VIBRATIONS OF CIRCULAR CYLINDRICAL SHELLS UNDER SEISMIC EXCITATION.

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Abstract

In the present paper experimental results regarding the dynamics of circular cylindrical shells are presented. A circular cylindrical shell having vertical axis, clamped at the base and connected to a disk on the top is seismically excited by means of a resonant forcing. The first axisymmetric mode is excited around the resonance at relatively low frequency and low amplitude excitation. A violent resonant phenomenon is observed and analyzed close to the previously mentioned resonance as well as an interesting saturation phenomenon.

Keywords

Shells, experiments, vibration, resonance, saturation.

1. Introduction

The continuous growing of the commercial use of Space facilities lead to the development of new and more efficient aerospace vehicles; therefore, new and accurate studies on light-weight, thin-walled structures are needed. A wide part of the technical literature in the past century was focalized on the analysis of thin-walled structures and tried to investigate their behaviour in many different operating conditions, i.e. under static or dynamic loads, either in presence or absence of fluid-structure interaction. Both linear and nonlinear models have been developed to forecast the response of such structures. Many studies were concerned with cylindrical shells that constitute main parts of aircrafts, rockets, missiles and generally aerospace structures.

The literature about vibration of shells is extremely wide and the reader can refer to Leissa [1] or more recently to Amabili and Païdoussis [2] for a comprehensive review of models and results present in literature.

Trotsenko and Trotsenko [3], studied vibrations of circular cylindrical shells with attached rigid bodies, by means of a mixed expansion based on trigonometric functions and Legendre polynomials; they considered only linear vibrations.

The literature analysis shows that in the past several methods were developed for investigating: *i*) linear vibrations of complex shells; *ii*) nonlinear vibrations of shells having simple shape and boundary conditions. Therefore, a contribution toward the knowledge of new dynamic phenomena on shells is welcome.

In the present paper, experiments are carried out on a circular cylindrical shell, made of a polymeric material (P.E.T.) and clamped at the base by gluing its bottom to a rigid support. The axis of the cylinder is vertical and a rigid disk is connected to the shell top end. In [4] this problem was fully analyzed from a linear point of view. Here nonlinear phenomena are investigated by exciting the shell using a shaking table and a sine excitation. Shaking the shell from the bottom induces a vertical motion of the top disk and, therefore, axial loads due to inertia forces. Such axial loads generally give rise to axial symmetric deformations; however, in some conditions it is observed that a violent resonant phenomenon takes place, with a strong energy transfer from low to high frequencies and huge amplitude of vibration. Moreover, an interesting saturation phenomenon is observed.

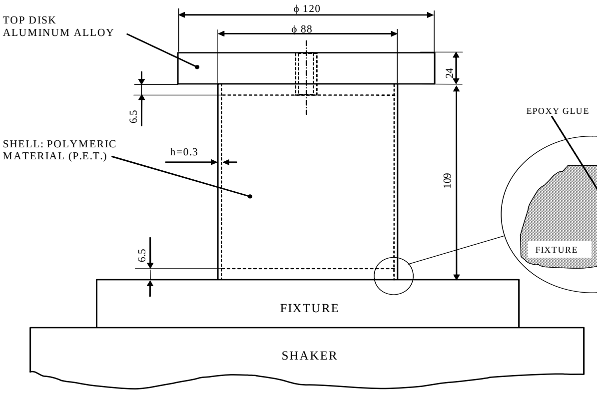


Figure 1.

2. Linear modal analysis

The system under investigation is described in Figures 1 and 2. A circular cylindrical shell, made of a polymeric material (P.E.T.) is clamped at the base by gluing its bottom to a rigid support (a disk that is rigidly bolted to a shaker, such disk is technically called “fixture”); the connection is on the lateral surface of the shell, in order to increase the gluing surface, see Figure 1. A similar connection is carried out on the top; in this case the shell is connected to a disk made of aluminium alloy, such a disk is not externally constrained; therefore, it induces a rigid body motion to the top shell end.

