MULTIPARAMETRICAL ANALYSIS AND OPTIMAL CHAOS SUPPRESSION IN DYNAMIC SYSTEMS BASED ON A TWO-STAGE PARAMETER CORRECTION SCHEME

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Abstract
The results of multiparametrical analysis of the optimal chaos suppression problem in non-autonomous dynamic systems based on a two-stage parameter correction scheme are presented in the paper. The efficiency of the scheme is proved by the comparison of numerical experiment results to analytical estimations gained from Melnikov method.

Key words
Chaos suppression, optimal correcton.

1 Introduction

Parametric space multidimensionality is characteristic of many chaotic systems. It substantially complicates the understanding of the unstable behavior disappearance (appearance) mechanisms. High parametric perturbation sensitivity of chaotic systems makes the investigation of such mechanisms even more complicated. That is why the parameters and forms of their perturbation optimal for chaos elimination are often uncertain. In this context investigations aimed at the development of effective means of multiparametrical analysis [Kuznetsov, Kuznetsov and Sataev, 1997; Seyranian and Mailybaev, 2003] and generalization of well-known chaos control methods [Warncke, Bauer and Martienssen, 1994; Barreto and Grebogi, 1995; Paula and Savi, 2008] applied to the multiparametrical case come into importance.

A multiparametrical branch close to the optimal chaos control problems and structural optimization is the optimal multiparametrical correction technique of chaotic systems [Gorelik, Talagaev and Tarakanov, 2006]. The possibilities of the technique are not limited only to carrying the optimal chaos suppression out [Talagaev and Tarakanov, 2008]. The characteristics of the class of optimal corrective functions generated by the technique provide the *modification* [Talagaev and Tarakanov, 2007] of the system limit set (chaotic attractor) from unstable state into the stable one regarding the small perturbations on parameters. Meanwhile it is possible to localize the unique limit set (satisfying the demands of transient process stability and optimality simultaneously) in the phase space. According to the control aims analysis in [Andrievskii and Fradkov, 2003], modification became a specific control aim having arisen from chaos control problem. Its difference from the chaos stabilization problem [Ott, Grebogi and Yorke, 1990] is that the system stability character, but not the quantitative characteristics of the target set (an unstable periodic orbit embedded into a chaotic attractor), is given beforehand. Such aim is typical for methods of non-feedback controlling chaotic motion [Chacon, 2002; Lenci and Rega, 2003; Dzhanoev, Loskutov, Cao and Sanjuan, 2007] where the efficiency of certain external periodic perturbations being used is estimated on the basis of Melnikov criterion [Mel'nikov, 1969; Guckenheimer and Holmes, 1990] and optimal bifurcation control [Cao and Chen, 2005].

In the paper we extend the results of previous investigations to set the general optimal multiparametric correction problem. The solution technique offered below in fact represents the basis of general scheme of multiparametrical analysis. Its value is in the possibility to investigate the optimal stabilizing perturbations complementing each other. Comparing them we show that optimal correction gives the full picture of chaotic dynamics disappearance. It includes enough information to provide optimal "chaos $\leftrightarrow$ order" transition in parameter space, along with the corrected system optimal dynamic regime identification and estimation of system dynamics sensitivity to the particular form of parametric perturbations.

2 General correction problem

Consider a class of dissipative non-autonomous systems

$$\dot{x} = F(x,p,t) = f(x,p) + g(x,t),$$

where $x = (x_1, x_2)$, f is a non-linear function, which describes the unperturbed system dynamics, g – time-periodic perturbation function, $p \in R^m$ – a vector of available parameters. It is suggested that: (i) among the points $x^e_{(k)} = (x^e_{1(k)}, 0)$, $k = \overline{1,s}$, determined by the condition $f(x,p) = 0$, there exists one hyperbolic saddle point; (ii) perturbation influence leads to the situation of transverse intersection of the stable and unstable separatrices of the hyperbolic point, causing the occurrence of homoclinic structure (interlacement). As it is known [Guckenheimer and Holmes, 1990], conditions (i)-(ii) cause the appearance of chaotic dynamics. Here the system realizes a chaotic attractor A_P, with its structure determined by the geometry of the set of unstable equilibrium states $E = \{x^e_{(k)}\}_{k=\overline{1,s}}$.

