OPTIMAL PERIODIC MOTIONS OF SYSTEMS WITH INTERNAL MASSES IN RESISTIVE MEDIA

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Abstract

The motion of a body controlled by movable internal masses in a resistive environment along a horizontal straight line is considered. Optimal periodic modes of motion are constructed for the internal masses to maximize the average speed of the velocity-periodic motion of the body. The maximum displacement allowed for the internal masses inside the body, as well as the relative velocities or accelerations of these masses are subjected to constraints. Three types of the resistance laws—piece-wise linear friction, quadratic friction, and Coulomb's dry friction—are considered.

Key words

Systems with internal masses, mobile microrobots, periodic motion, resistive media, friction, control, optimization

1 Introduction

A rigid body with internal masses that perform periodic motions can move progressively in a resistive medium with nonzero average velocity. This phenomenon can be used as a basis for the design of mobile systems able to move without special propelling devices (wheels, legs, caterpillars or screws) due to direct interaction of the body with the environment. Such systems have a number of advantages over systems based on the conventional principles of motion. They are simple in design, do not require gear trains to transmit motion from the motor to the propellers, and their body can be made hermetic and smooth, without any protruding components. The said features make this principle of motion suitable for capsule-type microrobots [Li, Furuta and Chernousko, 2006] designed for motion in strongly restricted space (e.g., inside narrow tubes) and in vulnerable media, for example, inside a human body for delivering a drug or a diagnostic sensor to an affected organ. Such systems can be driven to a prescribed position with high degree of accuracy, which enables them to be utilized in high-precision positioning systems in scanning electron and tunnel microscopes, as well as in micro- and nano-technological equipment [Breguet and Clavel, 1998; Schmoeckel and Worn, 2001; Vartholomeos and Papadopoulos, 2006].

Automatic transport systems moving due to periodic motion of internal masses are sometimes referred to as vibration-driven systems or vibration-driven robots. Some issues of the dynamics and parametric optimization of vibration-driven systems have been studied, e.g., in [Nagaev and Tamm, 1980; Gerasimov, 2003; Bolotnik et al., 2006].

At the Institute for Problems in Mechanics of the Russian Academy of Sciences, vibration-driven minirobots for motion inside small-diameter tubes were designed [Gradetsky et. al, 2003].

Chernousko has initiated a systematic study in control and optimization of motion of systems with internal movable masses [Chernousko, 2002; Chernousko, 2005; Chernousko, 2006]. He solved a number of parametric optimization problems for two-body systems moving along a dry rough surface or in a viscous medium. Both velocity-controlled and acceleration-controlled motions of the internal mass were considered, the structure of the control law being prescribed. An optimal control problem for a two-mass system moving along a dry rough plane, with the structure of the control law unknown in advance, was solved in [Figurina, 2007].

The present paper continues the studies in the optimization. Some simplifying restrictions that were imposed previously on the motion to be optimized are removed. In addition, an optimal control problem is solved for the motion of a rigid body with two internal masses along a dry rough plane. One of the masses moves horizontally along a straight line parallel to the line of motion of the body, while the other mass moves vertically. The vertically moving mass provides an additional possibility for the control of friction between the body and the supporting plane due to the change in the normal pressure force.

2 Two-body System

In this section, we consider a two-body system consisting of the main body and the internal body that can move relative to the main body along a straight line. The motion of the system along a horizontal straight line in resistive media is studied for various laws of friction.

2.1 Statement of the Problem

Consider a system of two interacting rigid bodies, the main body of mass M and the movable internal body of mass m (Fig. 1). In what follows we will refer to the main body and the internal body as body M and mass m. Body M interacts with a resistive environment. We will study periodic motions of mass m relative to body M under which the entire system moves progressively in the environment.

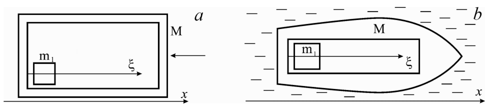

Figure 1. Two-mass system in a resistive medium

Let x denote the coordinate measuring the displacement of body M relative to the environment; ξ the coordinate measuring the displacement of mass m relative to body M; $v = \dot{x}$ the absolute velocity of body M; $u = \dot{\xi}$ the relative velocity of mass m; and $w = \dot{u}$ the relative acceleration of mass m.