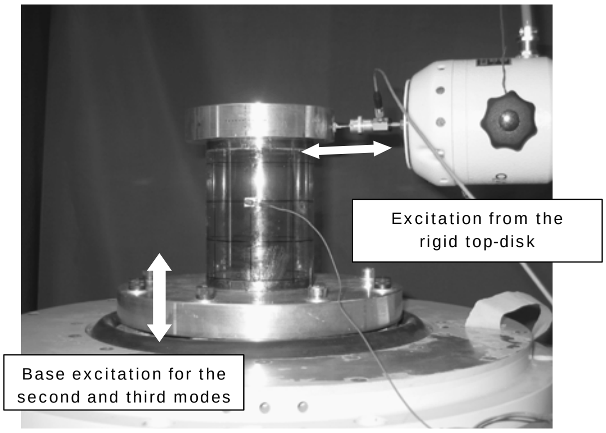


Figure 2

The use of P.E.T. is related to the experiments focused on nonlinear resonances, which gives rise to high amplitude of oscillation; in order to avoid plasticity for such high amplitudes the PET polymer is considered. This polymer showed a good linear behaviour, both from elastic and dissipation point of view, as proven by excellent curve fittings obtained from experimental data. Material characteristics are directly measured with specific tests (E and ρ) or found in literature (ν).

The fixture is bolted to a high power shaker (LDS V806, 9000N peak force, 100g maximum acceleration, 300kg payload, 1-3000Hz band frequency); such shaker is used to excite the shell from the base or to provide a stiff support when the excitation is provided with different devices.

When the shell excitation is not furnished from the base, two kinds of excitation sources are applied: a micro shaker (TIRAVIB, 10N peak force, see Figure 2) or a micro hammer.

The system exhibits both beam-like modes and shell-like modes (axisymmetric and asymmetric).

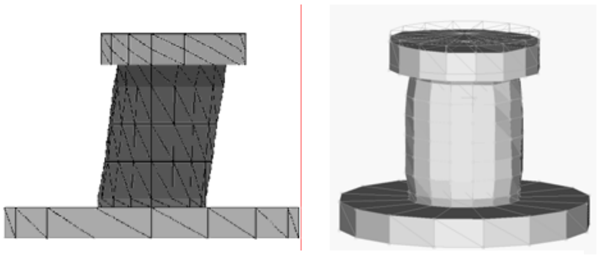
Experiments have been carried out by using different kind of excitations; indeed, the system characteristics make difficult to excite all modes together. The first beam like mode is excited using a shaker connected to the top disk, see Figure 2, this kind of excitation allows to transfer energy to the shell through the rigid body motion of the top-disk; the energy path will excite only modes having a rigid body motion of the top-disk orthogonal to the shell axis, i.e. mainly beam modes. A second type of excitation is provided by exciting the shell from the base motion (Figure 2); it was observed experimentally that such kind of excitation pumps energy to the first axisymmetric mode and to the second beam like mode; other modes were scarcely excited. The third kind of excitation was carried out by means of a micro-hammer; such kind of excitation allows to furnish energy directly to shell modes having $n>1$ (n is the number of nodal diameters). It is to note that the use of a micro hammer allows to furnish a small amount of energy to the system; therefore, all modes for which the motion of the top disk is present ($n=0$ or 1) are not excited, because the energy pumped in the system is not enough to induce a disk motion detectable from sensors (accelerometers). The combination of three types of excitation allowed to identify all modes within the frequency range of interest.

About 80 measurement points were used for mode shape identification, with a uniform distribution over the shell; the classical FRF based approach was used in lab experiments: the excitation was fixed and several measurements were carried out by using one micro-accelerometer and moving it on several positions. Simultaneous measurements with several accelerometers were avoided in order to reduce the added mass effect. Each FRF was evaluated after performing several measurements (at least three) in order to reduce noise, a "curve fitting" allowed to identify frequencies, damping and mode shapes (see e.g. [5]).