Possible changes of the dynamic system regime are linked to the presence of a special multiparametrical perturbation. For this purpose we take a set of parameters $\{p_1, p_2, ..., p_r\}$, $r \leq m$, where each p_j, $j = \overline{1,r}$, is perturbed by the function $h_j = h_j(t)$ according to the rule $p \mapsto \hat{p}(\cdot)$: $\hat{p}_j(t) = p_j(1 + h_j(t))$. The result of the perturbation is the transformation of the parameters into specific dynamic variables $\hat{p}(t) = \hat{p}(h(t))$, $h \in U$, where $U = \{ h(\cdot) \in L_2 \mid |h_j(t)| \leq a,\ j = \overline{1,r} \}$, which cause the initial system transformation:

$$\dot{x}(t) = F(x(t), p, t) \underset{p \mapsto \hat{p}(\cdot)}{\longrightarrow} \dot{x}(t) = F(x(t), \hat{p}(t), t), \tag{1}$$

called *multiparametrical correction.* The aim of correction is to provide the *corrected system* $\dot{x} = F(x, \hat{p}, t)$ stability at satisfying the natural physical demand of the "small" parameters correction that is optimal chaos suppression. After $p \mapsto \hat{p}(\cdot)$ the character of the corrected system dynamics stability depends on the value a in constraints $|h_j(t)| \leq a$, $j = \overline{1,r}$. If we formalize the "small" demand with the maxmin criterion the general correction problem will look like:

$$J_1(h,a) = \max_{t \geq 0} \max_{1 \leq j \leq r} |h_j(t)| \to \min_{a \in K}, \tag{2}$$

where $K = \{a > 0 \mid F(a)\}$, the formal condition $F(a)$ is itemized with regard to the chosen stability criterion of the system. Note that in (2) the minimum on the parameter a can be achieved at different $h \in U$. Solution (2) gives the value $a_{\min}$ – *minimal correction radius* of the parameters p_j, causing the transition to the stable dynamics with the corresponding modification of the system attractor.

In the paper the general problem investigation is conducted for three different situations corresponding to three basic forms of corrective influences. The object of correction is the Duffing-Holmes oscillator [Guckenheimer and Holmes, 1990] written as:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \alpha x_1 - \beta x_1^3 - \gamma x_2 + f\cos(\omega\, t). \tag{3}$$

Our interest to this sample model of non-linear dynamics was inspired by its multiparametricability. In particular, chaotic behavior (3) is defined by the combination of the values of five parameters: $\alpha, \beta > 0$ – system own parameters available for correction, γ – damping, f, ω – outer perturbation amplitude and frequency.

3 Correction on Melnikov chaos criterion

In case of small perturbation ($\gamma, f \ll 1$) Melnikov criterion allowing analytical estimation of the effectiveness of the given form of parametric influence on the border of chaos appearance can be applied to the model (3). The offered way of investigation in the field is the formulation and solving two types of optimization problem. Their solutions are found with regard to the connection between the parameters of the system and that allows supplementing our understanding of its dynamic characteristics.

Melnikov method is based on the analysis of the function Δ, which shows the distance between the stable and unstable perturbed separatrices. For (3), the calculation of Melnikov function at any given time θ gives [Guckenheimer and Holmes, 1990]:

$$\Delta_0(\theta) = A\sin(\omega\theta) - C, \tag{4}$$

$$A = \pi\omega f (2/\beta)^{1/2} \mathrm{sech}(\pi\omega/(2\alpha^{1/2})),$$

$$C = 4\gamma\alpha^{3/2}/(3\beta).$$

If Melnikov function $\Delta_0(\theta)$ changes its sign at θ, it will be the condition of chaos occurrence. In all the correction examples below we use the test values of the parameters

$$\alpha = 1,\ \gamma = 0.154,\ \beta = 4,\ f = 0.095,\ \omega = 1.1.$$

Let us denote Melnikov function corresponding to the corrected system as $\hat{\Delta}(\theta)$ and use Melnikov criterion to specify the formal condition $F(a)$ in the general task (2). Then the set of a providing stable dynamics will look as $K_M = \{a > 0 \mid sign(\hat{\Delta}(\theta)) = const\}$ (here $sign(\cdot)$ – "signum" function). Taking into consideration that the dependence on parameter α is more complex than on β, may (3) be corrected on parameter β only: $\beta \mapsto \hat{\beta} = \beta(1 + h(t))$, $h(t) \in U$. It will allow focusing attention to the possibility estimation of different forms of parametric perturbation.