We will confine ourselves to simple periodic motions of mass m such that during each period T, this mass first moves with a velocity u_1 from the left-hand extreme position $\xi = 0$ to the right-hand extreme position $\xi = L$ and then returns to the initial position with a velocity u_2. Positive parameter L characterizes the limits within which mass m is allowed to move relative to body M. This law of motion can be written as

$$u(t) = \begin{cases} u_1, & 0 \le t \le \tau, \\ -u_2, & \tau \le t \le T, \end{cases} \tag{1}$$

$$w(t) = u_1\delta(t) - (u_1 + u_2)\delta(t-\tau) + u_2\delta(t-T), \tag{2}$$

where

$$\tau = L/u_1, \quad T = L(u_1^{-1} + u_2^{-1}), \tag{3}$$

and $\delta(\cdot)$ is Dirac's delta function.

The motion of body M is governed by the equations

$$\begin{gathered} \dot{x} = v, \quad \dot{v} = -\mu w - r(v), \\ \mu = m/(M+m), \quad r(v) = -R(v)/(M+m), \\ vr(v) \ge 0, \end{gathered} \tag{4}$$

where $R(v)$ represents the resistance force applied to body M by the environment.

We will seek for the optimal parameters u_1 and u_2 such that the corresponding velocity-periodic motion of body M occurs with maximum average velocity. Thus we arrive at the optimization problem.

Problem 1. For the system of Eqs. (1)–(4), subject to the boundary conditions

$$x(0) = 0, \quad \dot{x}(0) = \dot{x}(T), \tag{5}$$

find the parameters u_1 and u_2 that satisfy the constraints

$$0 \le u_i \le U, \quad i = 1,\ 2 \tag{6}$$

and maximize the average velocity of mass M

$$V = x(T)/T. \tag{7}$$

The solve this problem use the following algorithm:

1. Substitute $w(t)$ of Eq. (2) into Eq. (4).
2. Solve the resulting equation subject to the initial conditions $x(0) = 0$, $\dot{x}(0) = v_0$ to obtain $x = x(t; u_1, u_2, v_0, L)$.
3. Find the initial velocity $v_0 = v_0^*$ using the periodicity condition

$$x(0; u_1, u_2, v_0, L) = x(T; u_1, u_2, v_0, L) \tag{8}$$

and the definition of Eq. (3) for T.
4. Substitute $x(T; u_1, u_2, v_0^*, L)$ into Eq. (7) to obtain

$$V = V(u_1, u_2, L). \tag{9}$$

5. Maximize the function $V = V(u_1, u_2, L)$ with respect to u_1 and u_2, subject to the constraints of Eq. (6).

In the subsequent subsections we will solve Problem 1 for three types of the resistance law $r(v)$. Piece-wise linear resistance, quadratic resistance, and Coulomb's friction will be considered.

2.2 Piece-wise Linear Resistance

The piecewise-linear resistance (anisotropic linear friction) is characterized by the law

$$r(v) = \begin{cases} k_+ v, & v \ge 0, \\ k_- v, & v < 0, \end{cases} \tag{10}$$

where k_- and k_+ are positive coefficients. The particular case $k_+ = k_-$ corresponds to linear viscous friction.

For $k_+ = k_- = k$, velocity-periodic motion of body M with nonzero average velocity is impossible for any periodic motion of mass m. To prove this, integrate Eq. (4) for v with respect to t from 0 to T to obtain

$$v(T) - v(0) = -\mu[u(T) - u(0)] - k[x(T) - x(0)]. \tag{11}$$

This relation implies that $x(0) = x(T)$ if the functions $v(t)$ and $u(t)$ are T-periodic; hence, $V = 0$.

For arbitrary k_- and k_+, the function V of Eq. (9) is given by

$$\begin{gathered} V = \frac{\mu(1-e_1)(1-e_2)(k_- - k_+)u_1u_2}{(1-e_1e_2)Lk_-k_+}, \\ e_1 = \exp(-k_-Lu_1^{-1}),\ e_2 = \exp(-k_+Lu_2^{-1}). \end{gathered} \tag{12}$$

From this expression it follows that $V > 0$ ($V < 0$) for $k_- > k_+$ ($k_- < k_+$). This means that body M moves on the average in the direction of the lower resistance.

