USAGE OF INVARIANT IMMERSION METHOD FOR IDENTIFICATION OF PARAMETERS FOR SYSTEMS WITH LIMITED OUTPUT (HARD LIMITATION IN OBSERVATION EQUATION).

V.M. Ponyatskiy *. I.V. Zamotaev **
V.B. Kisilev **

* Instrument Design Bureau, 1 Shcheglovskaya Zaseka, Tula 300001, Russia,
(e-mail: kbkedr@tula.net, pwmru@rambler.ru)
** Tula State University, 92 Lenin, Tula 300600, Russia

Abstract: In the paper, an approach based on the invariant embedding method for the non-stationary plant identification is considered. Synthesis of continuous and discrete time algorithms of the non-stationary dynamic plant identification is implemented. Program code implementation and testing of the algorithms obtained is done, and recommendations on their adjustment are formed. By use of the algorithms obtained, an estimation of the gain (transfer constant), time constant, and damping decrement of the servodrive and glider of a revolving unmanned aircraft is implemented. For a comparison, an estimated obtained by the linear Kalman filtering is also presented. The account of limitations on output coordinate provides more exact estimation of a transmission factor of the dynamic object.

Keywords: Dynamic Object, Observer, System Identification.

1. INTRODUCTION

The important task at research of the object of handle is the estimation of its characteristics by results of its trials on the basis of the measured informational signals $\mathbf{u}(t)$ and $\mathbf{z}(t)$ (see fig. 1). The task of an estimation of parameters of the object becomes complicated, if are available limited output (hard limitation in observation equation). For example, the estimation of characteristics of the servodrive of the pilotless spinned flying device is carried out on signals of handle and signals from an output of the sensor of angular position of rudders. Thus angular position of rudders hard limitations are imposed. Not the registration of these limitations can lead an inexact estimation of a transmission factor of the servodrive (Ponyatsky, 2008b).

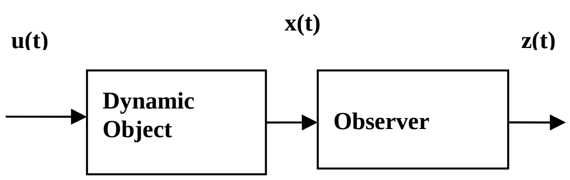


Fig. 1. The Dynamic Object.

Generally the control object can be described by the following system of the nonlinear differential equations:

$$\dot{\mathbf{x}}(t) = f[\mathbf{x}(t),\mathbf{u}(t),t] + G[\mathbf{x}(t),t]\mathbf{w}\,; \qquad (1)$$

$$\mathbf{z}(t) = \mathbf{H}(t)\mathbf{x}(t) + \mathrm{v}\,,$$

where $\mathbf{u}(t)$ is a vector of control actions; $\mathbf{z}(t)$ is a vector of observable signals; $\mathbf{x}(t)$ is an extended state vector including parameters of dynamic objects; $\mathbf{w}$ is a vector of shaping noise of intensity $\boldsymbol{\psi}_{\mathbf{w}}$; $\mathbf{v}$ is a vector of measurement noise of intensity $\boldsymbol{\psi}_{\mathbf{v}}$; $\mathbf{G}[\mathbf{x}(t),t]$ is a matrix of coefficients.

It is necessary on measured entry signals (a vector of handle **U)** and output signals (a vector of observation **z**), at known structure of the object (matrixes **f**, **G)** and a meter (a matrix of observation **H**) **to receive** an estimation of units of matrixes **f, G.**

2. INVARIANT IMBEDDING METHOD

The continuous algorithm of invariant imbedding for a simultaneous estimation of state and parameters of dynamic objects (1) has the following form (Sage and Melsa, 1974).

$$\dot{\mathbf{x}}_o(t) = f[\mathbf{x}_o(t),\mathbf{u}(t),t] + \mathbf{P}(t)\mathbf{H}(t)\Psi_v^{-1}(t)[\mathbf{z}(t) - \mathbf{H}(t)\mathbf{x}_o(t)]; \qquad (2)$$

$$\dot{\mathbf{P}}(t) = \mathbf{G}[\mathbf{x}_o(t),t]\psi_w\mathbf{G}^T[\mathbf{x}_o(t),t] + \mathbf{P}(t)\frac{\partial\mathbf{f}^T[\mathbf{x}_o(t),\mathbf{u}(t),t]}{\partial\mathbf{x}_o} + \mathbf{P}(t)\frac{\partial\mathbf{f}[\mathbf{x}_o(t),\mathbf{u}(t),t]}{\partial\mathbf{x}_o} - \mathbf{P}(t)\mathbf{H}^T(t)\mathbf{H}(t)\mathbf{P}(t),$$

where $\mathbf{x}_o(t)$ is an estimation of the extended state vector; $\mathbf{P}(t)$ is a correlation matrix of the filtering errors.

