Multistability in a semiconductor laser subject to optical feedback from a Fabry-Perot filter

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Abstract: We study the structure of the multistable locking region of a semiconductor laser subject to optical feedback from a Fabry-Perot filter. Key parameters organizing the degree of multistability are uncovered, and they include the feedback phase, the filter width and the frequency detuning.

Semiconductor lasers find many applications, for example, in optical telecommunication. Because of their high material gain they are very sensible to external perturbations, which may lead to instabilities and possibly even chaotic laser emission. We show that this sensitivity can also lead to a complex structure of multistable continuous wave (cw) and oscillatory emission. We consider here a laser subject to optical feedback from a Fabry-Perot filter — a frequently used set-up to control the dynamics of a laser via the spectral properties of the feedback light, which are determined by the filter detuning and the filter width. The system can be modelled by rate equations for the complex-valued laser field $E(t)$, the complex-valued filter field $F(t)$ and real-valued laser inversion $N(t)$, given by:

$$\begin{aligned} \dot{E}(t) &= (1+i\alpha)E(t)N(t)+\kappa F(t)\,, \\ T\dot{N}(t) &= P-N(t)-(1+2N(t))|E(t)|^2\,, \\ \dot{F}(t) &= \Lambda E(t-\tau)e^{-iC_p}+(i\Delta-\Lambda)F(t)\,. \end{aligned} \tag{1}$$

This model takes into account the time delay τ of the feedback light due to the propagation through the feedback loop and, hence, has the form of a delay differential equation. In (1) time is rescaled with respect to the photon decay time, which is typically in the order of picoseconds. The laser parameters are the linewidth enhancement factor $\alpha = 5.0$, the ratio between electron and photon decay time $T = 100$, and the pump parameter $P = 3.5$. The feedback is characterized by the feedback strength $\kappa = 0.01$, the feedback phase C_p, the linewidth of the filter $\Lambda = 0.07$, and the detuning $\Delta = \Omega_F - \Omega_0$, which is the difference between the filter frequency $\Omega_F = -0.07$ and the laser frequency Ω_0. Importantly, the feedback phase C_p takes into account the phase that the laser field accumulates as it propagates through the feedback loop.

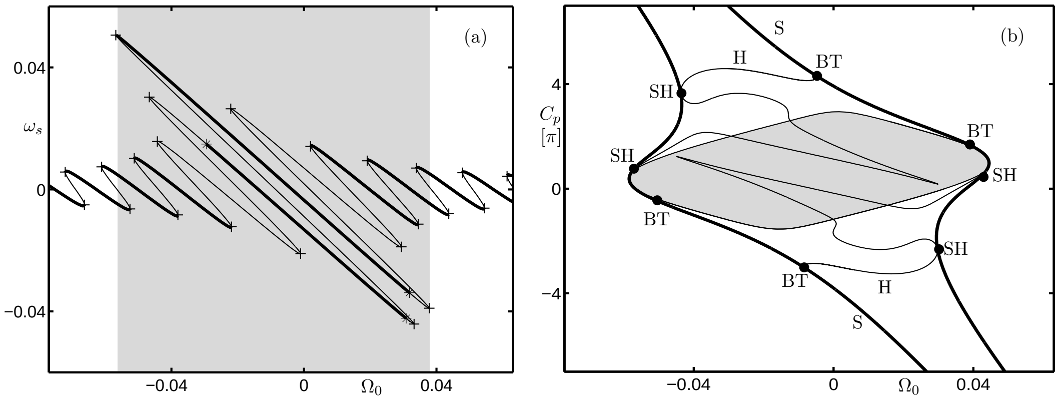

Figure 1: Branch of cw solutions (a) as a function of Ω_0 with stable (thick) and unstable (thin) parts; and bifurcation diagram with multistable locking region (shaded) in the (Ω_0, C_p)-plane (b), consisting saddle-node bifurcation curves (thick) and Hopf bifurcation curves (thin) that meet at saddle-node Hopf (SH) and Bogdanov-Takens (BT) points.

A cw solution of (1) has the form $(E(t);N(t);F(t)) = (|E_s|e^{i\omega_s t};N_t;|F_s|e^{(i\omega_s t+i\phi)})$, where the laser and the filter field have the same frequency ω_s, constant amplitudes $|E_s|$, $|F_s|$, a phase difference ϕ, and the laser inversion N_s is a constant. Figure 1 shows a bifurcation analysis as a function of the laser frequency Ω_0 and the feedback phase C_p, where the shading indicates multistable locking. The locking region is bounded by different bifurcation curves, which correspond to different locking-unlocking transitions, such as saddle-node (S) and Hopf (H) bifurcations. For the chosen set of parameters we find up to 3 simultaneously stable cw states. In addition there are more complicated stable oscillatory dynamics arising from Hopf bifurcations (not shown in Figure 1). Overall, we find a large degree of multistability between different types of solutions [1], which is organized by the feedback phase C_p as well as the feedback strength κ and the filter linewidth Λ.

[1] H. Erzgräber, B. Krauskopf, and D. Lenstra, “Bifurcation structure of semiconductor lasers with filtered optical feedback”, SIAM Journal on Applied Dynamical Systems, **6**(1) (2007) 1-28.