3.1 Static correction

Consider the correction variant (3) when $h(t) \equiv h = const$: $\beta \mapsto \hat{\beta} = \beta(1 + h)$. It still has a constraint $|h| \leq a$. However, due to $h(t) \equiv h$ it is clear that h may accept only one value, that is why the constraint turns to the equation $h = a$. According to (2), the optimal *static* correction problem will look like

$$a \to \min_{a \in K_M}. \tag{5}$$

The task (5) is the simplest from the point of view of forms of perturbation on the system parameter. It is necessary to find minimal corrective amendment h^* satisfying to $|h^*| = a_{\min} = \arg \min_{a \in K_M} a$ at which the corrected system dynamics is stable. In static correction h is an additional parameter in Melnikov function expression. That is why taking $\beta \mapsto \hat{\beta} = \beta(1+h)$ into account the expression (4) can be re-written as:

$$\hat{\Delta}(\theta) = \hat{A}\sin(\omega\theta) - \hat{C}, \quad (6)$$

$$\hat{A} = \pi\omega f (2/\hat{\beta})^{1/2}\text{sech}(\pi\omega/(2\alpha^{1/2})),$$

$$\hat{C} = 4\gamma\alpha^{3/2}/(3\hat{\beta}).$$

The values of parameters making system dynamic chaotic are initial in correction. It means that without the correction ($h = 0$) the inequality $\hat{A} > \hat{C}$ is performed for (6). Hence the correction should lead to performance of the reverse condition for (6), that is $\hat{A} \le \hat{C}$. However to solve the problem (5), it is important to know the values h placed on the regular and complex dynamics boundary. They are the minimal corrective amendment h^* of parameter β and $\hat{A} = \hat{C}$ is performed for them. Then from (6) we have

$$\hat{\beta} = D^2(\alpha, \gamma, f, \omega), \quad (7)$$

$$D(\alpha, \gamma, f, \omega) =$$

$$2\sqrt{2}\gamma\alpha^{3/2}(3\pi f\omega)^{-1}\cosh(\pi\omega/(2\alpha^{1/2})).$$

Having put $\hat{\beta} = \beta(1+h)$ in (7) we get solution in the form

$$h^* = \beta^{-1}D^2(\alpha, \gamma, f, \omega) - 1. \quad (8)$$

For the fixed configuration of $\alpha, \beta, \gamma, f, \omega$, the expression (8) allows finding the least corrective amendment able to provide the space transition of parameters from chaotic dynamics field into the regular one.

While considering the multiparametrical analytic condition (8), it is interesting to study the dependence of values placed on the "chaos $\leftrightarrow$ order" boundary on the frequency of chaos-making perturbation ω, that is the function $h^* = h^*(\omega) = \beta^{-1}D^2(\omega) - 1$ with other parameters values being fixed. The dependence is shown in the Fig. 1. It means that at some fixed frequency value $\tilde{\omega}$ the value of minimal corrective amendment $h^*(\tilde{\omega})$ definitely corresponds to the point on the curve. According to Melnikov criterion the condition $h > h^*(\omega)$ defines the field of complex (including chaotic) dynamics in the Fig. 1.

Note that in the frequency interval $(\underline{\omega}, \overline{\varpi})$ the value of the corrective amendment is negative, at the same time there exist perturbation frequency values $\underline{\omega} = 0.2645$ and $\overline{\varpi} = 1.6608$ when $h^*(\omega) = 0$, that is the system need not be corrected. In particular, the relation (8) for the frequency $\omega = 1.1$ gives the estimation $h^* = -0.588$. It is unsatisfactory from the point of view of "small" perturbation on the system parameter but its sign shows the necessity of lessening of parameter β under correction.

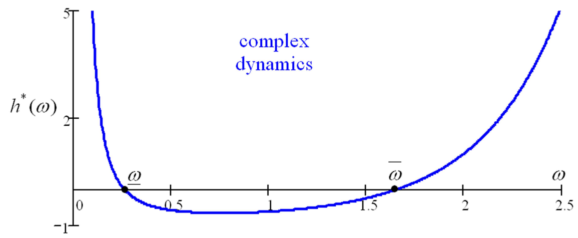

Figure 1. The regular and complex dynamics boundary at optimal static correction of the system (3) on parameter β.