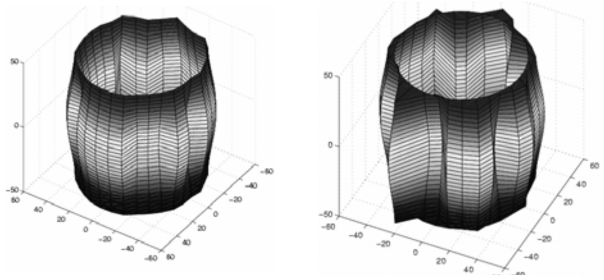
Mode		Experimental frequency
k	*n*	
first beam like mode *n*=1		95
1	0	314
second beam like mode *n*=1		438
1	6	791
1	7	816
1	5	890
1	8	950
1	9	1069

Table 1.

In Table 1, experimental results are presented: k means that the mode has k-1 nodal circumferences; n is the number of nodal diameters (beam modes can be considered $n=1$). In [4] a complete experimental, numerical and theoretical analysis is presented, the reader should read such paper for a full linear analysis of the problem under investigation. All modes are identified experimentally by using curve fitting techniques, present in LMS CADA-X, that give: frequency, modal damping ratio, and modal shape. A 3D geometry of the system has been created before measurements, in order to associate each measurement to the corresponding point (and degree of freedom) on the geometry. The mode shape identification is of crucial importance in the case of shells; indeed, the high modal density makes difficult to compare experimental and theoretical/numerical modes using natural frequencies only; therefore, the visualization of modes is mandatory.



first beam like mode *n*=1 (95Hz) mode (1,0) (314Hz)



mode (1,6) (791Hz) mode (1,7) (816Hz)

Figure 3. Mode shapes.

In Figure 3 experimental mode shapes are reported. The first mode is a beam-like mode; such kind of modes show both displacement and rotation of the top-disk. The rotation is lost in the experiments because accelerometers are located radially in this set of measurements. A final note is needed for shell like modes (n>1): a micro hammer and a micro accelerometer have been used, the latter one is extremely light (0.25 10^{-3} kg); however, it induces a mass effect which causes an increasing of the radial deformation where the accelerometer is located.

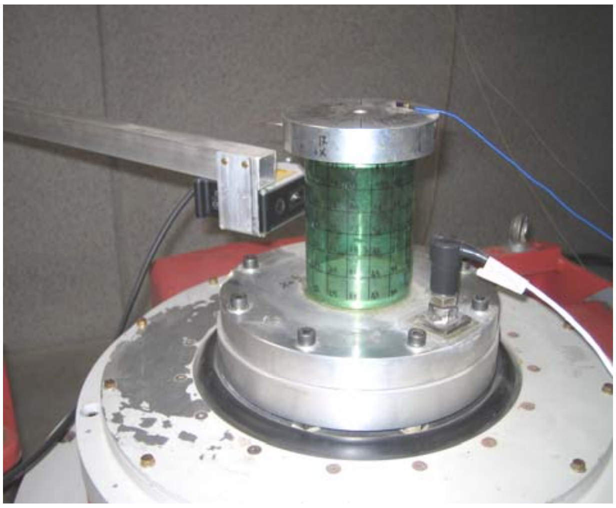


Figure 4

3 Seismic excitation: nonlinear analysis.

Tests are carried out using a seismic sine excitation, close to the resonance of the first axisymmetric mode (m=1, n=0).

The complexity and violence of vibrations due to nonlinear phenomena gave several problems to closed loop controllers of the shaking tables; therefore, an open loop approach was chosen, the excitation was set about 5-10 g.

Two accelerometers and a Laser telemeter are used to measure accelerations on base and top, and the displacement on the shell lateral surface: channel 1 (accelerometer Wilcoxon Research S 100 C) records the base acceleration (the excitation) due to the shaking table; channel 2 records the displacement of the shell in radial direction, using an Micro Epsilon optoNCDT 2200 Laser Telemeter; channel 3 records the top disk acceleration (PCB M352C65 micro accelerometer). The setup is shown in Figure 4.