The maximum magnitude of the function V of Eq. (12) occurs for $u_1 = u_2 = U$. In this case, $\tau = L/U = T/2$, in accordance with Eq. (3). Therefore, in the optimal mode, the internal mass moves in both directions with the maximal speed U, each stroke taking a half-period.

2.3 Quadratic Resistance

The quadratic resistance is characterized by

$$r(v) = \kappa|v|v, \tag{13}$$

where κ is a positive coefficient.

For this case, Eq. (8) can be reduced to the quadratic equation

$$\begin{gathered} (u_2 - u_1 + \kappa LY)Z^2 + (\kappa LY^2 - 2u_1Y)Z = u_1Y^2; \\ Z = v_0 - \mu u_1, \quad Y = \mu(u_1 + u_2). \end{gathered} \tag{14}$$

The initial velocity v_0^* is expressed by $v_0^* = Z^* + \mu u_1$, where Z^* is the solution of the quadratic equation. To simplify the calculations, we assume $v_0^* = 0$. For this assumption, the parameters u_1 and u_2 become related by

$$u_2 = (1 - \mu\kappa L)(1 + \mu\kappa L)^{-1}u_1. \tag{15}$$

We assume that $\mu\kappa L < 1$. If this condition is violated, the mode of motion with $v_0^* = 0$ does not occur.

The calculation of the average velocity of body M in accordance with Eq. (9) leads to the expression

$$V = -\frac{u_1u_2 \ln\left(1 - \mu^2\kappa^2L^2\right)}{\kappa L(u_1 + u_2)}. \tag{16}$$

The maximization of this function subject to the constraints of Eqs. (6) and (15) gives

$$\begin{aligned} u_1 &= U, \quad u_2 = (1 - \mu\kappa L)(1 + \mu\kappa L)^{-1}U \\ V_{\max} &= -\frac{U(1 - \mu\kappa L)}{2\kappa L} \ln\left(1 - \mu^2\kappa^2L^2\right). \end{aligned} \tag{17}$$

The quantity $V_{\max}$ is positive and, hence, for the quadratic resistance law, the progressive velocity-periodic motion of the system is possible even in the isotropic case, in contrast to the linear resistance.

2.4 Coulomb's friction

For Coulomb's dry friction, the function $r(v)$ of Eq. (4) is specified by

$$r(v) = \begin{cases} f_+g, & \text{if } v > 0 \text{ or } v = 0,\ \mu w < -f_+g, \\ -f_-g, & \text{if } v < 0 \text{ or } v = 0,\ \mu w > f_-g, \\ -\mu w, & \text{if } v = 0,\ -f_+g \le \mu w \le f_-g, \end{cases} \tag{18}$$

where f_+ and f_- are coefficients of friction that resist forward and backward motion of mass M, respectively; g is the acceleration due to gravity.

Introduce the dimensionless variables

$$\begin{gathered} x_i = u_i/u_0,\ i = 1, 2; \quad u_0 = \sqrt{Lf_-g/\mu}, \\ x_0 = v_0/(\mu u_0), \quad F = V/(\mu u_0), \\ c = f_+/f_-, \quad X = U/u_0. \end{gathered} \tag{19}$$

In terms of these variables, the solution of Problem 1 is reduced to the determination of the optimal values of x_0, x_1, x_2, and F for given X and c. The final result is given by the following expressions:

If $c < 1$ and $X < (c/2)^{1/2}$, **then**
$x_1 = x_2 = X, \quad x_0 = -X,$
$F = (1 - c)X^3/c.$

If $c < 1$ and $X \ge (c/2)^{1/2}$, **then**
$x_1 = x_2 = X, \quad x_0 = -X,$
$F = X - c(1 + c)(4X)^{-1}.$

If $c = 1$ and $X \le (2)^{-1/2}$, **then**
$|x_1| \le X, \quad |x_2| \le X,$
$x_0 = -x_2, \quad F = 0.$

If $c > 1$ and $X \le (2)^{-1/2}c$, **then**
$x_1 = 0, \quad x_2 = 0, \quad x_0 = 0,$
$F = 0.$

If $c \ge 1$ and $X > (2)^{-1/2}c$, **then**
$x_1 = X/c, \quad x_2 = X,$
$x_0 = (X^2 - c^2)(cX)^{-1},$
$F = (2X^2 - c^2)(2cX)^{-1}.$

From these expressions it follows that positive average velocity $V > 0$ of the two-body system can be achieved for any X if $c < 1$, i.e., if the coefficient of friction resisting the forward motion (in the positive direction of the x-axis) is less than the coefficient of friction resisting the backward motion. For $c \geq 1$, positive average velocity may occur only if $X > c/\sqrt{2}$, i.e., the maximum velocity allowed for the relative motion of mass m should be sufficiently large.