3.2 Dynamic correction with the known perturbation structure

Here we continue the studying of correction (3) on a single parameter: $\beta \mapsto \hat{\beta}(t) = \beta(1+h(t))$, though now corrective perturbation is time-changed. The structure of the constrained ($|h(t)| \le a$) corrective perturbation is known and represents a periodic time function $h(t) = a\cos(\Omega t)$, $a > 0$, Ω is frequency.

Having changed $\beta \mapsto \hat{\beta}(t)$ in (3) and calculated Melnikov function we get [Chacon, 2002]:

$$\hat{\Delta}(\theta) = \Delta_0(\theta) - aB\sin(\Omega\theta), \quad (9)$$

$$B = (6\beta)^{-1}\pi\Omega^2(\Omega^2+4)\csc h(\pi\Omega/(2\alpha^{1/2})).$$

May (9) be included in the condition, which determines the structure of set $K_M = \{a > 0 \mid sign(\hat{\Delta}(\theta)) = const\}$ in (2). Then we have the following problem

$$\max_{t \ge 0} |h(t)| \to \min_{a \in K_M}. \quad (10)$$

For the known corrective perturbation structure $h(t) = a\cos(\Omega t)$ the problem (10) is reduced to minimization of the parametric perturbation amplitude a on the set of values providing system dynamics stability.

The solution of (10) gives the analysis of the function (9). At resonance frequency correlation $\Omega = z\omega$, z – an integer, the condition of sign constancy of (9) expresses the inequality $(A - C) \le aB$. It means the value of the minimal amplitude $a_{\min} = \arg \min_{a \in K_M} \max_{t \ge 0} |h(t)|$ which provides optimal chaos suppression lies on the "chaos $\leftrightarrow$ order" boundary, that is $a_{\min} = (A-C)/B$. Thus the solution of (10) are the conditions: $h^*(t) = a_{\min}\cos(\Omega t)$, $a \ge a_{\min}$, $\Omega = z\omega$. The dynamics is chaotic at $0 < a < a_{\min}$.

However the estimation of $a_{\min}$ that we got is "local" as it is true only for fixed values of $\alpha, \beta, \gamma, f, \omega$ and z. For example, for the frequency $\omega = 1.1$ the calculation at $z = 1$ gives $a_{\min} = 0.095$, but at $z = 2$ we shall get $a_{\min} = 0.081$. In Fig. 2 we can see the general dependence $a_{\min}(\omega)$ at different z in the resonance correlation $\Omega = z\omega$ (we take into account resonances $z = 1, 2, 3, 4$). Values $\underline{\omega} = 0.2645, \overline{\varpi} = 1.6608$ are the roots of the equation $a_{\min}(\omega) = 0$. They correspond precisely to the values $\underline{\omega}, \overline{\varpi}$, got while solving optimal static correction problem in Ch. 3.1 at $h^*(\omega) = 0$.

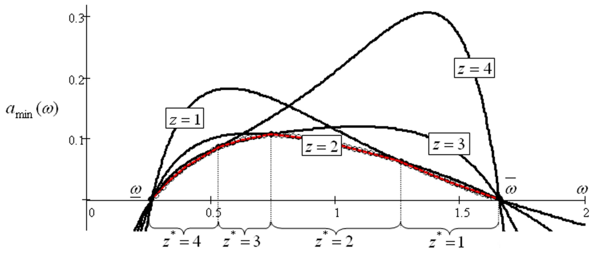

Figure 2. Optimal "chaos order" boundary, formed by the effective resonance correlation in the task (10) under the correction of the system (3) on parameter β.

