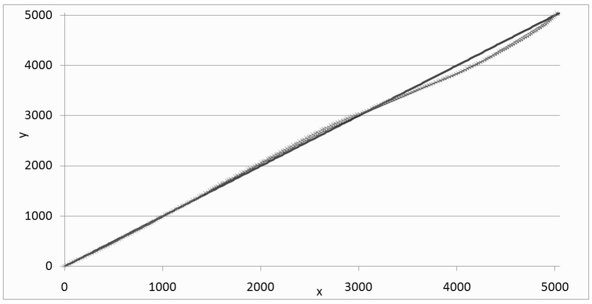

Figure 5. Paths under random noises $v_t = 10 \cdot (rand() \cdot 4 - 2)$.

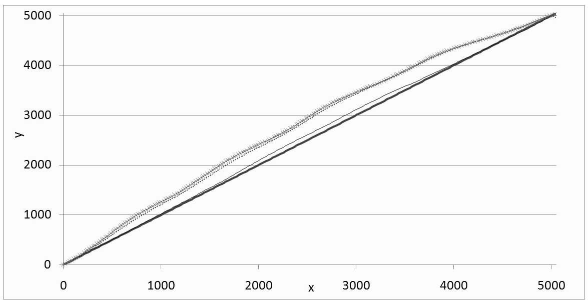

Figure 6. Paths under irregular noises $v_t = 0.1 \cdot \sin(t) + 19 \cdot \mathrm{sign}(50 - t \mod 100)$.

It is known that the Kalman filter (21) gives optimal estimates in the case of the Gaussian independent noise in the observation. Method (22) is sufficiently effective under zero-mean independent disturbances, what is why the behavior of the estimates is generated by algorithms (21), (22) is good under zero-mean random

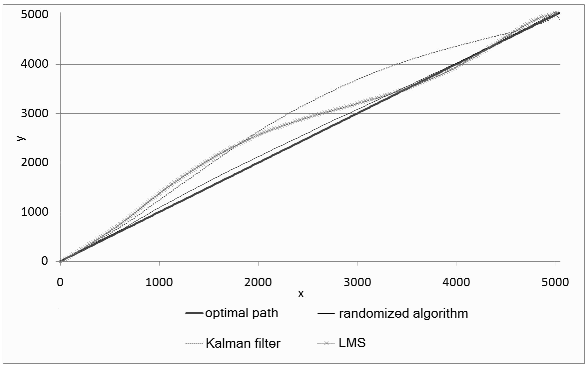

Figure 7. Paths under a constant noise $v_t = 20$.

disturbances despite a high level of noise in the observations (see Fig. 5). In the situations with an unknown constant noise or under a zero mean but not "rich" enough the mean values of the errors of algorithms (21) and (22) are comparable with a squared noise level (see Fig. 6, 7). At the same time the average level of errors for estimates of the randomized algorithm (20) is almost the same in all three situations, and it is several times better than the squared noise level. Table 1 shows the final results for the mean error values in typical computer experiments.

Table 1.

Noise v_t	(20)	(21)	(22)
$10 \cdot (\mathrm{rand}() \cdot 4 - 2)$	41.36	38.15	42.65
$0.1 \cdot \sin(t) +$ $+19 \cdot \mathrm{sign}(50 - t \mod 100)$	55.40	197.64	212.45
20	45.15	276.35	199.48

5 UAV for Multi-agent Group

In the future work we plan to use above theoretical results in our practical project: multi-agents group of UAV [Amelin, 2010], [Amelin, 2011].

For our UAV-agent we use a model of the lung glider "PAPRIKA". It is 1.2 m in length, 2 m wing span, 2-2.1 kg max take of weight, 600 g payload, 40–120 km/h velocity and 200 km range. We use the control system architecture with three levels.

Autopilot "Paparazzi" is a lower level. Autopilot is a set of devices with a microcontroller with real-time system. The main task of the autopilot is to control the actuators (servos, engine, additional equipment) based on given flight program and data from sensors (inertial, infrared sensors, pressure and velocity sensors, *etc*). On the middle layer the microcomputer Gumstix is used. It is $17mm \times 58mm \times 4.2mm$ sizes, Linux operating system, ARM Cortex-A8 processor with 600 Mhz clock frequency, 256 MB RAM and 256 MB NAND Flash. Microcomputer is the main on board device in the control system of UAV-agent. Interoperability between the main microcomputer and autopilot is organized by SIP. The upper level is a base station.

Connection between microcomputers of different UAVs carried out due to FM radio with a frequency of 2.4 GHz and the communication protocol 802.11 n

(Wi-Fi) which uses technology that connects the two nearest channels into one. Thus microcomputers in the UAVs will be able simultaneously receive and send information to each other. Communication with the base station carried out due to separate channel, or via GPRS over GSM modem. A GSM modem can be easily integrated with a microcomputer, but data packets should be compressed.

Due to the small UAVs weight the takeoff is carried out from human hands or from a catapult. Landing is carried out either through the built-parachute or due to "takeover of control" of the operator to manual control.