The algorithm of invariant imbedding for a simultaneous estimation of state $(\,y\,)$ and parameters $(\,K,\omega = 1/T\,)$ of the dynamic object in the form of aperiodic link is defined by the following vectors and matrixes (Ponyatsky, 2007a):

$$\mathrm{x}_o = |\mathrm{x}_{1o}\;\; K_o\;\; \omega_o|^T;\;\; \mathbf{H} = |1\;0\;0|;\;\; \mathbf{z} = z\,;\;\; \mathbf{v} = \nu_1;\;\; \boldsymbol{\psi}_{\mathbf{v}} = \psi_v\,;$$

$$\mathbf{w} = |w_1\;\; w_2\;\; w_3|^T\,;$$

$$\mathbf{G}[\mathbf{x}_o(t),t]=\begin{vmatrix} K_{ro}\omega_{ro} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}; \qquad \psi_{\mathbf{w}}=\begin{vmatrix} \psi_{w1} & 0 & 0 \\ 0 & \psi_{w2} & 0 \\ 0 & 0 & \psi_{w3} \end{vmatrix};$$

$$\mathbf{f}=\begin{vmatrix} -x_{1o}\omega_{ro}+K_{ro}\omega_{ro}u \\ 0 \\ 0 \end{vmatrix}; \frac{\partial \mathbf{f}}{\partial \mathbf{x}_o}=\begin{vmatrix} -\omega_{ro} & \omega_{ro}u & -x_{1o}+K_{ro}u \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

The identification equations will have the following form (2):

$$\dot{z}_o=-z_o\omega_{ro}+K_{ro}\omega_{ro}u+P_{11}(z-z_o)/\psi_v;$$
$$\dot{K}_o=P_{12}(z-z_o)/\psi_v; \qquad (3)$$
$$\dot{\omega}_o=P_{13}(z-z_o)/\psi_v,$$

where $K_{ro}=K_o+K_{pr}; \omega_{ro}=\omega_o+\omega_{pr}$; K_{pr}, ω_{pr} – program values of parameters.

The equation for variance of error:

$$\dot{p}_{11}=-p_{11}^2\psi_v^{-1}+2(-P_{11}\omega_{ro}+P_{12}\omega_{ro}u+$$
$$+P_{13}(-x_{1o}+K_{ro}u))+K_{ro}{}^2\omega_{ro}{}^2\psi_{w11};$$
$$\dot{p}_{12}=-p_{11}p_{12}\psi_v^{-1}-p_{12}\omega_{ro}+p_{22}\omega_{ro}u+$$
$$+p_{23}(-x_{1o}+K_{ro}u)+\psi_{w12};$$
$$\dot{p}_{13}=-p_{11}p_{13}\psi_v^{-1}-p_{13}\omega_{ro}+$$
$$+p_{23}\omega_{ro}u+p_{33}(-x_{1o}+K_{ro}u)+\psi_{w13};$$
$$\dot{p}_{22}=-p_{12}^2\psi_v^{-1}+\psi_{w22};$$
$$\dot{p}_{23}=-p_{12}P_{13}\psi_v^{-1}+\psi_{w23};$$
$$\dot{p}_{33}=-p_{13}^2\psi_v^{-1}+\psi_{w33}.$$

In case of presence of restrictions on an output coordinate (see fig. 2) for a vector of observations it is possible to write down following expression (Ponyatsky *et al.*, 2009):

$$z_g(t)=\begin{cases} z_{min}, x_1(t)<z_{min}; \\ x_1(t), z_{min}<x_1(t)<z_{max}; \\ z_{max}, x_1(t)>z_{max}. \end{cases}$$