The study of the frequency interval $(\underline{\omega}, \overline{\omega})$ and the values $a_{\min}(\omega) > 0$ (at different z) correspondent to it allows deepening our understanding of the peculiarities of chaotic dynamics correction by the function $h^*(t) = a_{\min}\cos(\Omega\, t)$. It is obvious from Fig. 2 that in reality $a_{\min}(\omega)$ can be achieved only at some z and for a certain frequency interval $w_i = (\omega_i, \omega_{i+1}) \in (\underline{\omega}, \overline{\omega})$, $i = 5 - z$, where $\underline{\omega} = \omega_1 < \omega_2 < ... < \omega_5 = \overline{\omega}$, $\cup_{i=1}^{4} w_i = (\underline{\omega}, \overline{\omega})$ (in Fig.2 $\omega_2 = 0.531$, $\omega_3 = 0.746$, $\omega_4 = 1.265$). For the function $h^*(t) = a_{\min}\cos(\Omega t)$ every interval w_i unambiguously defines the effective resonance correlation $\Omega_z = z^*\omega$, where $z^* \in \{1, 2, 3, 4\}$, $\omega \in w_{5-z^*}$, such that at every w_i there exists the value z^* for which $a^*_{\min}(\omega) = a_{\min}(\omega, z^*) \leq a_{\min}(\omega, z)$ is performed. Thus all the points $(\omega, a^*_{\min}(\omega))$, $\omega \in (\underline{\omega}, \overline{\omega})$ in Fig.2 make a curve which is the *optimal "chaos↔order" boundary*. As a result, if we know the interval w_i the given value ω belongs to, we can find z^* and the optimal amplitude $a_{\min}(\omega, z^*)$ of the chaos-suppressing corrective perturbation $h^*(t) = a_{\min}(\omega, z^*)\cos(z^*\omega\, t)$ and gain a more effective solution of the problem (10).

4 Optimal multiparametrical correction technique

Let us consider a more complicated variant of correction (problem (2)) when along with finding the correction radius $a_{\min}$ we should find the structure of the optimal corrective function $h^0(t) \in U$. The application of Melnikov criterion becomes a problem as the parametric perturbation structure is unknown. Thus, in this case the local instability of the system trajectories is used as a chaos criterion.

4.1 Problem statement

Consider the time interval $[0, T]$ where T is time, on which the solution of the system $\dot{x} = F(x, p, t)$ is determined. If the choice of T is made under the condition $T >> T^*$ (that is the moment is deliberately taken longer that the transient process $[0, T^*]$), it will be possible to study the general problem (2) on the finite interval $[0, T]$.

Let $x(t)$ and $\bar{x}(t) = x(t) + \tilde{x}(t)$ be two trajectories of the corrected system $\dot{x} = F(x, \hat{p}, t)$ with initial conditions $x(0) = x_0$ and $\bar{x}_0 = x_0 + \tilde{x}_0$ ($||\tilde{x}_0|| < \varepsilon$, ε – small enough) belonging to the attraction basin B_A of the attractorA_P. Attractor A_P is chaotic at $h \equiv 0$ and any trajectory beginning on it is unstable according to Lyapunov. It means that the upper Lyapunov exponent is positive: $\Lambda_1 = \max_{i=\overline{1,n}}\{\overline{\lim_{t\to T}}\,(t)^{-1}\ln||\tilde{x}_i(t)||\} > 0$.

Then in the problem (2) the correction radius value a in the constraints $|h_j(t)| \leq a$, $j = \overline{1, r}$, determines the evolution of small perturbation $\tilde{x}(t)$ along the corrected trajectory $x(t)$, chosen for the stability character analysis. In this case Λ_1 is the function $\Lambda_1 = \Lambda_1(a)$. This allows determining the set K in the form $K_\Lambda = \{a > 0 \mid \Lambda_1(a) < 0\}$.

Let us introduce an additional optimality criterion

$$J_2(h, a) = \int_0^T \sum\nolimits_{j=1}^{r} h_j^2(t)dt \to \min_{h\in U}. \qquad (11)$$

If a is fixed the condition (11) requires the desired corrective function h^0 to provide a minimum consumption of energy on a correction process. By strengthening (2) with (11), we have the *optimal multiparametrical correction* problem:

$$J_1(h^0, a) = \max_{0\leq t\leq T}\max_{1\leq j\leq r}|h_j^0(t)| \to \min_{a\in K_\Lambda,\ h^0\in U^0}, \qquad (12)$$

where the minimum is sought for admissible values a on the set K_Λ providing the corrected system stability on $[0, T]$ and for the function $h^0 \in U^0 = Arg\min_{h\in U} J_2(h, a)$. The optimal radius of correction $a_{\min}$ found for the components $h_j^0(t)$, $j = \overline{1, r}$, provides the transition of $h^0(\cdot) \mapsto h^*(\cdot)$ from the corrective function with the structure h^0 to the optimal function h^*. Thus the solution of (12) is the pair $(a_{\min}, h^*(t))$ which provides the stable regime of the corrected system determined by the $|h_j^*(t)| \leq a_{\min}$.