One of the important topic for the development of control UAVs programs is an optimization flight algorithms. One of possibilities is to use above described randomized algorithms. Other way is to accumulate energy and increase the flight range by using the thermal updrafts which are formed in the lower atmosphere due to disruption of warm air from the surface when it is heated by sunlight [Antal, Granichin and Levi, 2010].

6 Conclusion

This paper presents new approaches for constructing a confidence set of unknown parameters of a linear scalar control plant and for the prediction of varying parameters. From the theoretical point of view an important feature of suggested procedures is that they operate without any significant assumptions about the external noise. It is also of vital importance from the practical point of view since in practical applications when it is difficult to obtain a priori knowledge about the noise characteristics.

References

Amelin, K.S. (2010) Small UAV for the autonomous group *Stochastic Optimization in Informatics*, **6**, pp. 117–126.

Amelin, K.S. (2011) Software engineering of unmanned aerial vehicle for mobile autonomous group *Stochastic Optimization in Informatics*, **7**, pp. 93–115.

Amelin, K.S., and Granichin, O.N. (2011) Potential of randomization in a Kalman-type prediction algorithm at arbitrary external noise in observations. *Gyroscopy and Navigation*, **2**(3), pp. 277–284.

Amelin, K.S., and Granichin, O.N. (2012) Randomized controls for linear plants and confidence regions for parameters under external arbitrary noise. *In Proc. of the 2012 American Control Conference*, Montreal, Canada, pp. 851–856.

Antal, C., Granichin, O., and Levi, S. (2010) Adaptive autonomous soaring of multiple UAVs using SPSA. *In Proc. of the 49th IEEE Conference on Decision and Control*, Atlanta, GA, USA, pp. 3656–3661.

Bai, E.W., Nagpal, K.M., and Tempo, R. (1996) Bounded-error parameter estimation: Noise models and recursive algorithms. *Automatica*, **32**, pp. 985–999.

Campi, M.C., and Weyer, E. (2010) Non-asymptotic confidence sets for the parameters of linear transfer functions. *IEEE Trans. Automat. Control*, **55**(12), pp. 2708–2720.

Feldbaum, A.A., (1960) Dual Control Theory I-IV, *Automation and Remote Control*, **21**, pp. 874–880, 1033–1039, **22**, pp. 1–12, 109–121, 1961.

Fisher, R.A. (1935) *The Design of Experiments*. Oliver and Boyd, Edinburgh.

Fomin, V.N. (1985) *Methods of Control of Linear Discrete Objects*, Leningrad: Leningrad university publisher.

Garulli, A., Giarre, L., and Zappa, G. (2002) Identification of approximated Hammerstein models in a worst-case setting. *IEEE Trans. Autom. Control*, **47**(7), pp. 2046–2050.

Granichin O.N. (1988) A stochastic approximation algorithm with input perturbation for identification of a static nonstationary discrete object. *Vestnik Leningrad University: Mathematics (Vestnik Leningradskogo Universita. Matematika)*, **21**(3), pp. 56–58.

Granichin O.N. (1989) A stochastic recursive procedure with correlated noises in the observation, that employs trial perturbations at the input. *Vestnik Leningrad University: Mathematics (Vestnik Leningradskogo Universita. Matematika)*, **22**(1), pp. 27–31.

Granichin O.N. (1992) Unknown function minimum point estimation under dependent noise. *Problems of Information Transmission*, **28**(2), pp. 16–20.

Granichin, O.N. (2004) Linear regression and filtering under nonstandard assumptions (Arbitrary noise). *IEEE Trans. on Automat. Contr*, **49**, pp. 1830–1835.

Granichin, O.N. (2012) The nonasymptotic confidence set for parameters of a linear control object under an arbitrary external disturbance. *Automation and Remote Control*, **73**, no. 1, pp. 20–30.

Granichin, O.N., and Fomin, V.N. (1986) Adaptive control using test signals in the feedback channel, *Automation and Remote Control*, **47**, no. 2, part 2, pp. 238–248.

Bucy, R.S., and Kalman, R.E. (1961) New results in linear filtering and prediction theory. *J.Basic.Eng. ASME*, **83**, no. 1, pp. 95–108.

Levy, L.J. (1997) The Kalman filter: navigation's integration workhorse. *GPS World*, **8**, no. 9, pp. 65–71.

Ljung, L. (1999) *System Identification – Theory for the User. 2nd ed. Englewood Cliffs*, NJ: Prentice Hall.

Polyak, B.T., and Sherbakov, P.S. (2002) *Robust Stability and Control.* Moscow: Nauka.

Polyak, B.T., and Topunov, M.V. (2008) Suppression of bounded exogenous disturbances: output feedback. *Autom. and Remote Control*, **69**(5), pp. 801–818.

Vakhitov, A., Granichin, O., and Vlasov, V. (2010) Adaptive control of SISO plant with time-varying coefficients based on random test perturbation. *In Proc. of the 2010 American Control Conference*, Baltimore, MD, USA, pp. 4004–4009.