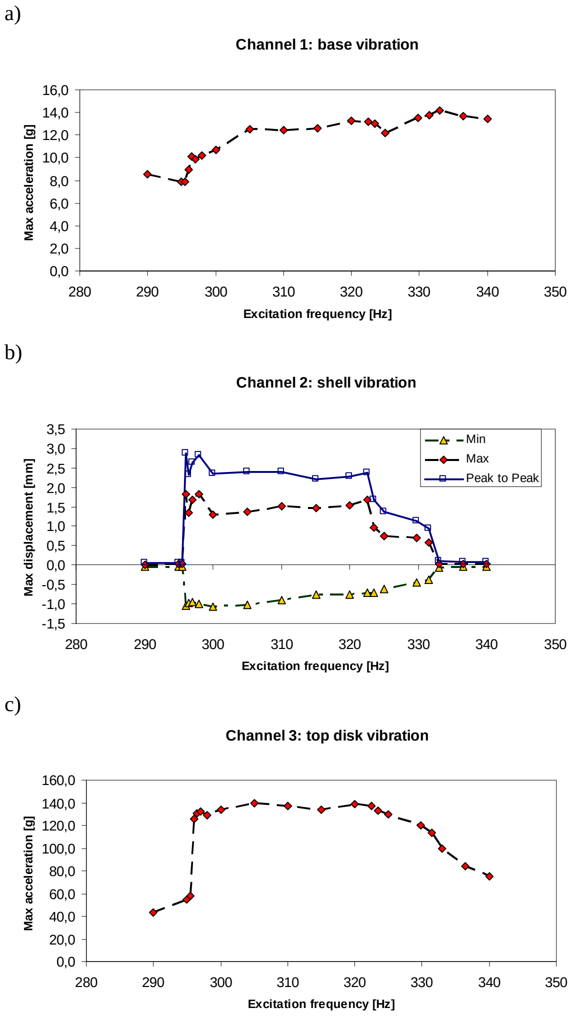


Figure 5. a) max amplitude of vibration (channel 1), b) max min and peak to peak of vibration (channel 2), c) max amplitude of vibration (channel 3)

Figures 5 a-c represent the amplitudes of vibration in terms of acceleration or displacement obtained when the excitation frequency is reduced. Channel 1 shows that the maximum excitation is between 8 and 14 g. The top disk (channel 3) vibration is magnified by the first axisymmetric mode resonance. However, close to 330 Hz, reducing the frequency, the classical resonance behaviour is not present and the response is flat up to 295 Hz. In the same frequency range the shell experiences a violent vibration that appears suddenly, the amplitude passes from few microns to some millimetres; note that this corresponds to huge accelerations; for example if the amplitude is 3 mm, and we suppose it is purely harmonic (actually there are super-harmonic components) at 300 Hz, an approximate estimation of the acceleration is about 1100 g! In some previous experiments we measured up to 2000 g.

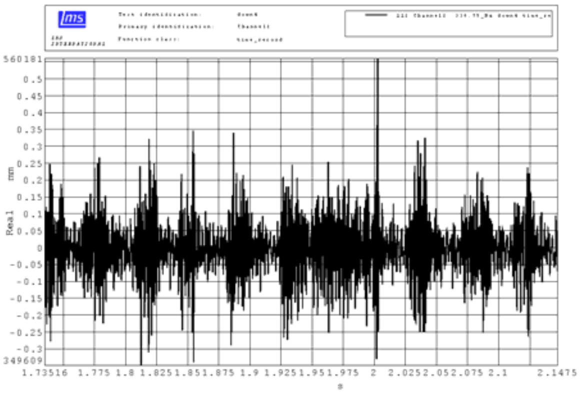


Figure 6. Time history from channel 2 (shell displacement). Excitation freq 330.75 Hz.

In Figure 6 one can observe that the response is not regular nor stationary; moreover, some spikes are visible.

Figure 7 clarifies that, when the nonlinear resonance takes place the response looses the periodicity, such analysis clearly shows that the phenomenon is extremely nonlinear.