4.2 Two-stage correction scheme

The order of optimality criteria in (12) allows solving it in two steps through the following optimal multiparametrical correction scheme

$$J_2(h(\cdot), a) \xrightarrow{I} \min_{h(\cdot)\in U} \Rightarrow J_2(h^0(\cdot), a) \to$$
$$J_1(h^0(\cdot), a) \xrightarrow{II} \min_{a\in K_\Lambda, h^0\in U^0} \Rightarrow J_1(h^*(\cdot), a_{\min}). \qquad (13)$$

Realization of the scheme (13) presupposes the combination of optimal control theory methods with computer simulations of the system dynamic behaviour.

At the *stage I*it is necessary to define the dynamic features of the desired correction function $h^0(t)$ by solving the problem $J_2(h^0(\cdot), a) = \min_{h(\cdot)\in U} J_2(h(\cdot), a)$. For the *fixed a* on the basis of the maximum principle [Pontryagin, Boltyanski, Gamkrelidze and Mischenko, 1962] we seek for the structure of $h^0(t)$, $t \in [0, T]$, that transfers the corrected system with the initial condition $x_0 \in B_A$ to the set $M_E = \{(x, h)|\ f(x, \hat{p}) = 0\}$ at the interval $[0, T]$ with the condition (11). Dynamics peculiarities on the set M_E at $h \equiv 0$ coinciding with E are shown in [Talagaev and Tarakanov, 2008].

Introduce Hamilton-Pontryagin function

$$H(x, h, \psi, t) = \psi^T F(x, \hat{p}, t) - ||h||^2/2.$$

For the process

$$C^0 = \{x^0(t), (h^0(t) = h(x^0(t), \psi(t)), a), t \in [0, T]\}$$

where $x^0(t)$ is corresponding solution of (1) under $h^0(t)$, with the boundary conditions $x_0 \in B_A, x^0(T) \in M_E$ to be optimal it is necessary [Pontryagin, Boltyanski, Gamkrelidze and Mischenko, 1962]:

1: there exists a non-zero $\psi(t) \in R^n$ satisfying the equation $\dot{\psi}(t) = -(\partial/\partial x)H(x^0(t), h^0(t), \psi(t), t)$;

2: the function $h^0(t)$ satisfies the maximum condition

$$H(x^0(t), h^0(t), \psi(t), t) = \max_{h \in U} H(x^0(t), h, \psi(t), t); \tag{14}$$

3: the transversality conditions $\psi_0 \perp \Omega(x_0)$ and $\psi(T) \perp \Omega(x^0(T))$ are performed.

The condition (14) allows finding the optimal structure of the vector-function $h^0(t) = (h_1^0(t), ..., h_r^0(t))$ in the form

$$h_j^0(t) = sat(\tilde{h}_j(t)) = \begin{cases} \tilde{h}_j(t), & |\tilde{h}_j(t)| \le a, \\ a \cdot sign(\tilde{h}_j(t)), & |\tilde{h}_j(t)| > a, \end{cases} \tag{15}$$

where $\tilde{h}_j(t) = \psi^T (\partial/\partial h_j) F(x(t), \hat{p}(t), t)$ is the solution of equation $(\partial/\partial h_j) H(x(t), h, \psi(t), t) = 0$. As the components $h_j^0(t)$ are independent from each other the minimization of a at simultaneous correction on several parameters is possible.

The process C^0 defined by the maximum principle is found from the $2n$-system with the help of (15)

$$\begin{cases} \dot{x} = (\partial/\partial \psi) H(x, h, \psi, t), \\ \dot{\psi} = -(\partial/\partial x) H(x, h, \psi, t), \end{cases} \tag{16}$$

with orthogonal initial conditions

$$x_0 \in B_A,\ \psi_0 = \left\{\psi \in R^n \mid \sum_{i=1}^n \psi_i(0) x_i(0) = 0\right\}. \tag{17}$$

Dynamic features of the class of corrective functions (15) were discussed in [Talagaev and Tarakanov, 2008; Talagaev and Tarakanov, 2007]. It was shown that the process C^0 provides attractiveness of the corrected trajectories $x^0(t)$ to the set M_E. It is significant that C^0 preserves the conditions of the system attractor existence (localized on M_E) with its stability dependent from the value a.