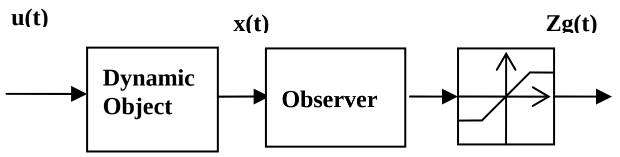


Fig. 2 The circuit design of dynamic installation with restriction on an output coordinate

Then the algorithm of an estimation of a transfer ratio and time constant under constraints in an output coordinate will have a following appearance:

$$\dot{z}_o=-z_o\omega_{ro}+K_{ro}\omega_{ro}u+P_{11}(z-z_{go})/\psi_v;$$
$$\dot{K}_o=P_{12}(z-z_{go})/\psi_v; \qquad (4)$$
$$\dot{\omega}_o=P_{13}(z-z_{go})/\psi_v,$$

where $z_{go}(t)$ - estimation of an output coordinate,

$$z_{go}(t)=\begin{cases} z_{min}, & x_{1o}(t)<z_{min}; \\ x_{1o}(t), & z_{min}<x_{1o}(t)<z_{max}; \\ z_{max}, & x_{1o}(t)>z_{max}. \end{cases}$$

The equations for a dispersion of a mistake in this case look like:

$$\dot{p}_{11}=-p_{11}\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}p_{11}\psi_v^{-1}+2(-P_{11}\omega_{ro}+P_{12}\omega_{ro}u+$$
$$+P_{13}(-x_{1o}+K_{ro}u))+K_{ro}{}^2\omega_{ro}{}^2\psi_{w11};$$
$$\dot{p}_{12}=-p_{11}\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}p_{12}\psi_v^{-1}-p_{12}\omega_{ro}+p_{22}\omega_{ro}u+$$
$$+p_{23}(-x_{1o}+K_{ro}u)+\psi_{w12};$$
$$\dot{p}_{13}=-p_{11}\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}p_{13}\psi_v^{-1}-p_{13}\omega_{ro}+$$
$$+p_{23}\omega_{ro}u+p_{33}(-x_{1o}+K_{ro}u)+\psi_{w13};$$
$$\dot{p}_{22}=-p_{12}\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}p_{12}\psi_v^{-1}+\psi_{w22};$$
$$\dot{p}_{23}=-p_{12}\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}p_{13}\psi_v^{-1}+\psi_{w23};$$
$$\dot{p}_{33}=-p_{13}\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}p_{13}\psi_v^{-1}+\psi_{w33}.$$

where

$$\tilde{\mathbf{H}}=\begin{cases} 0, & x_{1o}(t)<z_{min}; \\ \mathbf{H}, & z_{min}<x_{1o}(t)<z_{max}; \\ 0, & x_{1o}(t)>z_{max}. \end{cases}$$

The offered algorithm of identification based on a method of invariant imbedding, allows to directly receive state estimations and values of time-dependent parameters of nonlinear dynamic object