It is of interest to have analyzed the character of the optimal motion of body M. From Eq. (4) it follows that, if mass m moves according to Eqs. (1) and (2), the velocity v of body M cannot change in sign in the intervals $(0, \tau)$ and (τ, T). Therefore, this change can occur only at the instant τ. According to Eqs. (2) and (4), at this instant, the velocity v instantaneously increases by $u_1 + u_2$ and, hence,

$$v(t) \leq 0,\ t \in (0, \tau); \quad v(t) \geq 0,\ t \in (\tau, T) \tag{20}$$

For the systems with the piece-wise linear and quadratic resistance laws, considered previously, body M cannot remain in a state of rest on part of the intervals $(0, \tau)$ or (τ, T), which is not the case for systems with Coulomb's dry friction. According to Eq. (2) and (4), the absolute value of v does not increase in the said intervals and, hence, the segments of rest can only be adjacent to the right-hand ends of the respective intervals. Thus, there are four possible modes of motion of body M in the interval $(0, T)$ characterized by

A: no intervals of rest;
B: one interval of rest (t_1, τ);
C: one interval of rest (t_2, T);
D: two intervals of rest, (t_1, τ) and (t_2, T),

where t_1 and t_2 are the initial instants of the intervals of rest, satisfying the inequalities $0 < t_1 < \tau$ and $\tau < t_2 < T$.

It turns out that for $c < 1$, maximum velocity of body M occurs for mode D, if $X < (c/2)^{1/2}$, or for mode B, if $X \geq (c/2)^{1/2}$. For $c \geq 1$ and $X > 2^{-1/2}c$, the maximum velocity occurs for mode A.

3 Three-body System

In this section, an optimal control problem is solved for a three-body system consisting of the main body and two internal masses, one of which moves horizontally along the line parallel to the line of motion of the main body, while the other mass moves vertically. The main body moves along a dry rough plane. The motion of the internal mass along the vertical is used to control the force of friction between the main body and the supporting plane due to the change in the normal pressure force. The results of this section were obtained together with Dr. Figurina.

3.1 Statement of the Optimal Control Problem

Consider a mechanical system distinguished from the system studied in Section 2 by an additional internal mass m_2 that can move vertically. Let ξ_1 denote the displacement of mass m_1 along the horizontal and ξ_2 the displacement of mass m_2 along the vertical. Let body M move along a rough horizontal plane. Coulomb's friction is assumed to act between the body and the plane, the coefficient of friction being independent of the direction of the motion.

Proceed to the dimensional variables, using $M+m_1+m_2$, l, and $\sqrt{l/g}$ as the units of mass, length, and time, respectively. The unit of length l can be chosen arbitrarily, since the model of the system does not involve a characteristic length. In the normalized variables the motion of body M is governed by the equation

$$\begin{gathered}
\dot{x} = v, \quad \dot{v} = -\mu_1 w_1 - r(v), \\
r(v) = \begin{cases} fN\mathrm{sgn}(v), & \text{if } v \neq 0, \\ -\mu_1 w_1, & \text{if } v = 0,\ |\mu_1 w_1| \leq fN, \\ -fN\mathrm{sgn}(w_1), & \text{if } v = 0,\ |\mu_1 w_1| > fN, \end{cases} \\
N = 1 + \mu_2 w_2,
\end{gathered} \tag{21}$$

where

$$\begin{gathered}
\mu_i = m_i/(M + m_1 + m_2), \\
\ddot{\xi}_i = w_i, \quad i = 1,\ 2.
\end{gathered} \tag{22}$$

The quantity N in Eq. (21) represents the normal pressure force exerted on body M by the supporting plane.