At *stage II* the problem is being solved

$$J_1(h^*(\cdot), a_{\min}) = \min_{a \in K_\Lambda, h^0 \in U^0} J_1(h^0(\cdot), a).$$

The minimization of correction radius a is performed through numerical testing of chaos suppression quality:

step 1: two close initial conditions $x_0^0 = (x_{01}^0, x_{02}^0)^T$, $\bar{x}_0^0 = (\bar{x}_{01}^0 + \varepsilon, \bar{x}_{02}^0 + \varepsilon,)^T$, $\varepsilon << 1$, are specified and we calculate the corresponding ψ_0, $\bar{\psi}_0$ according to (17);

step 2: for x_0^0, ψ_0 and $\bar{x}_0^0$, $\bar{\psi}_0$ on the given interval $[0, T]$, the system (16) is integrated with (15) by the 4-order Runge-Kutta method. A comparison of the processes C^0 and $\bar{C}^0$ shows the small perturbation evolution $\tilde{x}^0(t) = \bar{x}^0(t) - x^0(t)$ along the trajectory$x^0(t)$, necessary for defining the corrected system stability character;

step 3: the dynamics of the value $\Lambda_1(t, a_0) = T^{-1} \ln(\varepsilon^{-1} ||\bar{x}^0(t) - x^0(t)||)$ at $[0, T]$ is tracked for the chosen value $a_0 < 1$. If $\Lambda_1(t, a_0) < 0$ for all $t \to T$, the calculations Λ_1 will be repeated for the sequence $a_l < ... < a_1 < a_0$ until $a_{\min} = \min_{a \in K_\Lambda} \{a_m\}_{m=\overline{0,l}}$ is found.

This procedure does not need linearization and uses an approximate method of calculation of the upper Lyapunov exponent described in [Andrievskii and Fradkov, 2003]. The found optimal process $C^* = \{x^*(t), (h^*(t), a_{\min}), t \in [0, T]\}$ provides an optimal dynamic modification of the chaotic attractor into the unique invariant set correspondent to the stable regime of the corrected system.

4.3 Correction scheme realization and comparison of the results

In the general case in (3) α and β are correctable:

$$\dot{x}_1 = x_2,\ \dot{x}_2 = \hat{\alpha}(t) x_1 - \hat{\beta}(t)\, x_1^3 - \delta\, x_2 + f \cos(\omega\ t), \tag{18}$$

where $\hat{\alpha}(t) = \alpha(1 + h_1(t))$, $\hat{\beta}(t) = \beta(1 + h_2(t))$. Then

$$H = \psi_1 x_2 + \psi_2(\hat{\alpha} x_1 - \hat{\beta}\, x_1^3 - \delta\, x_2 + f \cos(\omega\ t)) - 0.5(h_1^2 + h_2^2).$$

From the optimality conditions of the stage I of the scheme (13), we get a system of conjugate variables

$$\dot{\psi}_1 = \psi_2(3\hat{\beta}(t) x_1^2 - \hat{\alpha}(t)), \quad \dot{\psi}_2 = -\psi_1 + \delta \psi_2, \tag{19}$$

with the initial condition $\psi_0 = (-x_{02}, x_{01})^T$ taken for $x_0 = (x_{01}, x_{02})^T$ from (17) and the components of the optimal corrective function

$$h_j^0(t) = sat(\tilde{h}_j(t)),\ \tilde{h}_1(t) = \alpha\, x_1 \psi_2,\ \tilde{h}_2(t) = -\beta\, x_1^3 \psi_2. \tag{20}$$

The dynamics of the corrected system (18) subject to the condition $|h_j^*(t)| \le a_{\min} = 0.05$, $j = \overline{1, 2}$, is shown in the Fig.3,4. In course of correction radius minimization the system (18)-(19) was integrated with (20) at $x_0 = (0.1, 0.1)^T$, $\varepsilon = 3 \cdot 10^{-5}$, $t \in [0, 1000]$ and integration step $d = 0.01$. For contrast in Fig.3,4 we show the solution of the problem (10) achieved in course of correction β by the function $h^*(t) = a_{\min}(\omega, z^*) \cos((\omega\, z^*) t)$. Its minimal amplitude value $a_{\min}(\omega, z^*) = 0.081$ placed on the optimal