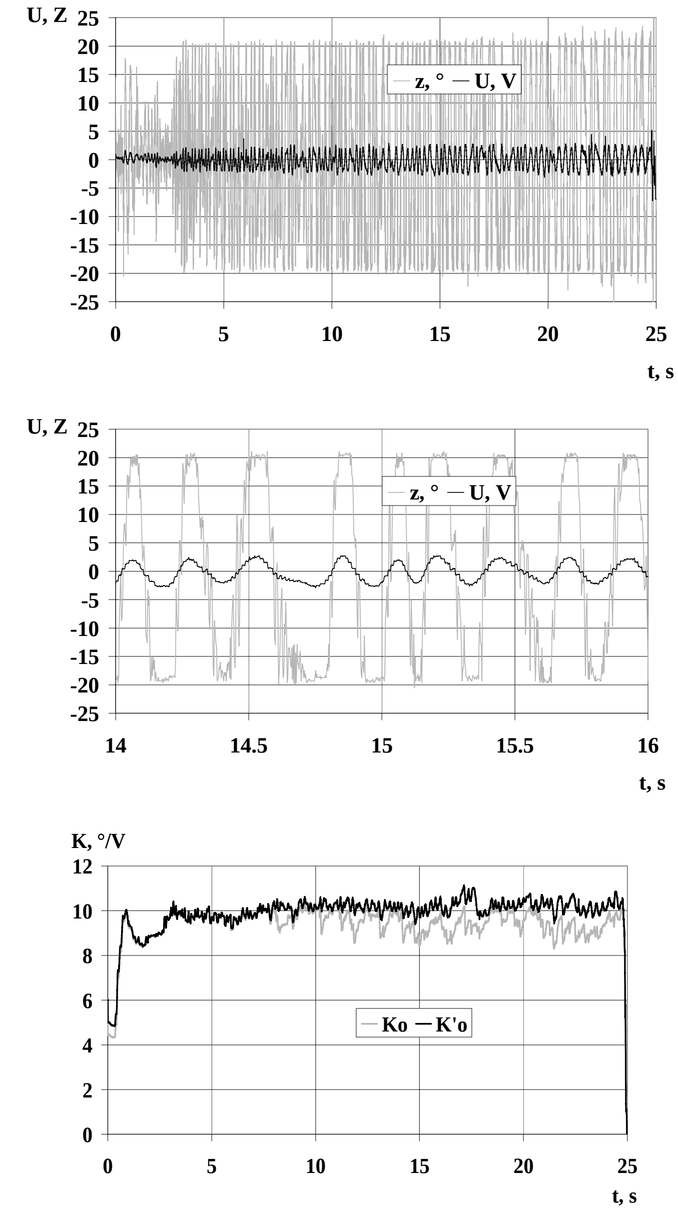
3. RESULTS

The considered method can be used at identification of models of dynamic objects using results of processing bench and full-scale experiment. For operative estimation of the characteristics of servo as a part of the aircraft control system the identification is performed in the form of linear time-dependent models of the low order. The identification of parameters of full nonlinear model is required to analyze the servo design operation. In this case the identification can be carried out stage by stage: first the processing of the bench test results with tentative assessment of the object parameters which don't depend on external effects is carried out. The final estimation of parameters of full model is performed after flight tests.

A program system has been worked out in the form of the program for OS Windows in medium of programming Visual C++ and Visual C#, realizing the classical algorithm of the dynamic objects identification in the form of time-dependent linear models (Ponyatsky and Oberman, 2003b; Ponyatsky et al., 2009).

It is possible to carry that it is included in composition of the closed contour of a control system to habit of a prototype system, i.e. management of the aircraft is carried out according to measured coordinates. Besides research is spent, as a rule, in conditions of passive experiment. However arrival signals on the servodrive and the sail plane are non-stationary both on amplitude, and on frequency, as operating signals промоделированы the rotational speed of the aircraft changing on flying time. Handicapes of measurement of coordinates of the aircraft during its management negatively influence quality of management, but allow to carry out an estimation of parameters of the servodrive and the sail plane (Ponyatsky *et al.*, 2008a; Ponyatsky 2008b;Ponyatsky *et al.*, 2009).

The results of estimation of time constant and the servo transfer ratio on a signal from the servo input and on a signal from the controls angular position sensor are given in Figure 3. As follows from given data, the estimation obtained by means of the invariant imbedding method (T_{iim}) gives a smoother law of the parameter change with lower fluctuations in comparison with the Kalman filtration (T_{kf}) (Ponyatsky, 2003a; Ponyatsky and Nadezhdin, 2007b). Results of an estimation of a transfer ratio of the servodrive in view of (K'_{iim}) and without taking into account restrictions on an output coordinate (K_{iim}) are resulted. The account of restrictions on an output coordinate provides more exact estimation of a transfer ratio of the servodrive.



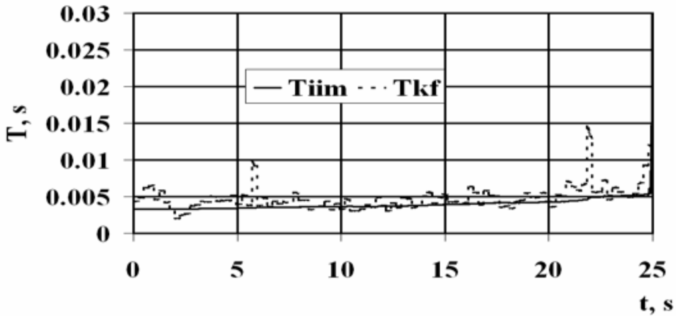


Fig.3. Results of estimation of time constant and servo transfer ratio by means of the invariant imbedding method (T_{iim}) and the Kalman filtration (T_{kf}); K'_{iim} - in view of restrictions on an output coordinate; K_{iim} - without taking into account restrictions on an output coordinate