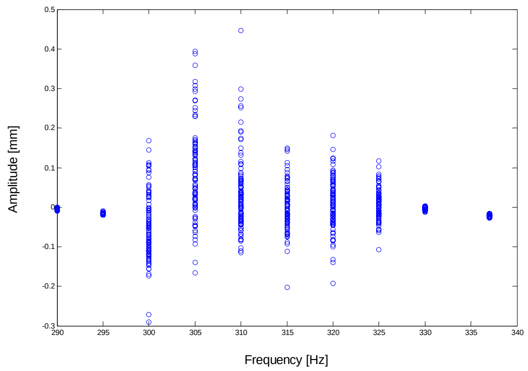


Figure 7. Bifurcation diagram of the Poincaré maps.

The scenario represented in Figure 5 is referred to the maximum amplitude of vibration, which is not strictly related to the mechanical energy, in particular when the response is not regular. In order to have an enhanced view on the phenomenon, useful for a physical interpretation, in Figure 8 the scenario is represented in terms of RMS. It is extremely interesting that, the RMS the top disk response is completely flat in a wide frequency range (Fig. 8 c), even though the excitation is not constant (Fig. 8 a), and the shell adsorbs a certain amount of energy (Fig. 8b).

The phenomenon disappears for low amplitudes of excitation, the threshold is currently under investigation.

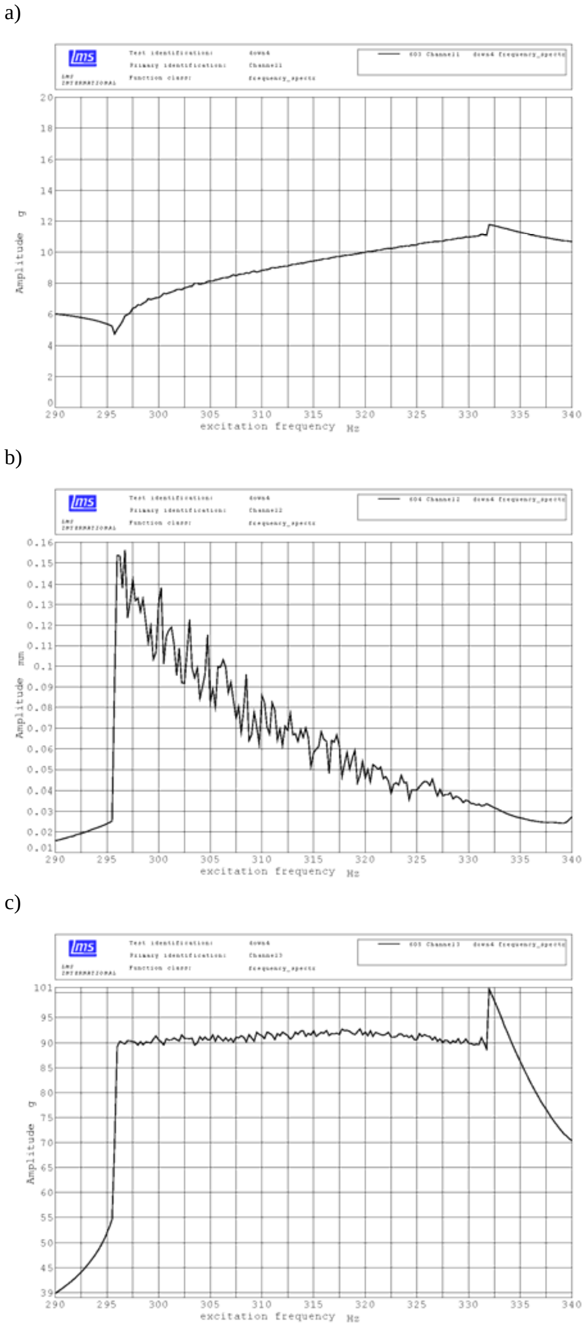


Figure 8. RMS of the response.

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