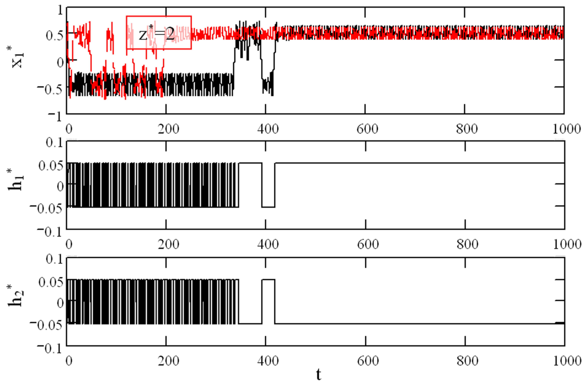

Figure 3. The solution of the problem (12): optimal dynamics of the corrected system (18) with $a_{\min}=0.05$.

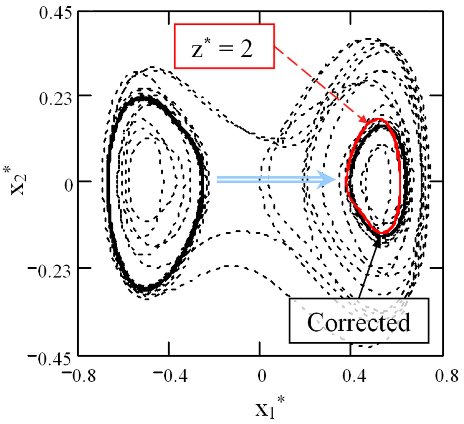

Figure 4. The solution of the problem (12): trajectories of the corrected system (18) in the phase space with $a_{\min}=0.05$.

chaos boundary (Fig.2) is achieved for the resonance $z^*=2$ at $\omega=1.1$.

The comparison of problem (12) with (5) and (10) gives us interesting information. The first feature is the *saturation effect* of the optimal functions $h_1^*(t)$, $h_2^*(t)$ on the upper/lower boundary edges (see Fig.3), arising after the optimal transient process termination and complete stabilization of the system. The saturation character is independent from the number of parameters under correction. For instance, if in problem (12) the correction of the system is done at β only ($h_1^*(t)\equiv 0$), the stable regime will occur at the greater value $a_{\min}=0.152$, but $h_2^*(t)$ will still be saturated on the lower boundary edge. This fact agrees with the solution of problem (5) where the corrective amendment to parameter β for perturbation frequency $\omega=1.1$ is $h^*=-0.588$. Its sign corresponds to the function $h_2^*(t)$ saturation on the lower boundary edge.

The second peculiarity is the *localization* of the unique orbit of the system (18). Its stabilization is caused by optimal correction of the parameters (see. Fig.4). The comparison of Fig.4 with the dynamics in Fig.3 shows that the most of the time $[0,T^*]$ the transient process looks like a regular trajectory movement along the orbit localized in the vicinity of the equilibrium state $(-\sqrt{\alpha/\beta},0)=(-0.5,0)$ (multistability may cause similar movement in the vicinity of $(\sqrt{\alpha/\beta},0)$). However in the end of the transient process the stabilized regime looses its stability. The system finds a new type of motion stable at $T>T^*$ (periodic oscillations) along the closed orbit, which includes the point$(0.5,0)$. In the Fig.4 it is shown that the localization of the optimal stable orbit almost precisely coincides to the solution of (10) for $z^*=2$.

5 Conclusion

In the paper we present the mechanism of multiparametrical analysis of the problem of optimal transition from chaotic dynamics to the regular one in nonautonomous systems. The two-stage optimal multiparametrical correction scheme constitutes the framework of the analysis as the most general and flexible variant of general correction problem solution. We compare its results to analytical estimations which represent particular cases of solution of the general problem with Melnikov criterion. The offered correction technique applicable to a wide class of chaotic systems, allows comparing the efficiency of different forms of parametric influence and finding the most appropriate ways of influence on the parameters which provides optimal chaos suppression.

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