5. CONCLUSIONS

With use of the developed programm complex the estimation of a transfer ratio, time constant and a damping ratio of the servodrive and the sail plane of the pilotless aircraft twirled on a roll is lead (Ponyatsky *et al.*, 2008a; Ponyatsky 2008b; Ponyatsky *et al.*, 2009). Comparison of these estimations with the estimations received with use of algorithms of identification on the basis of linear filtering Калмана (Ljung, 1991; Ponyatsky, 2003a; Ponyatsky, 2004a; Fatuev *et al.*, 2004b), has shown, that the method of invariant plunging gives more smooth and with smaller fluctuations an estimation of factors (Ponyatsky and Nadezhdin, 2007b). However, the method of filtering Калмана is more resistant to, and at use of a method of invariant plunging the task of initial oncoming of a vector of the expanded condition of installation is necessary. In case of the account of restrictions in an output coordinate the algorithm of invariant plunging (4) in comparison with (3) provides more exact definition of a transfer ratio of installation of management.

REFERENCES

Sage A.P., Melsa J.L. (1974). *Control system Identification*. Science, Moscow.

Luing L. (1991). *System Identification. Theory for the users*. Science, Moscow.

Ponyatsky V.M. (2003a). Computer technology of dynamic object linear non-stationary model parameters identification by Kalman filtering. *Works for the international Scientific-Practical Conference «The Third Okunev readings»*. Baltic State Technical University, Saint-Petersburg, Part 3, pp. 146-150.

Ponyatsky V.M., Oberman M.S. (2003b). Program for dynamic object linear non-stationary model parameters identification by Kalman filtering. *Works for the international Scientific-Practical Conference «The Third Okunev readings»*. Baltic State Technical University, Saint-Petersburg, Part 3, pp. 151-158.

Ponyatsky V.M. (2004a). Identification of multidimensional non-stationary dynamic objects with Kalman filtration method. *Issues of nuclear science and techniques. Series:*

Mathematical simulation of physical processes. Moscow, Vol. 1, pp.81-86.

Fatuev V.A., Judaev A.V., Ponjatsky V.M., Kargin A.V., Oberman M.S. (2004b). Structural and parametrical Identification of multidimensional non-stationary dynamic systems. *Proceedings of the III International conference "System Identification of and control Problems"*. Institute of Control Sciences, Moscow, pp. 159-186.

Ponyatsky V.M. (2007a). Application of invariant imbedding method to estimate the electrotechnical objects parameters. *Works of the fourth All-Russia scientifically-practical conference "Electrotechnical objects control systems»*. State university, Tula, Russian, pp. 108-109.

Ponyatsky V.M., Nadezhdin E.N. (2007b) Neural networks for parameter identification of servo-drives of the flying device. *9th IFAC Workshop "Adaptftion and Learning in Control and Signal Processing" ALCOSP '2007.* Abstracts. Saint Petersburg, Institute for Problems of Mechanical Engineering. P. 37. http://www.ifac-papersonline.net/

Ponjatsky V.M., Zamotaev I.V., Kisilev V.B. (2008a). Method of invariant immersion of non-stationary dynamic object identification. *Proceedings of the* VII *International conference "System Identification of and control Problems"*. Institute of Control Sciences, Moscow. P

Ponjatsky V.M (2008b) Assessment of the experimental characteristics of an air Vehicle a method of invariant imbedding. *The first All-Russia scientific and technical conference « Fundamental bases of ballistic designing »* Baltic State Technical University, Saint-Petersburg, Part 1, pp. 29-33

Ponjatsky V.M., Zamotaev I.V., Kisilev V.B. (2009). Mathematical tols and Software of Identification of a non-stationary dynamic plant by use of the Method Invariant Imbedding. *Proceedings of the* VIII *International conference "System Identification of and control Problems"*. Institute of Control Sciences, Moscow.