We will construct T-periodic motions of masses m_1 and m_2 that satisfy the constraints

$$\begin{gathered}
|w_1| \leq W_1, \quad -W_2^- \leq w_2 \leq W_2; \\
W_2^- = \min\left(1/\mu_2,\ W_2\right), \\
\mu_1 W_1 > f(1 - \mu_2 W_2^-)
\end{gathered} \tag{23}$$

and maximize the average speed of the corresponding velocity-periodic motion of body M. The period T is fixed.

In Eq. (23), positive quantities W_1 and W_2 constrain the magnitudes of the relative accelerations of the internal masses due to limited power of the actuators. The lower bound W_2^- for the relative acceleration of mass m_2 is due to the requirement that body M have permanent contact with the supporting plane. For $\mu_2 w_2 < -1$, the normal pressure force N would have been negative, which is impossible for the unilateral contact. The condition $\mu_1 W_1 > f(1 - \mu_2 W_2^-)$ is necessary for body M to be able to be moved from a state of rest.

Thus we arrive at the optimal control problem:

Problem 2. For the system of Eqs. (21) and (22) considered in the time interval $[0, T]$, find the control functions $w_1(t)$ and $w_2(t)$ that satisfy the constraints of Eq. (23), generate the motion subject to the boundary conditions

$$x(0) = 0, \quad v(0) = 0, \quad v(T) = 0, \\ \xi_i(0) = \xi_i(T) = 0, \quad \dot{\xi}_i(0) = \dot{\xi}_i(T) \tag{24}$$

and maximize the average velocity of body M:

$$V = x(T)/T \to \max. \tag{25}$$

3.2 Solution of the Problem

It has been proven that the optimal control is provided by piece-wise constant functions

$$w_1(t) = \begin{cases} -W_1, & t \in [0, \tau_1), \\ W_1, & t \in [\tau_1, \delta_*), \\ \bar{w}_1, & t \in [\delta_*, T], \end{cases} \\ w_2(t) = \begin{cases} -W_2^-, & t \in [0, \tau_2), \\ W_2, & t \in [\tau_2, \delta_*), \\ \bar{w}_2, & t \in [\delta_*, T]; \end{cases} \tag{26}$$

and that body M moves forward ($v > 0$) for $t \in (0, \delta_*)$ and remains in a state of rest for $t \in [\delta_*, T]$. In the optimal mode, body M never moves backward.

Using these observations, Eq. (21), and the boundary conditions $v(0) = v(\delta_*) = 0$, we express the parameter τ_1 in terms of τ_2 and δ_*:

$$\tau_1 = \frac{\delta_*[U_1 + f(1 + U_2)] - 2f\widetilde{U}_2\tau_2}{2U_1}; \\ U_i = \mu_1 W_i, \quad U_2^- = \mu_2 W_2^-, \\ \widetilde{U}_2 = \frac{U_2^- + U_2}{2}. \tag{27}$$

Using the definitions $\ddot{\xi}_i = w_i$ of Eq. (22), the conditions $\dot{\xi}_i(0) = \dot{\xi}_i(T)$ of Eq. (24), expressions (26) for the control functions, and relations (27), we calculate the constant quantities $\bar{w}_1$ and $\bar{w}_2$:

$$\bar{w}_1 = \frac{f[\delta_*(1 + U_2) - 2\widetilde{U}_2\tau_2]}{\mu_1(T - \delta_*)}, \\ \bar{w}_2 = \frac{2\widetilde{U}_2\tau_2 - U_2\delta_*}{\mu_2(T - \delta_*)}. \tag{28}$$

The average velocity of body M is expressed by

$$V(T) = \frac{U_1^2 - k^2(1 + U_2)^2}{4U_1 T}{\delta_*}^2 + \\ f\widetilde{U}_2\frac{U_1 + f(1 + U_2)}{U_1 T}\tau_2\delta_* - f\widetilde{U}_2\frac{U_1 + f\widetilde{U}_2}{U_1 T}\tau_2^2. \tag{29}$$

The optimal values of τ_2 and δ_* are determined by the maximization of the function V with respect to these parameters under the constraints

$$\delta_*[U_1 + f(1 + U_2)] \le U_1 T + 2f\widetilde{U}_2\tau_2, \\ 2\widetilde{U}_2\delta_* - U_2^- T \le 2\widetilde{U}_2\tau_2 \le U_2 T, \\ (1 + U_2)\delta_* \le \frac{T}{2} + 2\widetilde{U}_2\tau_2, \\ \max\left\{0, \ \frac{\delta_*[f(1 + U_2) - U_1]}{2f\widetilde{U}_2}\right\} \le \tau_2 \le \delta_* \le T. \tag{30}$$

These inequalities are derived from the conditions $0 \le \tau_i \le \delta \le T$, expressions (27) and (28) , and the constraints

$$|\mu_1\bar{w}_1| \le U_1, \quad U_2^- \le \mu_2\bar{w}_2 \le U_2, \tag{31}$$

$$|\mu_1\bar{w}_1| \le f(1 + \mu_2\bar{w}_2), \tag{32}$$

The constraints of Eq. (31) express those of Eq. (23) for the controls of Eq. (26) in the time interval $[\delta_*, T]$. Inequality (32) follows from the condition that body M remains in a state of rest for $t \in [\delta_*, T]$ and Coulomb's friction law $r(v)$ of Eq. (21).

For the optimal controls $w_i(t)$ constructed in accordance with the algorithm described, the functions $\xi_i(t)$, characterizing the optimal motions of the internal masses, are defined by

$$\xi_i(t) = \frac{t}{T}\int_0^T \eta w_i(\eta)d\eta + \int_0^t (t - \eta)w_i(\eta)d\eta. \tag{33}$$

3.3 Limiting Cases

To assess the effect of introducing the internal mass moving vertically on the maximum average speed of the system, solve Problem 2 for two limiting cases, $W_2 = \infty$ and $W_2 = 0$. In the former case, the function $w_2(t)$, which controls mass m_2, is unbounded from above, while in the latter case, $w_2(t) \equiv 0$ and, hence, the motion of the internal mass along the vertical does not occur.

3.3.1 $W_2 = \infty$ In this case, the optimal controls for $t \in [0, T]$ are given by

$$w_1(t) = W_1 \mathrm{sgn}(t - T/2), \\ w_2(t) = {\mu_2}^{-1}[T\delta(t - T) - 1], \tag{34}$$

where $\delta(\cdot)$ is Dirac's delta function.

For this control, body M speeds up from $v = 0$ to $v = \mu_1 W_1 T/2$ with acceleration $\mu_1 W_1$ in the time interval $[0, T/2]$ and slows down to $v = 0$ with acceleration $-\mu_1 W_1$ in the interval $[T/2, T]$. The average velocity over the period T is $V = \mu_1 W_1 T/4$. Mass m_2 moves upward for $t \in (0, T/2)$ and downward for $t \in (T/2, T)$, with the velocity $\dot{\xi}_2$ uniformly decreasing at a rate of μ_2^{-1} from $\mu_2^{-1}T/2$ to $-\mu_2^{-1}T/2$ in the interval $(0, T)$. At the instant T, mass m_2 undergoes an elastic impact to restore the initial velocity. It is important that the motion of mass m_2 with acceleration $-\mu_2^{-1}$ provides zero normal pressure force and, hence, zero friction force between body M and the plane.

3.3.2 $W_2 = 0$

This case was considered in detail in [Figurina, 2007]. It was shown that the average velocity is determined by

$$V = \frac{\mu_1 W_1 T}{16}\left(1 - \frac{f^2}{\mu_1^2 W_1^2}\right) \tag{35}$$

A comparison with the case $W_2 = \infty$ indicates that the activation of mass m_2 enables at least 4-fold increase in the maximum average velocity of body M.

4 Conclusion

Optimal velocity-periodic motions of mobile systems driven by periodic motions of the internal masses were calculated for various laws of resistance of the environment. It was shown that for any constraints on the displacements of the internal masses inside the main body, the maximum average speed of the body can be made arbitrarily high, provided that sufficiently large velocities or accelerations are allowed for the internal masses. For systems moving along a horizontal dry rough plane, the introduction of an internal mass moving vertically can lead to a significant increase in the average velocity due to the control of the normal pressure force. The principle of motion discussed in the paper was successfully implemented experimentally [Gradetsky et. al, 2003; Li, Furuta and Chernousko, 2005; Li, Furuta and Chernousko, 2006]. The experiments demonstrated good agreement with the theoretical predictions.

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